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AN APPROACH TO SOLVE MOTORAIL TRANSPORTATION LOADING LAYOUT OPTIMIZATION PROBLEM: THE CHINA CASE

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Abstract. In this paper, we formulate a cost and safety oriented approach to optimize the motorail transport car loading layout optimization problem. The model has 2 objective functions, one maximizes the wagon loading weight utilization rate, and another minimizes the gravitational center height after loading the cars on the wagons. The calculation results show that: the heaviest cars should be loaded in the middle of the decks, the cars weight loaded on both sides should be gradually reduced; the total weight of cars loaded on lower deck should be larger than the total weight of cars loaded on the upper deck.

Keywords: motorail transportation, railway, loading layout optimization problem, loading weight utilization ratio, gravitational center height, cost and safety oriented.

Notations

ACT – auto-carrier transportation;
CLP – container loading problem;
DB – *Deutsche Bahn* (in German);
DSS – decision support system;
LP – linear programming;
MTP – motorail transportation problem;
SUV – sport utility vehicle.

Introduction

Motorail belongs to a new transportation model, which provides long distance transportation service for passengers and their cars on one train. Usually, passengers are accommodated in special sleeping wagons, while their vehicles are loaded on special car transportation wagons, the sleeping wagons and the car transportation wagons form the motorail train. The cars are loaded on the wagons under several constraints, e.g., the weight limitation, the height limitation, the width limitation, etc. (Table 1). Due to the long operation distance, most of the operation lines are carried out during night time, which provides a sleeping arrival at the destination for passengers (Lutter, Werners 2014, 2015; Lutter 2016). At the beginning of the 1950s, motorail transportation was popular around the world, this kind of transportation services can be found in 13 European countries, e.g., Germany, France, Italy, Austria, Czech Republic, Slovakia, Australia, Chile as well as the US, and offered by national companies and private rail transportation companies.

In order to satisfy passengers' long-distance self-driving travel in China, Beijing Railway Bureau began to operate the self-driving traveling automobiles service in 2014. After that, the other railway bureaus such as Shanghai, Zhengzhou, Chengdu, Lanzhou, etc., also began to operate this new service. Different from the loading constraints of motorail, the cars loaded on the special wagons only under 2 constraints, length and height limitations (Table 1). Furthermore, this kind of service accommodates passengers in high-speed trains, while their cars are loaded on special car transportation trains. The departure and arrival time for passengers and their cars are quite different (usually the cars depart and arrive early). Therefore, it is necessary to provide motorail service in China (Huang, Shuai 2015).

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For each particular vehicle, the researchers try to find out which wagon, loading deck and position are suitable to load the vehicle, and solving the motorail capacity problem (Lutter, Werners 2014, 2015; Lutter 2016). The loaded vehicles include general cars for family as well as large groups of motorcycles, which are transported to their destination via motorail services, especially during summer holiday. Therefore, the models applied in the 3 papers are suitable to solve the motorcycles and cars mixed loading problem. In China, people will choose cars, especially the SUVs, not the motorcycles to reach their destinations. Furthermore, as continuously increasing vehicle lengths and widths, growing requests for equipment, the present capacities of car transportation wagons become a key issue. In this paper, we focus on formulating a suitable motorail transportation car loading layout optimization approach in China.

The remainder of this paper is organized as follows. In section 1 we give brief previous theoretical researches review about MTP, ACT problem, containers loading onto freight trains problems, as well as the cargo layout and packing problem in a container or a railway wagon. In section 2 we give a detailed introduction about the motorail transportation car loading layout optimization model. The next section 3 is devoted to algorithm designing to solve the model. Then section 4 is applied to the description and results discussion of the case study. The final section presents the major conclusions and the further tasks need to be completed, followed by recommendations.

1. Previous theoretical researches

In this section, we present a theoretical researches review of the railway transportation loading problem literatures, which provides a background and establishes a framework for this paper. We also point out the contributions of the research presented in this paper. 1st, we will introduce the 3 papers devoted to MTP. Then, we will introduce ACT problem as well as the containers loading onto freight trains problems. Next, the cargo layout and packing problem in a container or a railway wagon related papers will be introduced. After that, we will introduce the contributions and objectives about the approach applied in this paper.

