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AN APPROACH TO MULTIPLE ATTRIBUTE GROUP DECISION MAKING WITH INTERVAL INTUITIONISTIC TRAPEZOIDAL FUZZY INFORMATION

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Abstract. In this paper, we investigate the multiple attribute group decision making (MAGDM) problems in which both the attribute weights and the expert weights take the form of real numbers, attribute values take the form of interval intuitionistic trapezoidal fuzzy numbers. Firstly, some operational laws of interval intuitionistic trapezoidal fuzzy numbers are introduced. Then some new aggregation operators including interval intuitionistic trapezoidal fuzzy ordered weighted geometric (IITFOWG) operator and interval intuitionistic trapezoidal fuzzy hybrid geometric (IITFHG) operator are proposed and some desirable properties of these operators are studied, such as commutativity, idempotency and monotonicity. An IITFWG and IITFHG operators-based approach is developed to solve the MAGDM problems in which both the attribute weights and the expert weights take the form of real numbers and attribute values take the form of interval intuitionistic trapezoidal fuzzy numbers. Finally, some illustrative examples are given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords: multiple attribute group decision making (MAGDM), interval intuitionistic trapezoidal fuzzy numbers, interval intuitionistic trapezoidal fuzzy weighted geometric (IITFWG) operator, interval intuitionistic trapezoidal fuzzy ordered weighted geometric (IITFOWG) operator, interval intuitionistic trapezoidal fuzzy hybrid geometric (IITFHG) operator.

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1. Introduction

Atanassov (1986), Atanassov and Gargov (1989) introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the concept of fuzzy set (Zadeh 1965). The intuitionistic fuzzy set has received more and more attention since its appearance (Hui *et al.* 2009; Lin *et al.* 2007; Liu 2007, 2009; Ye 2009a, b; Li 2008, 2010; Li *et al.* 2009; Wei 2008a, b, 2009, 2010a, b, c, 2011a, b, c, d, e, f, g; Wei *et al.* 2011b; Wei, Zhao 2011; Zhang, Liu 2010; Nowak 2011; Ulubeyli, Kazaz 2009). Xu and Yager (2006) developed some geometric aggregation operators with intuitionistic fuzzy information. Xu (2007a) further developed some arithmetic aggregation operators with intuitionistic fuzzy information. Wei (2008a) utilized the maximizing deviation method for intuitionistic fuzzy multiple attribute decision making with incomplete weight information. Wei (2010b) developed the GRA method for intuitionistic fuzzy multiple attribute decision making with incomplete weight information. Later, Atanassov and Gargov (1989), Atanassov (1994) further introduced the interval-valued intuitionistic fuzzy set (IVIFS), which is a generalization of the IFS. The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers. Xu (2007b) and Xu and Chen (2007) developed some aggregation operators with interval-valued intuitionistic fuzzy information. Xu (2008) and Wei (2009) proposed some aggregation functions for dynamic multiple attribute decision making in intuitionistic fuzzy setting or interval-valued intuitionistic fuzzy setting. Wei (2010a) developed some induced geometric aggregation operators with intuitionistic fuzzy information or interval-valued intuitionistic fuzzy information. Li (2010) proposed linear programming method for MADM with interval-valued intuitionistic fuzzy sets. Wei *et al.* (2011a) developed correlation coefficient for interval-valued intuitionistic fuzzy multiple attribute decision making with incomplete weight information. Shu *et al.* (2006) gave the definition and operational laws of intuitionistic triangular fuzzy number and proposed an algorithm of the intuitionistic fuzzy fault-tree analysis. Wang (2008) gave the definition of intuitionistic trapezoidal fuzzy number and interval intuitionistic trapezoidal fuzzy number. Wang and Zhang (2008) gave the definition of expected values of intuitionistic trapezoidal fuzzy number and proposed the programming method of multi-criteria decision-making based on intuitionistic trapezoidal fuzzy number with incomplete certain information. Wang and Zhang (2009) developed the Hamming distance of intuitionistic trapezoidal fuzzy numbers and intuitionistic trapezoidal fuzzy weighted arithmetic averaging (ITFWAA) operator, then proposed multi-criteria decision-making method with incomplete certain information based on intuitionistic trapezoidal fuzzy number.

