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METHOD FOR AGGREGATING TRIANGULAR FUZZY INTUITIONISTIC FUZZY INFORMATION AND ITS APPLICATION TO DECISION MAKING

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Abstract. On the foundation of the theory of the intuitionistic fuzzy set, this paper uses the triangular fuzzy number to denote the membership degree and the non-membership degree and proposes the triangular intuitionistic fuzzy number. Then the operation rules of triangular intuitionistic fuzzy numbers are defined. The weighted arithmetic averaging operator and the weighted geometric average operator are presented and used to the decision making area after defined the score function and the accuracy function. An effective solution is offered for multi-attitude decision-making problem and an active try is made.

Keywords: triangular fuzzy number; triangular intuitionistic fuzzy number; integration operator; decision-making.

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1. Introduction

Decision-making is the process of finding the best option from all of the feasible alternatives. Sometimes, decision-making problems considering several criteria are called multi-criteria decision-making (MCDM) problems. The MCDM problems may be divided into two kinds of problem. One is the classical MCDM problems (Hwang and Yoon 1981; Kaklauskas *et al.* 2006; Zavadskas *et al.* 2008a, 2008b; Lin *et al.* 2008; Ginevičius *et al.* 2008), among which the ratings and the weights of criteria are measured in crisp numbers. Another is the fuzzy multiple criteria decision-making (FMCDM) problems (Bellman and Zadeh 1970; Liu and Wang 2007; Liu and Guan 2008; Liu 2009a, 2009b), among which the ratings and the weights of criteria evaluated on imprecision, subjective and vagueness are usually expressed

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by linguistic terms, fuzzy numbers or intuition fuzzy numbers. In 1965, the fuzzy set theory was proposed by Prof. Zadeh (Zadeh 1965), it was a very good tool to research the FMCDM problems. Gau and Buehrer (1993) proposed the concept of vague set. On the foundation of it, Chen (1994) and Hong and Choi (2000) researched the fuzzy multi-criteria decision-making problem based on the Vague sets. Bustine and Burillo (1996) points out that Vague set is the intuitionistic fuzzy set. Because the intuitionistic considered the membership degree, the non-membership degree and the hesitancy degree synchronously, it is more flexible and practical than the traditional fuzzy set at the aspect of dealing with the vagueness and uncertain. Atanassov *et al.* (1986) extended the intuitionistic fuzzy set and proposed the concept of the interval intuitionistic fuzzy set. Atanassov (1989) defined a lot of basic operation rules of the interval intuitionistic fuzzy set and they thought that the fuzzy set is the special condition of the intuitionistic fuzzy set. Bustince and Burillo (1995) researched the correlation degree and decomposition theorem of the interval intuitionistic fuzzy set. Hung, Wu (2002) calculated the correlation coefficient using centroid method. Deschrijver and Kerre (2003) researched the relation of interval intuitionistic fuzzy set, L-fuzzy set, intuitionistic fuzzy and interval fuzzy set. Xu (2007) proposed the methods for aggregating interval-valued intuitionistic fuzzy information. Most related literatures are all the researches of interval intuitionistic fuzzy set and concentrates on the basic theory. Lots of problems about the intuitionistic fuzzy are worth to research.

The triangular fuzzy number is easier to show the fuzzy problem than the interval number. So, using the triangular fuzzy number in the intuitionistic fuzzy set is easier to deal with the fuzzy and the uncertain information. The application of the triangular fuzzy number in the area of the intuitionistic fuzzy set is rare. This paper discusses the intuitionistic fuzzy set based on the triangular fuzzy number and proposed the triangular intuitionistic fuzzy number. And two integration operations of the triangular intuitionistic fuzzy number are given. The score function and accuracy function of the triangular intuitionistic fuzzy number are also proposed. Then this paper represents a simple priority method. At last, this theory is used to decision making area.