An integer linear optimization model is established to load vehicles (cars, as well as motorcycles) to transportation wagons under realistic technical and legal constraints, which is called MTP (Lutter, Werners 2014, 2015). The authors try to solve the MTP during the motorail tickets booking process as well as during the vehicles loading process at motorail terminals. The model is designed to solve the order acceptance decisions when weights of the vehicles are known within an interval. The optimization results calculated by the commercial solver – FICO® Xpress Solver (*https://www.fico.com/en/products/fico-xpress-solver*) show that solving the MTP is far from being trivial, if we want to apply the approach to the real-world large optimality gaps or running times, the model need to be improved. In order to improve the optimization results, Lutter (2016) proposes and evaluates 2 different MTP integer LP formulations. One uses less variables, provides a tighter LP relaxation, belongs to a simplified original problem. The other uses branch-and-price, which is formulated as a column-generation approach. The author compares and evaluates the solution quality and speed of the 2 approaches based on the real-world data sets.

Except for the 3 literatures mentioned above, to the best of our knowledge, no other direct literatures on motorail optimization exist. As one of the problems, which is related to MTP, ACT and CLP focuses on dealing with simultaneous loading and auto-carriers routing problem, which is formulated as the real-world problem on delivering cars to car dealers on demand. ACT deals with the vehicles' selection and allocation parking positions planning on car transporters, and the routing decisions affect the auto-carriers' itinerary planning. Usually, ACT problems have the goals to minimize the total transportation costs, and the penalty costs when cars are not delivered by due date. The ACT was introduced by Agbegha *et al.* (1998) for the 1st time, and then further developed by Tadei *et al.* (2002). Both the above 2 papers consider simplified loading constraints, but differently, Agbegha *et al.* (1998) focus on the vehicles' loading sequence planning with minimum associated costs, in order to solve the ACT, a geometric assignment problem is formulated to allocate vehicles to parking positions. Tadei *et al.* (2002) propose a decomposition of routing and load planning decisions, including 3 sub-problems: (1) considering the auto-carrier loading

Table 1. Cars loading constraints of DB motorail and self-driving traveling trains in China (Huang, Shuai 2015)

	Cars loading constraints of DB motorail						
	Maximum car height [m]			Maximum car length [m]		Maximum car width [m]	Maximum seats number
	TW < 1.35	TW 1.35…1.55	TW > 1.55	Ordinary car	Trailer		
FW	2.05	1.96	–	5.30	10.30	2.05	9
CW	1.95	1.95	–	5.30	–	2.05	9
Cars loading constraints of self-driving automobiles transport in China							
Small car				Large car			
length ≤ 4.9 m; height ≤ 1.7 m				length ≤ 5.5 m; height ≤ 1.9 m			

Notes: FW – flatcar wagon; CW – caravan wagon; TW – top width of the car.

equipment and computing each auto-carrier's length; (2) considering the vehicle shape and computing each vehicle's length; (3) imposing the total equivalent vehicle lengths to be no larger than the equivalent auto-carrier length and modelling the loading as a single capacity constraint. The ACT is treated and solved as integer programs by using a standard commercial solver. In a recent publication, as far as we know, Dell'Amico *et al.* (2015) give us the ACT that incorporates the loading constraints more detailed than the previous researches, which is commonly tackled heuristically because of its computational complexity, an iterated local search algorithm is designed in order to solve the problem. A rolling horizon algorithm is introduced by Cordeau *et al.* (2015) to plan the transport of vehicles to automotive dealers. The model is established by minimizing the total travel distances, auto-carrier operation's fixed costs, service costs, as well as the penalties for late deliveries. In order to solve the model, a branch-and-bound search is applied to check the loading feasibility, the complete algorithm is applied repeatedly in a rolling horizon framework in order to deal with the problem's dynamic nature.

Another related problem focuses on deals with the containers' loading problem onto freight trains; usually we called CLPs, which are characterized by a detailed treatment on physical loading requirements. Just like the MTP, time tabling planning and routing decisions problem are not part of the optimization problem. Most of the research papers on CLP focus on providing real-world decision support at container terminals, but some of the containers are stored at container terminals before the loading process, other containers arrive just-in-time, so the loading plans need to be revised during the loading process. Furthermore, when focusing on real-world decision support, the run times are very important, which is also suitable for optimization process at motorail terminals. Despite these commonalities, MTP is distinctively different from CLP regarding physical loading constraints, as well as the handling of transportation goods. Feo, González-Velarde (1995) developed an integer programming model for the CLP by considering physical loading constraints and associated wagon configurations for the 1st time. After that, Corry, Kozan (2006, 2008) introduce an integer programming formulation for CLP. In the 2 papers, a dynamic load planning model is proposed to optimize the CLP at intermodal terminals. A local search combined with simulated annealing heuristics algorithm is proposed in order to solve the problem. Bruns, Knust (2012) propose 3 different integer LP models for the CLP, which considering the length and weight constraints. Bruns *et al.* (2014) establish their CLP model by considering container weight, container overhang and wagon failure, a commercial solver is applied to solve the model formulations. In a recent publication, as far as we know, Anghinolfi *et al.* (2014) propose a mathematical model for the CLP with an innovative transfer system for loading and unloading containers. A greedy randomized adaptive search heuristic algorithm is developed in order to solve the problem, the algorithm is able to generate good quality solutions in short time.