Geometric means (Herrera *et al.* 2001; Xu, Da 2002; Wei 2010c; Wei *et al.* 2010a, b, c, d) is widely used as a tool to aggregate input data. Considering that, in the existing literature, the geometric mean is generally considered as a fusion technique of numerical data, interval data, intuitionistic fuzzy data and interval-valued intuitionistic fuzzy data, in the real-life situations, the input data sometimes cannot be obtained exactly, but interval intuitionistic trapezoidal fuzzy data can be given. Therefore, "how to aggregate interval intuitionistic trapezoidal fuzzy data by using the geometric mean?" is an interesting research topic and is worth paying attention to. The aim of this paper is to propose some new geometric aggregation operators including interval intuitionistic trapezoidal fuzzy ordered weighted geometric (IITFOWG) operator and interval intuitionistic trapezoidal fuzzy hybrid geometric (IITFHG) operator and studied some desirable properties of these operators. An IITFWG and IITFHG operators-based approach is developed to solve the MAGDM problems in which both the attribute weights and the expert weights takes the form of real numbers, attribute values takes the form of interval intuitionistic trapezoidal fuzzy numbers. Finally, an illustrative example is given to verify the developed approach.

2. Preliminaries

In the following, we shall introduce some basic concepts related to intuitionistic trapezoidal fuzzy numbers and interval intuitionistic trapezoidal fuzzy numbers.

Definition 1 (Wang 2008). Let $\tilde{a}$ is an intuitionistic trapezoidal fuzzy number, its membership function is:

$$\mu_{\tilde{a}}(x)=\begin{cases}\dfrac{x-a}{b-a}\mu_{\tilde{a}},\ a\le x<b;\\ \mu_{\tilde{a}}, b\le x\le c;\\ \dfrac{d-x}{d-c}\mu_{\tilde{a}},\ c<x\le d;\\ 0,\ others\end{cases} \tag{1}$$

its non-membership function is:

$$\nu_{\tilde{a}}(x)=\begin{cases}\dfrac{b-x+\nu_{\tilde{a}}(x-a_1)}{b-a_1},\ a_1\le x<b;\\ \nu_{\tilde{a}}, b\le x\le c;\\ \dfrac{x-c+\nu_{\tilde{a}}(d_1-x)}{d_1-c},\ c<x\le d_1;\\ 0,\ \text{others},\end{cases} \tag{2}$$

where $0\le\mu_{\tilde{a}}\le 1; 0\le\nu_{\tilde{a}}\le 1$ and $\mu_{\tilde{a}}+\nu_{\tilde{a}}\le 1; a,b,c,d\in R$.

Then $\tilde{a}=\left\langle\left(\left[a,b,c,d\right];\mu_{\tilde{a}}\right),\left(\left[a_1,b,c,d_1\right];\nu_{\tilde{a}}\right)\right\rangle$ is called an intuitionistic trapezoidal fuzzy number. For convenience, let $\tilde{a}=\left(\left[a,b,c,d\right];\mu_{\tilde{a}},\nu_{\tilde{a}}\right)$.

If $\tilde{\mu}_A(x)\subset[0,1]$ and $\tilde{\nu}_A(x)\subset[0,1]$ are interval numbers, and $0\le\sup\left(\tilde{\mu}_A(x)\right)+\sup\left(\tilde{\nu}_A(x)\right)\le 1, \forall x\in X$, for convenience, let $\tilde{\mu}_A(x)=\left[\underline{\mu},\overline{\mu}\right]$, $\tilde{\nu}_A(x)=\left[\underline{\nu},\overline{\nu}\right]$. Then $\tilde{a}=\left(\left[a,b,c,d\right];\tilde{\mu}_{\tilde{a}},\tilde{\nu}_{\tilde{a}}\right)=\left(\left[a,b,c,d\right];\left[\underline{\mu},\overline{\mu}\right],\left[\underline{\nu},\overline{\nu}\right]\right)$ is called an interval intuitionistic trapezoidal fuzzy number (Wan 2011).