2. The intuitionistic fuzzy set

Suppose X is a nonempty set, $A=\{<x,u_A(x),v_A(x)>x\in X\}$ is a intuitionistic fuzzy set (Atanassov 1986), where, $u_A(x)$ is the membership degree of x belongs to X and $v_A(x)$ is the non-membership degree of x belongs to X, $u_A : X \to [0,1]$, $v_A : X \to [0,1]$ and $0 \le u_A(x)+v_A(x) \le 1,\ \forall x \in X$. In addition, $1-u_A(x)-v_A(x)$ denotes the hesitancy degree of x belongs to X.

If $A=\{<x,u_A(x),v_A(x)>x\in X\}$ and $B=\{<x,u_B(x),v_B(x)>x\in X\}$ are intuitionistic fuzzy numbers, the operation rules of the intuitionistic fuzzy number are as follows (Atanassov 1989, 1994):

$$A+B=\{<x,u_A(x)+u_B(x)-u_A(x)u_B(x),v_A(x)v_B(x)>x\in X\}; \tag{1}$$

$$A\cdot B=\{<x,u_A(x)u_B(x),v_A(x)+v_B(x)-v_A(x)v_B(x)>x\in X\}; \tag{2}$$

$$nA = \{< x, 1-[1-u_A(x)]^n, [v_A(x)]^n > x \in X\}; \tag{3}$$

$$A^n = \{< x, [u_A(x)]^n, 1-[1-v_A(x)]^n > x \in X, n > 0\}. \tag{4}$$

The intuitionistic fuzzy set is flexible and practical to deal with the fuzzy and uncertain information. But things are usually complex and uncertain, so it is hard to express the membership degree and the non-membership degree using the exact real number value. Using the triangular fuzzy number to show them is a feasible method.

3. The triangular intuitionistic fuzzy number

This paper extends the intuitionistic fuzzy set and uses the triangular fuzzy number to express the membership degree $\bar{u}_A(x)$ and the non-membership degree $\bar{v}_A(x)$. So an intuitionistic fuzzy number is got based on the triangular fuzzy number. And it is called triangular intuitionistic fuzzy number. The general form of the triangular intuitionistic fuzzy number is marked as $([a^L, a^M, a^U], [b^L, b^M, b^U])$, $[a^L, a^M, a^U]$ and $[b^L, b^M, b^U]$ are triangular fuzzy numbers.

Suppose $I_1 = ([a_1{}^L, a_1{}^M, a_1{}^U], [b_1{}^L, b_1{}^M, b_1{}^U])$ and $I_2 = ([a_2{}^L, a_2{}^M, a_2{}^U], [b_2{}^L, b_2{}^M, b_2{}^U])$ are two triangular intuitionistic fuzzy numbers, according to formulas (1)–(4) and operation rules of triangular fuzzy numbers, then

$$I_1 + I_2 = ([a_1{}^L, a_1{}^M, a_1{}^U], [b_1{}^L, b_1{}^M, b_1{}^U]) + ([a_2{}^L, a_2{}^M, a_2{}^U], [b_2{}^L, b_2{}^M, b_2{}^U]) = ([a_1{}^L + a_2{}^L - a_1{}^L a_2{}^L, a_1{}^M + a_2{}^M - a_1{}^M a_2{}^M, a_1{}^U + a_2{}^U - a_1{}^U a_2{}^U], [b_1{}^L b_2{}^L, b_1{}^M b_2{}^M, b_1{}^U b_2{}^U]); \tag{5}$$

$$I_1 \cdot I_2 = ([a_1{}^L, a_1{}^M, a_1{}^U], [b_1{}^L, b_1{}^M, b_1{}^U]) \cdot ([a_2{}^L, a_2{}^M, a_2{}^U], [b_2{}^L, b_2{}^M, b_2{}^U]) = ([a_1{}^L a_2{}^L, a_1{}^M a_2{}^M, a_1{}^U a_2{}^U], [b_1{}^L + b_2{}^L - b_1{}^L b_2{}^L, b_1{}^M + b_2{}^M - b_1{}^M b_2{}^M, b_1{}^U + b_2{}^U - b_1{}^U b_2{}^U]); \tag{6}$$