More meticulously and detail, some papers have also been devoted to the cargo layout and packing problem in a container or a railway wagon. When dealing with these problems, researchers usually idealized the loaded cargo, containers and wagons as cuboids with different sizes, and establish one-dimensional (e.g., Mathur 1998), 2-dimensional (e.g., Pisinger, Sigurd 2005), 3-dimensional (e.g., Bortfeldt, Mack 2007) or 2-dimension's orthogonal (e.g., Clautiaux *et al.* 2008) encasement equilibrium model to solve the packing problems. The objective functions of these models focus on minimizing loading costs, or maximizing the utilization of the container space, or both. And generally design heuristic algorithm to solve the models. Gai (2000) establishes an optimized model to solve the loading problem of several uneven-weight goods on a wagon, the model focuses on minimizing the total loaded cargo's gravitational center in the wagon. Lei *et al.* (2011) establish a loading and packing optimized model to load multiple goods on a single vehicle, which focuses on maximizing loading capacity and the integrated utilization rate of the wagon. In order to solve the same model, Guo *et al.* (2011) formulate a 1st fit based partition algorithm, as well as the ratio of weight and volume based algorithm to solve the model; Zhu *et al.* (2013) design heuristic information based greedy construction algorithm to solve the model, propose loading sequence update based and goods layout location adjustment based local optimization scheme, and improve the optimization scheme by establishing new tower-sets to realize global search.

Researches above mainly aimed at cars and cargos loading problem, usually have the target of maximizing train carrying capacity utilization ratio, maximizing wagon volume utilization ratio, minimizing transport costs as well as the transportation time. Actually, motorail trains carry diverse passenger vehicles, with different classification, size, weight and transhipment condition, which tremendously increase the difficulty during the loading process. Meanwhile, owners need to buy transportation insurance for their cars, safety incidents in transportation process would be extremely unfavourable. In consequence, it is necessary to research car loading layout optimization to reduce wagon application number, decrease transportation costs and ensure transportation safety. These goals mentioned above are the optimization objective in the following model and algorithm, we try to formulate a cost and safety oriented approach to optimize the motorail transport car loading layout optimization problem.

2. Model formulation on motorail transportation loading layout problem

Before introducing our optimization model on motorail transportation loading layout optimization problem, the important assumptions, parameters, sets and decision variables, cars loading illustration of both decks in a wagon, as well as the inputs in this paper are explained below. The assumptions include:

- the cars are loaded on the motorail trains at the departure terminals, transported to the arrival terminals without additional stops during the whole process, so there is no additional cars loading and unloading operations along the operation lines;
- in order to prevent the occurrence of security incidents during the transportation processes, e.g., cars fuel leakage (the tanks may be full of gas), body collision, etc., we only adopt horizontal loading mode. Tilted loading mode, climbing loading mode as well as the straddle loading mode are not allowed;
- in order to deal with the loading related data conveniently, we could approximately treat the cars as cuboids with specific length, width and height. Treat the car's geometric center as its gravitational center. Furthermore, we also simplify the wagon as a cuboid with fixed length, width and height. The MTP in this paper can be regarded as loading the small cuboids into larger cuboids;
- to ensure the transportation safety, only single-row loading mode is taken into consideration on every deck for each kind of wagons. Generally, there are 2 loading decks on a wagon, the cars can be loaded on each deck, possibly.

All parameters, sets as well as the decision variables used in optimization model are presented in the following Table 2.