Definition 2 (Wan 2011). Let $\tilde{a}_1=\left(\left[a_1,b_1,c_1,d_1\right];\left[\underline{\mu}_1,\overline{\mu}_1\right],\left[\underline{\nu}_1,\overline{\nu}_1\right]\right)$ and $\tilde{a}_2=\left(\left[a_2,b_2,c_2,d_2\right];\left[\underline{\mu}_2,\overline{\mu}_2\right],\left[\underline{\nu}_2,\overline{\nu}_2\right]\right)$ be two interval intuitionistic trapezoidal fuzzy number, and $\lambda\ge 0$, then

1) $\tilde{a}_1\tilde{a}_2=\left(\left[a_1a_2,b_1b_2,c_1c_2,d_1d_2\right];\right.$
$\left.\left[\underline{\mu}_1\cdot\underline{\mu}_2,\overline{\mu}_1\cdot\overline{\mu}_2\right],\left[\underline{\nu}_1+\underline{\nu}_2-\underline{\nu}_1\cdot\underline{\nu}_2,\overline{\nu}_1+\overline{\nu}_2-\overline{\nu}_1\cdot\overline{\nu}_2\right]\right);$

2) $\tilde{a}_1^{\lambda}=\left(\left[a_1^{\lambda},b_1^{\lambda},c_1^{\lambda},d_1^{\lambda}\right];\left[\underline{\mu}_1^{\lambda},\overline{\mu}_1^{\lambda}\right],\left[1-\left(1-\underline{\nu}_1\right)^{\lambda},1-\left(1-\overline{\nu}_1\right)^{\lambda}\right]\right)$.

Definition 3 (Wan 2011). Let $\tilde{a}=\left(\left[a,b,c,d\right];\left[\underline{\mu},\overline{\mu}\right],\left[\underline{\nu},\overline{\nu}\right]\right)$ be an interval intuitionistic trapezoidal fuzzy number, a score function S of an interval intuitionistic trapezoidal fuzzy number can be represented as follows:

$$S\left(\tilde{a}\right)=\frac{a+b+c+d}{4}\bullet\frac{\underline{\mu}-\underline{\nu}+\overline{\mu}-\overline{\nu}}{2},\ S\left(\tilde{a}\right)\in\left[-1,1\right]. \tag{3}$$

Definition 4 (Wan 2011). Let $\tilde{a}=\left(\left[a,b,c,d\right];\left[\underline{\mu},\overline{\mu}\right],\left[\underline{\nu},\overline{\nu}\right]\right)$ be an interval intuitionistic trapezoidal fuzzy number, an accuracy function H of an interval intuitionistic trapezoidal fuzzy number can be represented as follows:

$$H\left(\tilde{a}\right)=\frac{a+b+c+d}{4}\bullet\frac{\underline{\mu}+\underline{\nu}+\overline{\mu}+\overline{\nu}}{2},\ H\left(\tilde{a}\right)\in\left[0,1\right] \tag{4}$$

to evaluate the degree of accuracy of the interval intuitionistic trapezoidal fuzzy number $\tilde{a}$, where $H\left(\tilde{a}\right)\in\left[0,1\right]$. The larger the value of $H\left(\tilde{a}\right)$, the more the degree of accuracy of the interval intuitionistic trapezoidal fuzzy number $\tilde{a}$.