$$\lambda I_1 = \lambda([a_1{}^L, a_1{}^M, a_1{}^U], [b_1{}^L, b_1{}^M, b_1{}^U]) = ([1-(1-a_1{}^L)^\lambda, 1-(1-a_1{}^M)^\lambda, 1-(1-a_1{}^U)^\lambda], [(b_1{}^L)^\lambda, (b_1{}^M)^\lambda, (b_1{}^U)^\lambda]), \lambda \geq 0; \tag{7}$$

$$I_1{}^\lambda = ([a_1{}^L, a_1{}^M, a_1{}^U], [b_1{}^L, b_1{}^M, b_1{}^U])^\lambda = ([(a_1{}^L)^\lambda, (a_1{}^M)^\lambda, (a_1{}^U)^\lambda][1-(1-b_1{}^L)^\lambda, 1-(1-b_1{}^M)^\lambda, 1-(1-b_1{}^U)^\lambda]), \lambda \geq 0. \tag{8}$$

It is easy to see that the calculation results in definition 1 are triangular intuitionistic fuzzy numbers. According to the Eq.(5)–(8), the following calculation rules can be gained:

$$I_1 + I_2 = I_2 + I_1; \tag{9}$$

$$I_1 \cdot I_2 = I_2 \cdot I_1; \tag{10}$$

$$\lambda(I_1 + I_2) = \lambda I_1 + \lambda I_2,\ \lambda \geq 0; \tag{11}$$

$$\lambda_1 I_1 + \lambda_2 I_1 = (\lambda_1 + \lambda_2) I_1,\ \lambda_1, \lambda_2\ \geq 0; \tag{12}$$

$$I_1^{\lambda_1} \cdot I_1^{\lambda_2} = (I_1)^{\lambda_1 + \lambda_2},\ \lambda_1, \lambda_2\ \geq 0. \tag{13}$$

4. Aggregation operators of the triangular intuitionistic fuzzy number and its score function and accuracy function

4.1. Aggregation operators of the triangular intuitionistic fuzzy number

On the foundation of the above calculation rules, the weighted arithmetic average operator of the triangular intuitionistic fuzzy numbers is given.

Definition 1: suppose $I_i (i = 1,\ 2,\ \ldots, n)$ is a set of triangular intuitionistic fuzzy numbers, and suppose $f : \Omega^n \to \Omega$, if

$$f_\omega(I_1, I_2, \ldots, I_n) = \sum_{i=1}^{n} \omega_i I_i. \tag{14}$$

Where, Ω is the set of all the triangular intuitionistic fuzzy numbers, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $I_i (i = 1,\ 2,\ \ldots, n)$, $\omega_i \in [0,1]$, $\sum_{i=1}^{n} \omega_i = 1$. Then the f is called the weighted arithmetic average operator of the triangular intuitionistic fuzzy number. Specially, if $\omega = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$, the f is called the arithmetic averaging operator of the triangular intuitionistic fuzzy number.

Definition 2: suppose $I_i (i = 1,\ 2,\ \ldots, n)$ is a set of triangular intuitionistic fuzzy numbers, and suppose $g : \Omega^n \to \Omega$, if

$$g_\omega(I_1, I_2, \ldots, I_n) = \prod_{i=1}^{n} (I_i)^{\omega_i}. \tag{15}$$

Where, Ω is the set of all the triangular intuitionistic fuzzy numbers, $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weight vector of $I_i (i = 1,\ 2,\ \ldots, n)$, $\omega_i \in [0,1]$, $\sum_{i=1}^{n} \omega_i = 1$. Then the g is called the weighted geometric average operator of the triangular intuitionistic fuzzy number. Specially, if $\omega = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$, the g is called the geometric averaging operator of the triangular intuitionistic fuzzy number.