In order to formulate our optimization model conveniently, we now introduce cars loading illustration of both decks in a wagon by establishing a space Cartesian coordinate system, shows in Figure 1. The floor of the lower deck will be selected as the X–Y plane, X axis is the train operation direction. X axis measures the length of the cars and wagons, Y axis measures the width of the cars and the wagons, Z axis measures the height of the cars and the decks of the wagons. Figure 1 shows us both lower and upper loading decks of a transportation wagon, the minimum safety length gap, width gap and height gap between the loaded cars and the wagon body, as well as the detail information about the loaded cars.

The inputs in this model include 3 sets: *set* 1 presents the information of cars need to be loaded, e.g., length, width, height and weight, shows in Equation (1); *set* 2 presents the information of all the possible wagons, including

Table 2. Parameters, sets and decision variables

Index sets	
$I=\{1,\ldots,i,\ldots,\bar{I}\}$	set of cars, each car is indexed by i
$K=\{1,\ldots,k,\ldots,\bar{K}\}$	set of wagon types, each kind of wagon is indexed by k
$E=\{0,1\}$	set of loading decks of wagon, indexed by e, upper loading deck (e = 1) and lower loading deck (e = 0)
Parameters	
l_i	length of car i
w_i	width of car i
h_i	height of car i
q_i	weight of car i
n_e^k	the e kind of loading decks of the selected k kind of loading wagon
L_k	length of the k kind of wagon
W_k	width of the k kind of wagon
H_k	height of the k kind of wagon
Q_k	loading weight of the k kind of wagon
S_k	wagon weight of the k kind of wagon
Δx	deviation between X axis and the average abscissa about the total loaded cars
Δy	deviation between Y axis and the average ordinate about the total loaded cars
Δz	deviation between Z axis and the average height about the total loaded cars
Δl	minimum safety length gap between 2 loaded cars
Δw	minimum safety width gap between loaded cars and wagon body
Δh	minimum safety height gap between loaded cars and wagon roof body
$g_{i\to n_e^k}$	center of gravity coordinates of car i loading on the n_e^k, $g_{i\to n_e^k}=\left(x_{i\to n_e^k}, y_{i\to n_e^k}, z_{i\to n_e^k}\right)$
G_k	center of gravity coordinates of the wagon k, $G_k=\left(x_k, y_k, z_k\right)$
Decision variables	
$\alpha_{i\to n_e^k}$	indicates if car i loads on the n_e^k, then $\alpha_{i\to n_e^k}=1$; otherwise $\alpha_{i\to n_e^k}=0$

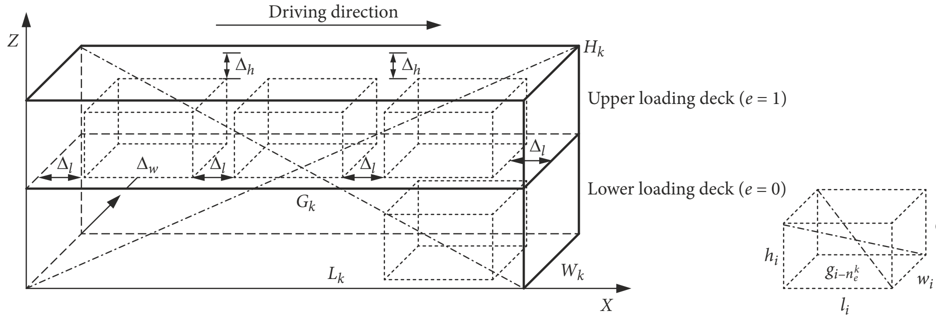

Figure 1. Loading illustration of both decks in a wagon

length, width, height, loading weight, wagon weight, center of gravity coordinates, shows in Equation (2); *set* 3 presents other limitation conditions, shows in Equation (3):

$$set\ 1 = \{l_i, w_i, h_i, q_i\},\ \forall i; \quad (1)$$

$$set\ 2 = \{L_k, W_k, H_k, Q_k, S_k, x_k, y_k, z_k\},\ \forall k; \quad (2)$$

$$set\ 3 = \{\Delta x, \Delta y, \Delta z, \Delta l, \Delta w, \Delta h\}. \quad (3)$$