Theorem 1: suppose $I_i = ([a_i^L, a_i^M, a_i^U], [b_i^L, b_i^M, b_i^U])$ $(i = 1, 2, \cdots, n)$ is a set of triangular intuitionistic fuzzy numbers, then the result is triangular intuitionistic fuzzy number aggregated by Eq.(14), and

$$f_\omega(I_1, I_2, \ldots, I_n) = ([1 - \prod_{i=1}^{n} (1 - a_i^L)^{\omega_i},$$
$$1 - \prod_{i=1}^{n} (1 - a_i^M)^{\omega_i}, 1 - \prod_{i=1}^{n} (1 - a_i^U)^{\omega_i}], [\prod_{i=1}^{n} (b_i^L)^{\omega_i}, \prod_{i=1}^{n} (b_i^M)^{\omega_i}, \prod_{i=1}^{n} (b_i^U)^{\omega_i}]). \tag{16}$$

Where, $\omega=(\omega_1,\omega_2,\ldots,\omega_n)^T$ is the weight vector of $I_i(i=1,2,\ldots,n)$, $\omega_i\in[0,1]$, $\sum_{i=1}^{n}\omega_i=1$. Mathematical induction is used to prove the Eq.(16). The proving procedures are as follows:

(1) When $n = 1$, it is right obviously.

(2) When $n = 2$,

$$\omega_1 I_1=([1-(1-a_1{}^L)^{\omega_1},1-(1-a_1{}^M)^{\omega_1},1-(1-a_1{}^U)^{\omega_1}],[(b_1{}^L)^{\omega_1},(b_1{}^M)^{\omega_1},(b_1{}^U)^{\omega_1}]),$$

$$\omega_2 I_2=([1-(1-a_2{}^L)^{\omega_2},1-(1-a_2{}^M)^{\omega_2},1-(1-a_2{}^U)^{\omega_2}],[(b_2{}^L)^{\omega_2},(b_2{}^M)^{\omega_2},(b_2{}^U)^{\omega_2}]).$$

So

$$f_\omega(I_1,I_2)=\omega_1 I_1+\omega_2 I_2=([1-(1-a_1{}^L)^{\omega_1}+1-(1-a_2{}^L)^{\omega_2}-(1-(1-a_1{}^L)^{\omega_1})(1-(1-a_2{}^L)^{\omega_2}),$$
$$1-(1-a_1{}^M)^{\omega_1}+1-(1-a_2{}^M)^{\omega_2}-(1-(1-a_1{}^M)^{\omega_1})(1-(1-a_2{}^M)^{\omega_2}),$$
$$1-(1-a_1{}^U)^{\omega_1}+1-(1-a_2{}^U)^{\omega_2}-(1-(1-a_1{}^U)^{\omega_1})(1-(1-a_2{}^U)^{\omega_2})],$$

$$[(b_1{}^L)^{\omega_1}(b_2{}^L)^{\omega_2},(b_1{}^M)^{\omega_1}(b_2{}^M)^{\omega_2},(b_1{}^U)^{\omega_1}(b_2{}^U)^{\omega_2}])=$$
$$([1-(1-a_1{}^L)^{\omega_1}(1-a_2{}^L)^{\omega_2},1-(1-a_1{}^M)^{\omega_1}(1-a_2{}^M)^{\omega_2},1-(1-a_1{}^U)^{\omega_1}(1-a_2{}^U)^{\omega_2}],$$
$$[(b_1{}^L)^{\omega_1}(b_2{}^L)^{\omega_2},(b_1{}^M)^{\omega_1}(b_2{}^M)^{\omega_2},(b_1{}^U)^{\omega_1}(b_2{}^U)^{\omega_2}]).$$

(3) when $n = k$, Eq. (16) is tenable,

$$f_\omega(I_1,I_2,\ldots,I_k)=([1-\prod_{i=1}^{k}(1-a_i{}^L)^{\omega_i},1-\prod_{i=1}^{k}(1-a_i{}^M)^{\omega_i},$$
$$1-\prod_{i=1}^{k}(1-a_i{}^U)^{\omega_i}],[\prod_{i=1}^{k}(b_i{}^L)^{\omega_i},\prod_{i=1}^{k}(b_i{}^M)^{\omega_i},\prod_{i=1}^{k}(b_i{}^U)^{\omega_i}]).$$

Then, when $n = k + 1$,

$$f_\omega(I_1,I_2,\ldots,I_{k+1})=f_\omega(I_1,I_2,\ldots,I_k)+\omega_{k+1}I_{k+1}=$$
$$([1-\prod_{i=1}^{k}(1-a_i{}^L)^{\omega_i},1-\prod_{i=1}^{k}(1-a_i{}^M)^{\omega_i},1-\prod_{i=1}^{k}(1-a_i{}^U)^{\omega_i}],[\prod_{i=1}^{k}(b_i{}^L)^{\omega_i},\prod_{i=1}^{k}(b_i{}^M)^{\omega_i},$$
$$\prod_{i=1}^{k}(b_i{}^U)^{\omega_i}])+([1-(1-a_{k+1}{}^U)^{\omega_{k+1}},1-(1-a_{k+1}{}^M)^{\omega_{k+1}},1-(1-a_{k+1}{}^U)^{\omega_{k+1}}],$$
$$[(b_{k+1}{}^L)^{\omega_{k+1}},(b_{k+1}{}^M)^{\omega_{k+1}},(b_{k+1}{}^U)^{\omega_{k+1}}])=$$
$$([1-\prod_{i=1}^{k+1}(1-a_i{}^L)^{\omega_i},1-\prod_{i=1}^{k+1}(1-a_i{}^M)^{\omega_i},1-\prod_{i=1}^{k+1}(1-a_i{}^U)^{\omega_i}],[\prod_{i=1}^{k+1}(b_i{}^L)^{\omega_i},\prod_{i=1}^{k+1}(b_i{}^M)^{\omega_i},\prod_{i=1}^{k+1}(b_i{}^U)^{\omega_i}]).$$

Namely, when $n = k + 1$, Eq.(16) is tenable. According to (1) (2) (3), Eq.(16) is tenable for all n.

Theorem 2: suppose $I_i = ([a_i^L, a_i^M, a_i^U], [b_i^L, b_i^M, b_i^U])$ $(i = 1, 2, \cdots, n)$ is a set of triangular intuitionistic fuzzy numbers, then the result is triangular intuitionistic fuzzy number aggregated by Eq.(15), and

$$g_\omega(I_1, I_2, \ldots, I_n) = ([\prod_{i=1}^{n}(a_i^L)^{\omega_i}, \prod_{i=1}^{n}(a_i^M)^{\omega_i}, \prod_{i=1}^{n}(a_i^U)^{\omega_i}],$$
$$[1-\prod_{i=1}^{n}(1-b_i^L)^{\omega_i}, 1-\prod_{i=1}^{n}(1-b_i^M)^{\omega_i}, 1-\prod_{i=1}^{n}(1-b_i^U)^{\omega_i}]). \tag{17}$$

The proving procedures of the theorem 2 are similar to the theorem 1, so this paper omits them. The emphasis points of the arithmetic averaging operator of the triangular intuitionistic fuzzy number and the geometric averaging operator are different. The former emphasizes on the personal importance and the latter pays attention to the collective effect.

4.2. The score function and accuracy function of the triangular intuitionistic fuzzy number

Definition 3 (Chen and Tan 1994): suppose $I = (\mu_A, \nu_A)$ is a intuitionistic fuzzy number, then $S(I) = \mu_A - \nu_A$ is called the score function of *I*.

Definition 4 (Xu 2007): suppose $I = ([a^L, a^U], [b^L, b^U])$ is a interval-valued intuitionistic fuzzy number, then $S(I) = (a^L - b^L + a^U - b^U)/2$ is called the score function of *I*.

Definition 5 (Hong and Choi 2000): suppose $I = (\mu_A, \nu_A)$ is a intuitionistic fuzzy number, then $H(I) = \mu_A + \nu_A$ is called the accuracy function of *I*.