Then the motorail transportation car loading layout optimization problem optimization model can be formulated as follows:

$$\max Z = \sum_k \left(\sum_e \sum_i \left(\frac{\alpha_{i \to n_e^k} \cdot q_i}{Q_k} \right) \right); \quad (4)$$

$$\min H = \frac{\sum_k \left(S_k \cdot z_k + \sum_e \sum_i \left(\alpha_{i \to n_e^k} \cdot q_i \cdot z_{i \to n_e^k} \right) \right)}{\sum_k \left(S_k + \sum_e \sum_i \left(\alpha_{i \to n_e^k} \cdot q_i \right) \right)} \quad (5)$$

subject to:

$$\left| \frac{\sum_e \sum_i \left(\alpha_{i \to n_e^k} \cdot q_i \cdot x_{i \to n_e^k} \right)}{\sum_k \sum_e \left(\alpha_{i \to n_e^k} \cdot q_i \right)} - x_k \right| \le \Delta x,\ \forall k; \quad (6)$$

$$\left| \frac{\sum_e \sum_i \left(\alpha_{i \to n_e^k} \cdot q_i \cdot y_{i \to n_e^k} \right)}{\sum_k \sum_e \left(\alpha_{i \to n_e^k} \cdot q_i \right)} \right| \le \Delta y,\ \forall k; \quad (7)$$

$$\left| \frac{\sum_e \sum_i \left(\alpha_{i \to n_e^k} \cdot q_i \cdot z_{i \to n_e^k} \right)}{\sum_k \sum_e \left(\alpha_{i \to n_e^k} \cdot q_i \right)} - z_{i \to n_e^k} \right| \le \Delta z,\ \forall k; \quad (8)$$

$$\sum_i \left(\alpha_{i \to n_e^k} \cdot l_i \right) + \Delta l \cdot \left(\sum_i \alpha_{i \to n_e^k} + 1 \right) \le L_k,\ \forall n_e^k; \quad (9)$$

$$W_k - \alpha_{i \to n_e^k} \cdot w_i \ge 2 \cdot \Delta w,\ \forall n_e^k, \forall i; \quad (10)$$

$$H_k - \sum_i \sum_e \left(\alpha_{i \to n_e^k} \cdot h_i \right) \ge 2 \cdot \Delta h,\ \forall n_e^k; \quad (11)$$

$$\sum_i \sum_e \left(\alpha_{i \to n_e^k} \cdot q_i \right) \le Q_k,\ \forall k. \quad (12)$$

The cost-oriented objective function (Equation (4)) of the model maximizes the wagon loading weight utilization rate, and denominator of objective (Equation (4)) shows we need minimum transportation wagons, this is critical to reduce the operation costs during the whole transport process. Another safety-oriented objective function (Equation (5)) in the optimization model minimizes the vehicle gravitational center height after loading the cars on the wagons. The total gravitational center height should be as small as possible in order to ensure the transport safety during the transportation process. There are 3 groups of constraints: Group 1 includes Equations (6)–(8), require the total gravitational center about the loaded cars should be coincident with the gravitational center of the wagons as much as possible, which means, the deviation between X, Y, Z axis and the gravitational center of total loaded cars should be limited; Group 2 includes Equations (9)–(11), indicates that length, width and height of loaded cars should meet the needs of the limitation of the wagons. Constraint (Equation (9)) shows that, if there are N cars loaded on one deck of any wagon, there are $N + 1$ safety length gaps Δl in total, so the summation of total safety length gaps and the total length of total loaded cars should not exceed the length of the deck. Constraint (Equation (10)) shows that, the total safety width between loaded cars and wagon body after loading cars should not exceed the double minimum safety width gaps. Constraint (Equation (11)) indicates that summation of safety height between loaded cars and wagon body on 2 decks should not exceed the double minimum safety width gaps limitation; Group 3 (Equation (12)) is loading weight limitation, which requires the total loaded cars' weight should not exceed the wagon's rated loading weight.

3. Algorithm for motorail transportation loading layout optimization model

Algorithm input data: *set* 1, *set* 2 and *set* 3. The outputs include: loading position coordinates of each car, wagon type and amount, values of objective function (Equations (4) and (5)), as well as the loaded cars' lateral distribution location X_k on each wagon deck:

$$X_k = \frac{\sum_i \sum_e \left(\alpha_{i \to n_e^k} \cdot q_i \cdot x_{i \to n_e^k} \right)}{\sum_i \sum_e \left(\alpha_{i \to n_e^k} \cdot q_i \right)},\ \forall k. \quad (13)$$

If there are $\overline{I}$ cars need to transport during one motorail transportation service, and there are $\sum_k \sum_e n_e^k$ kinds of wagons can be applied to load the cars, the possible loading layout schemes should be $\overline{I}!\times\left(\sum_k \sum_e n_e^k\right)!$. Therefore, an algorithm with highly constrains need to be designed in order to solve the model and obtain the best car loading layout, rapidly. The heuristic algorithm designed in this paper will work as follows:

Step 1: calculate all possible cars loading schemes:
- Step 1.1: choose one kind of wagon from *set* 2;
- Step 1.2: choose one car from *set* 1, load it on the selected wagon in Step 1.1, randomly. All loading processes must satisfy the constrains – Equations (6)–(8). Judge the wagon left loading capacity according to constrains – Equations (9)–(12) after loading on a car, meanwhile update *set* 1;
- Step 1.3: if the wagon still remains loading capacity, turn to Step 1.2; otherwise turn to Step 1.4;
- Step 1.4: update *set* 1, if there is no more cars need to load, turn to Step1.5; otherwise turn to Step1.1;
- Step 1.5: record each loaded car's position coordinates, the wagon type and its serial number n_e^k, and mark this as a kind of feasible loading scheme F_t;
- Step 1.6: repeat Step 1.1 to Step 1.5, find all feasible loading schemes $\sum_t F_t$, save them to *set* 4.

Step 2: for all the feasible schemes F_t in *set* 4, calculate objective function (Equation (1)) values, select the scheme(s) with maximum $Z_{\max}$ value, mark it (or them) as F_t', save it (or them) to *set* 5. If there is only one single scheme exists in *set* 5, turn to **Step 5**; otherwise turn to **Step 3**;

Step 3: for all the feasible schemes F_t' in *set* 5, calculate objective function (Equation (2)) values, select the scheme(s) with minimum $H_{\min}$ value, mark it (or them) as F_t'', save it (or them) to *set* 6. If there is only one single scheme exists in *set* 6, turn to **Step 5**; otherwise turn to **Step 4**;

Step 4: for all feasible schemes F_t'' in *set* 6, calculate the lateral bias $\sum_k \left(X_k - x_k\right)$, select the scheme with minimum value as the final loading scheme of cars $\overline{I}$. There must be only one optimal solution exists after **Step 4**, turn to **Step 5**;

Step 5: end.

4. Case study and results discussing

We now give our case study by taking $\overline{I}=29$ cars as a numerical example in order to test our motorail transportation car loading layout optimization model and algorithm in this paper. The information about the cars need to be loaded is shown in Table 3, includes the length, width, height, weight and number for each type of cars. Table 4 presents us the information about the wagons that can be applied for motorail in China, includes length, width, height of the upper and lower decks, wagon weight, loading weight, etc. Table 5 shows the other needed initial data. Some other related data can be calculated according to the data in Table 3 and Table 4. A DSS to compute the model has been implemented, the DSS is written in MATLAB R2012b optimizer with *Microsoft Excel 2013 Visual Basic* to resolve the motorail transportation car loading layout optimization problem. The model and algorithm are run on an *Intel I 5* 2.4 GHZ with 2 GB RAM in the environment of *Microsoft Win 8.0*. 3.5 min will be occupied to carry on the $\overline{I}=29$ cars numerical example.

The results of the calculation show that we need 3 wagons to carry the 29 cars totally, include 2 type JSQ6 and one type JSQ5. We record the wagons as Wagon I, Wagon II and Wagon III. The total height of the wagon after loading cars, average horizontal safety gap between 2 cars, utilization rate of loading capacity, gravitational center height of the wagon after loading cars, average lateral safety gap between cars and wagon body, the value of objective function (Equation (4)) as well as the value of objective function (Equation (5)) will be presented in Table 5. The loaded location in the *X* axis for each car will be presented in Table 6.

After analysing the calculation results in Tables 5 and 6, we can find 2 basic conclusions, which are critical to the actual practice about motorail transportation loading layout optimization. The conclusions provide shorter loading time consumption as well as lower loading costs if the managers and operators follow the conclusions. We next present the 2 conclusions, and prove the conclusions with strict mathematical proofs.

Conclusion 1: for each deck on any wagons, the cars' optimal layout should be loaded like this: the heaviest cars should be loaded in the middle of the decks, the cars weight loaded on both sides should be gradually reduced. If the smaller weight difference of the 2 sides loaded cars, the car loading layout is better. This kind of loading approach is propitious to ensure the gravitational center located around the point of transversal and axial center line on the wagon, which is beneficial for transportation safety.