Definition 6 (Xu 2007): suppose $I = ([a^L, a^U], [b^L, b^U])$ is a interval-valued intuitionistic fuzzy number, then $H(I) = (a^L + b^L + a^U + b^U)/2$ is called the accuracy function of *I*.

According to definition 3–6, we can be deduced the score function and accuracy function of the triangular intuitionistic fuzzy number.

Definition 7: suppose $I = ([a^L, a^M, a^U], [b^L, b^M, b^U])$ is a triangular intuitionistic fuzzy number, then *S*(*I*) is called the score function of *I*,

$$S(I) = (a^L - b^L + a^M - b^M + a^U - b^U)/3, \tag{18}$$

where $S(I) \in [-1, 1]$. Obviously, if the bigger *S*(*I*), the bigger *I*, especially, if *S*(*I*) = 1, the value of *I* is the biggest one $I = ([1, 1, 1], [0, 0, 0])$. If *S*(*I*) = –1, the value of *I* is the smallest one $I = ([0, 0, 0], [1, 1, 1])$.

But the score function can not compare two triangular intuitionistic fuzzy numbers in a special situation. For example, if $I_1 = ([0.3, 0.4, 0.5], [0.3, 0.4, 0.5])$, $I_2 = ([0.2, 0.3, 0.4], [0.2, 0.3, 0.4])$, then $S(I_1) = S(I_2) = 0$ and the score function can not compare I_1 and I_2. In order to resolve this problem, an accuracy function is given in this paper.

Definition 8: suppose $I = ([a^L, a^M, a^U], [b^L, b^M, b^U])$ is a triangular intuitionistic fuzzy number, then *E*(*I*) is called the accuracy function of *I*,

$$E(I) = (a^L + b^L + a^M + b^M + a^U + b^U)/3, \tag{19}$$

where $E(I) \in [0, 1]$.

In the example above, if $I_1 = ([0.3, 0.4, 0.5], [0.3, 0.4, 0.5])$, $I_2 = ([0.2, 0.3, 0.4], [0.2, 0.3, 0.4])$ then the accuracy functions $E(I_1) = 0.8$, $E(I_2) = 0.6$. The accuracy function can compare I_1 and I_2.

This paper thinks that the following conclusion is right: when the values of score function of the triangular intuitionistic fuzzy numbers are the same, the bigger the value of the accuracy function, the bigger the corresponding triangular intuitionistic fuzzy numbers. So I_1 is better than I_2.

On the foundation of the above analyses, a priority method is proposed for the triangular intuitionistic fuzzy numbers.

Definition 9: suppose I_1 and I_2 are any two triangular intuitionistic fuzzy numbers, then

$$\begin{cases} (1)\ \text{if}\ \ S(I_1) < S(I_2)\ \ \text{then}\ \ I_1 \prec I_2 \\ (2)\ \text{if}\ \ S(I_1) > S(I_2)\ \ \text{then}\ \ I_1 \succ I_2 \\ (3)\ \text{if}\ \ S(I_1) = S(I_2)\ \ \text{and}\ \ E(I_1) < E(I_2)\ \ \text{then}\ \ I_1 \prec I_2 \\ (4)\ \text{if}\ \ S(I_1) = S(I_2)\ \ \text{and}\ \ E(I_1) > E(I_2)\ \ \text{then}\ \ I_1 \succ I_2 \\ (5)\ \text{if}\ \ S(I_1) = S(I_2)\ \ \text{and}\ \ E(I_1) = E(I_2)\ \ \text{then}\ \ I_1 \approx I_2 \end{cases}$$

5. Decision making method based on the triangular intuitionistic fuzzy number

In a problem of multi-attribute decision making, suppose $S = \{s_1, s_2, \cdots, s_m\}$ is a set of projects. $R = \{r_1, r_2, \ldots, r_n\}$ is a set of attributes. $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weighted vector of the attributes, where, $\omega_i \in [0,1], \sum_{i=1}^{n} \omega_i = 1$. Suppose the characteristic information of project s_i is denoted by the triangular intuitionistic fuzzy number $s_i = \{< r_j, u_i(r_j), v_i(r_j) > r_j \in A\}, i = 1, 2, \ldots, m$, where, $u_i(r_j)$ denotes the satisfaction degree of project s_i to attribute r_j and $v_i(r_j)$ denotes the non-satisfaction degree of project s_i to attribute r_j. Here $u_i(r_j)$ and $v_i(r_j)$ are triangular fuzzy numbers. $u_i(r_j) = [a_{ij}{}^L, a_{ij}{}^M, a_{ij}{}^U]$, $v_i(r_j) = [b_{ij}{}^L, b_{ij}{}^M, b_{ij}{}^U]$. Then the corresponding triangular intuitionistic fuzzy numbers is denoted as: $I_{ij} = ([a_{ij}{}^L, a_{ij}{}^M, a_{ij}{}^U]\ [b_{ij}{}^L, b_{ij}{}^M, b_{ij}{}^U])$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$. So the decision making matrix is obtained: $D = (I_{ij})_{m \cdot n}$.

The steps of the decision making based on triangular fuzzy inuitionistic fuzzy numbers are as follows:

Step 1: according to the weighted arithmetic averaging operator or the weighted geometric average operator to integrate all the elements $I_{ij}\,(j = 1, 2, \ldots, n)$ of the i-th row in the decision making matrix D. Then the comprehensive triangular intuitionistic fuzzy value for s_i shows as follows.

$I_i^f = f_\omega(I_{i1}, I_{i2}, \ldots, I_{in})$ (for the weighted arithmetic averaging operator) or

$I_i^g = g_\omega(I_{i1}, I_{i2}, \ldots, I_{in})$ (for the weighted geometric averaging operator)

Step 2: calculate the value of the score function $S(I_i)$ and the value of the accuracy function $E(I_i)$ using the formulas of the score function and the accuracy function, where, $i = 1, 2, \ldots, m$.

Step 3: according to the definition 6, confirm the order of the project $s_i (i = 1, 2, \ldots, m)$. Then the best project can be gained.

6. Example analysis

A company intends to select one person to take the department manager position from four candidates ($s_1 - s_4$). Five indicators must be evaluated. They are shown as follows: ideological and moral quality(r_1), professional ability(r_2), creative ability (r_3), knowledge range (r_4) and leadership ability (r_5). The weights of the indicators are $W = (0.10, 0.25, 0.25, 0.15, 0.25)$. The leaders and people evaluate each indicator of each candidate. Suppose the evaluation information (I_{ij}) can be denoted by the triangular intuitionistic fuzzy number after transforming. The evaluation data is shown in Table 1.

Table 1. The transformed evaluation information of the indicators of each candidate (the decision making matrix *D*)

	r_1	r_2	r_3	r_4	r_5
s_1	([0.4, 0.5, 0.6], [0.1, 0.2, 0.3])	([0.3, 0.5, 0.6], [0.1, 0.3, 0.4])	([0.2, 0.3, 0.5], [0.3, 0.4, 0.5])	([0.4, 0.5, 0.6], [0.2, 0.3, 0.4])	([0.15, 0.2, 0.4], [0.3, 0.4, 0.6])
s_2	([0.25, 0.3, 0.4], [0.4, 0.45, 0.5])	([0.45, 0.5, 0.55], [0.1, 0.2, 0.3])	([0.4, 0.5, 0.55], [0.15, 0.2, 0.35])	([0.55, 0.6, 0.65], [0.1, 0.15, 0.2])	([0.35, 0.4, 0.5], [0.3, 0.35, 0.4])
s_3	([0.5, 0.55, 0.6], [0.2, 0.25, 0.3])	([0.4, 0.45, 0.5], [0.1, 0.2, 0.25])	([0.45, 0.5, 0.55], [0.1, 0.15, 0.2])	([0.3, 0.4, 0.5], [0.25, 0.3, 0.4])	([0.5, 0.6, 0.7], [0.05, 0.1, 0.15])
s_4	([0.3, 0.35, 0.4], [0.4, 0.45, 0.5])	([0.2, 0.3, 0.4], [0.3, 0.35, 0.45])	([0.4, 0.45, 0.5], [0.2, 0.25, 0.3])	([0.5, 0.55, 0.65], [0.2, 0.25, 0.3])	([0.3, 0.35, 0.4], [0.2, 0.25, 0.3])