Proof 1: we assume that the loaded cars' weight on one deck satisfy $m_1 \ge m_2 \ge \ldots \ge m_i \ge \ldots \ge m_n$, the cars' location on the *X* axis are $x_1, x_2, \ldots, x_i, \ldots, x_n$. L presents length of the wagon. According to the constraint (Equation (6)) of the model, we can find that the value of $\left|\frac{\sum_{i=1}^{n} m_i \cdot x_i}{\sum_{i=1}^{n} m_i} - \frac{L}{2}\right|$ should be as small as possible, which means, the value of the equation $\frac{\left|m_1 \cdot \left(2x_1 - L\right) + \ldots + m_n \cdot \left(2x_n - L\right)\right|}{2 \cdot \sum_{i=1}^{n} m_i}$

Table 3. Information for the cars need to be transported

Car type code	Length [mm]	Width [mm]	Height [mm]	Weight [t]	Number
BM	4894	1894	1365	1.840	3
BX	4880	1983	1709	2.265	1
P	4970	1931	1418	1.760	2
B	5099	1920	1840	2.295	1
C	4846	1939	1705	2.170	1
R	4999	2073	1835	2.570	2
FE	4527	1937	1213	1.380	2
F	4480	1840	1500	1.370	3
A	4441	1930	1244	1.700	2
L	5273	1930	1712	2.883	1
RO	5099	1948	1550	2.360	1
BE	5145	1926	1521	2.585	2
CA	5131	1852	1501	1.840	3
V	4715	1866	1481	1.642	2
M	5915	1980	1573	2.855	1
J	4672	2835	1416	1.690	2

Note: data source – *https://www.google.com*.

Table 4. Information for the wagons used to motorail in China (Huang, Shuai 2015)

Wagon Type	Weight [t]	Loading weight [t]	Length [mm]	Width [mm]	Upper deck height [mm]	Lower deck height [mm]	Gravitational center height [mm]
JSQ1K	33.3	16.8	21738	3195	1719	1690	82.50
JSQ2K	31.5	15.0	21738	3197	3161 (single deck)		123.25
JSQ3K	31.0	17.0	21738	3197	1752	1694	97.75
JSQ4K	30.1	12.0	18138	3201	3118 (single deck)		105.00
JSQ5	37.0	20.0	26030	3066	1710	1820	100.75
JSQ6	37.4	22.0	26066	3086	1590	2070	100.75
J5SQ	25.5	13.0	17938	3065	1591	1770	113.00
J6SQ	25.7	13.0	17938	3065	1591	1770	113.00

Table 5. Statistics information about the wagons and loaded cars

	Wagon I	Wagon II	Wagon III
Wagon type	JSQ6	JSQ5	JSQ6
Total height of the wagon after loading cars [t]	19.852	18.060	18.730
Average horizontal safety gap between 2 cars [mm]	234	681	375
Utilization rate of loading capacity [%]	90.24	90.30	85.14
Gravitational center height of the wagon after loading cars [mm]	700.47	613.15	651.39
Average lateral safety gap between cars and wagon body [mm]	61.81	20.25	199.05
Objective function (Equation (4)) value [%]	88.50		
Objective function (Equation (5)) value [mm]	655.57		

Table 6. Loaded location in the X-axis for each [car/m]

Wagon I				Wagon II				Wagon III			
Upper deck		Lower deck		Upper deck		Lower deck		Upper deck		Lower deck	
Type	Location	Type	Location	Type	Location	Type	Location	Type	Location	Type	Location
V	2.718	P	2.593	A	3.598	BM	2.547	F	2.835	B	2.705
J	7.771	RO	7.736	CA	9.761	BX	7.534	F	7.910	R	7.909
CA	13.033	L	13.030	CA	16.269	M	13.042	FE	13.009	BE	13.136
J	18.294	BE	18.347	A	22.432	C	18.532	FE	18.131	R	18.363
V	23.349	P	23.473	–	–	BM	23.483	F	23.230	BM	23.466

should be as small as possible. So we should let $\left|(2x_1 - L) \le (2x_2 - L) \le \ldots \le (2x_n - L)\right|$, x_1 is supposed to equal to $\frac{L}{2}$, other cars should be successively located on both sides of x_1. We can also obtain this conclusion according to the results of the case study: taking coordinates of car's gravitational center on X axis as abscissa, each car's weight as ordinates, we can fit 6 loading curves, shows in Figure 2.

Conclusion 2: for each wagon with upper and lower decks, the total weight of the cars loaded on lower deck should be larger than the total weight of the cars loaded on the upper deck. This kind of loading layout is helpful to reduce the total gravitational center height in a wagon. The weight difference between the 2 decks is bigger, the layout is more safer during the whole transport process. We can also find the conclusion in Figure 2, the 3 upper curves present the loading layout on 3 lower decks (3 dotted lines in Figure 2), the 3 lower curves present the loading layout on 3 upper decks (3 solid lines in Figure 2).

Proof 2: we assume the total cars weight loaded on 2 decks are m_1 and m_2, $m_1 \le m_2$. The cars' average gravitational center height is h_1 and h_2, respectively. The height of the lower deck is l. If the upper cars' weight is m_1, lower is m_2, then the total gravitational center height of the cars loaded on the wagon should be $\eta_1 = \frac{m_1 \cdot h_1 + m_2 \cdot (h_2 + l)}{m_1 + m_2}$; otherwise, the total gravitational center height of the cars loaded on the wagon should be $\eta_2 = \frac{m_1 \cdot (h_1 + l) + m_2 \cdot h_2}{m_1 + m_2}$. We can easily find that $\eta_1 - \eta_2 = \frac{(m_1 - m_2) \cdot l}{m_1 + m_2} \le 0$ always holds, which indicates that, the total weight on upper deck should be lighter than the total weight on lower deck.

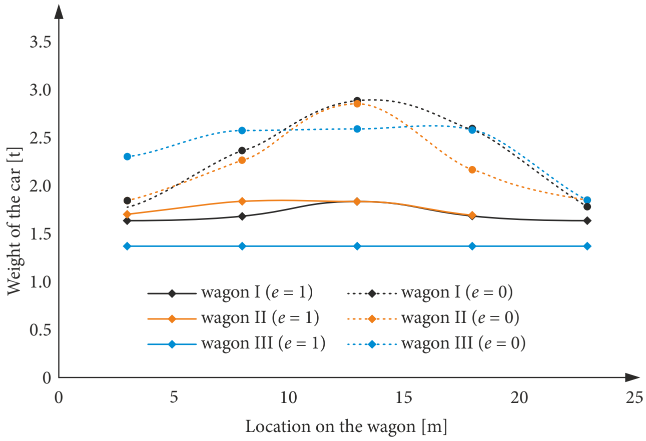

Figure 2. 6 loading curves about the case study

Conclusions and further study works

The motorail transportation operators only provide car and passenger transport service in China, not include motorcycle transport service in other countries, so the car loading layout optimization model is also different from the models applied in other countries. Compared with the car transport train, there are more complex loading limitation factors because the cars transported by motorail are already put into daily use, the cars may be dangerous because the tanks are full of gas, so the cars' loading layout schemes must be safety-oriented. In addition, the cars transported by motorail may have different types, the length, width, height as well as the weight are variety, so the optimization model must be cost-oriented.

The model presented in this paper has 2 objective functions. One is cost-oriented, which maximizes the wagon loading weight utilization rate, this is critical to reduce the operation costs during the whole transport process; Another is safety-oriented, which minimizes the vehicle gravitational center height after loading the cars on the wagons. There are 3 groups of constraints: Group 1 require the total gravitational center about the loaded cars should be coincident with the gravitational center of the wagons as much as possible; Group 2 indicates that length, width and height of loaded cars should meet the needs of the limitation of the wagons; Group 3 is loading weight limitation, which requires the total loaded cars' weight should not exceed the wagon's rated loading weight.

According to the calculation results of the case study presented in this paper, we can find that: For each deck on any wagons, the heaviest cars should be loaded in the middle of the decks, the cars weight loaded on both sides should be gradually reduced. If the smaller weight difference of the 2 sides loaded cars, the car loading layout is better. This kind of loading approach is propitious to ensure the gravitational center located around the point of transversal and axial center line on the wagon, which is beneficial for transportation safety; For each wagon with upper and lower decks, the total weight of the cars loaded on lower deck should be larger than the total weight of the cars loaded on the upper deck. This kind of loading layout is helpful to reduce the total gravitational center height in a wagon. The weight difference between the 2 decks is bigger, the layout is more safer during the whole transport process.

The model and algorithm designed in this paper only suitable for loading layout motorail transport without any other stops during the transport process. When then line planning has other midway stops with loading and unloading operations, the model is not inappropriate, more restriction conditions are supposed to add into the model, this belongs to one of our further research tasks. In addition, the algorithm presented material describe transportation only across one country, next we can discuss the motorail transportation through all continent between different countries.

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