Based on the weighted arithmetic averaging operator, the procedures to confirm the best candidate are as follows:

(1) According to the Eq. (16), integrate all the element $I_{ij} (j = 1, 2, \ldots, 5)$ of the i-th row. So the comprehensive value $I_i^f (i = 1, 2, 3, 4)$ of candidate $s_i (i = 1, 2, 3, 4)$ is calculated:

$I_1^f = ([0.269, 0.388, 0.532], [0.192, 0.333, 0.455])$;

$I_2^f = ([0.413, 0.477, 0.542], [0.167, 0.239, 0.332])$;

$I_3^f = ([0.436, 0.508, 0.581], [0.103, 0.170, 0.227])$;

$I_4^f = ([0.338, 0.399, 0.471], [0.237, 0.288, 0.349])$.

(2) According to (18), the value of the score function $S(I_i^f)$ can be calculated:

$S(I_1^f) = 0.07,\ S(I_2^f) = 0.231,\ S(I_3^f) = 0.341,\ S(I_4^f) = 0.111.$

(3) According to the value of the score function, the order of the candidates can be confirmed:

$s_3 \succ s_2 \succ s_4 \succ s_1.$

So, the best candidate is s_3.

In addition, we can do decision based on the weighted geometric average operator. The computational results are as follows:

$I_1^g = ([0.245, 0.350, 0.518], [0.220, 0.343, 0.474]);$

$I_2^g = ([0.399, 0.462, 0.533], [0.200, 0.262, 0.348]);$

$I_3^g = ([0.427, 0.498, 0.567], [0.123, 0.185, 0.245]);$

$I_4^g = ([0.314, 0.384, 0.455], [0.248, 0.298, 0.363]);$

$S(I_1^g) = 0.025,\ S(I_2^g) = 0.195,\ S(I_3^g) = 0.313,\ S(I_4^g) = 0.081.$

According to the value of the score function, the order of the candidates can be confirmed:

$s_3 \succ s_2 \succ s_4 \succ s_1.$

So the best candidate is also s_3.

In this example, the condition that the values of the score functions are same does not appear, so the accuracy function is not used. If the condition that the values of the score functions are same appears, the accuracy function should be used to calculate the final result to confirm the order of the candidates.

7. Conclusion

On the foundation of the theory of the intuitionistic fuzzy set, this paper extends the traditional research. This paper uses the triangular fuzzy to denote the membership degree and the non-membership degree and proposes the triangular intuitionistic fuzzy number. Then the operation rules of triangular intuitionistic fuzzy numbers are defined. The weighted geometric averaging operator and the weighted arithmetic average operator are presented and are used to the decision making area. An effective solution is offered for multi-attitude decision making problem and an active try is made. The example proves that the integration methods proposed in this paper are feasible effective.

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NUMANOMŲ NEAPIBRĖŽTŲJŲ AIBIŲ TEORIJA IR JOS TAIKYMAS PRIIMANT SPRENDIMUS

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Santrauka

Remiantis numanomų neapibrėžtųjų aibių teorija, šiame straipsnyje aprašomas trečiojo laipsnio neapibrėžtasis skaičius, apibūdinantis priklausomumo laipsnį. Toliau nustatomos operavimo su tokiais skaičiais taisyklės. Pasiūlytas svertinis aritmetinio vidurkio operatorius, naudojamas priimant sprendimus. Siūlomi efektyvūs daugiaaspekčių problemų sprendimo būdai.

Reikšminiai žodžiai: numanomas neapibrėžtasis skaičius, numanomas trečiojo laipsnio neapibrėžtasis skaičius, integracijos operatorius, sprendimų priėmimas.

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