As presented above, the score function S and the accuracy function H are, respectively, defined as the difference and the sum of the membership function $\tilde{\mu}_A\left(x\right)$ and the non-membership function $\tilde{\nu}_A\left(x\right)$. Based on the score function S and the accuracy function H, in the following, Wan (2011) give an order relation between two interval intuitionistic trapezoidal fuzzy number, which is defined as follows:

Definition 5. Let $\tilde{a}_1=\left(\left[a_1,b_1,c_1,d_1\right];\left[\underline{\mu}_1,\overline{\mu}_1\right],\left[\underline{\nu}_1,\overline{\nu}_1\right]\right)$ and $\tilde{a}_2=\left(\left[a_2,b_2,c_2,d_2\right];\left[\underline{\mu}_2,\overline{\mu}_2\right],\left[\underline{\nu}_2,\overline{\nu}_2\right]\right)$ be two interval intuitionistic trapezoidal fuzzy number, $s\left(\tilde{a}_1\right)$ and $s\left(\tilde{a}_2\right)$ be the scores of $\tilde{a}$ and $\tilde{b}$, respectively, and let $H\left(\tilde{a}_1\right)$ and $H\left(\tilde{a}_2\right)$ be the accuracy degrees of $\tilde{a}$ and $\tilde{b}$, respectively, then if $S\left(\tilde{a}\right)<S\left(\tilde{b}\right)$, then $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a}<\tilde{b}$; if $S\left(\tilde{a}\right)=S\left(\tilde{b}\right)$, then if $H\left(\tilde{a}\right)=H\left(\tilde{b}\right)$, then $\tilde{a}$ and $\tilde{b}$ represent the same information, denoted by $\tilde{a}=\tilde{b}$; (2) if $H\left(\tilde{a}\right)<H\left(\tilde{b}\right)$, $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a}<\tilde{b}$ [10–11].

3. Some geometric aggregation operators with interval intuitionistic trapezoidal fuzzy numbers

In the following, some geometric aggregation operators with interval intuitionistic trapezoidal fuzzy number are developed as follows:

Definition 6 (Wan 2011). Let $\tilde{a}_j\left(j=1,2,\cdots,n\right)$ be a collection of interval intuitionistic trapezoidal fuzzy number, and let IITFWG: $Q^n\rightarrow Q$, if

$$\begin{aligned}&\mathrm{IITFWG}_{\omega}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)\\&=\prod_{j=1}^{n}\left(\tilde{a}_j\right)^{\omega_j}\\&=\left(\left[\prod_{j=1}^{n}\left(a_j\right)^{\omega_j},\prod_{j=1}^{n}\left(b_j\right)^{\omega_j},\prod_{j=1}^{n}\left(c_j\right)^{\omega_j},\prod_{j=1}^{n}\left(d_j\right)^{\omega_j}\right];\right.\\&\left.\left[\prod_{j=1}^{n}\underline{\mu}_j^{\;\omega_j},\prod_{j=1}^{n}\overline{\mu}_j^{\;\omega_j}\right],\left[1-\prod_{j=1}^{n}\left(1-\underline{\nu}_j\right)^{\omega_j},1-\prod_{j=1}^{n}\left(1-\overline{\nu}_j\right)^{\omega_j}\right]\right),\end{aligned}\tag{5}$$

where $\omega=\left(\omega_1,\omega_2,\cdots,\omega_n\right)^T$ be the weight vector of $\tilde{a}_j\left(j=1,2,\cdots,n\right)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$, then IITFWG is called the interval intuitionistic trapezoidal fuzzy weighted geometric(IITFWG) operator.

Definition 7. Let $\tilde{a}_j\left(j=1,2,\cdots,n\right)$ $\left(j=1,2,\cdots,n\right)$ be a collection of interval intuitionistic trapezoidal fuzzy number. An interval intuitionistic trapezoidal fuzzy ordered weighted geometric (IITFOWG) operator of dimension n is a mapping IITFOWG: $Q^n\to Q$, that has an associated vector $w=\left(w_1,w_2,\cdots,w_n\right)^T$ such that $w_j>0$ and $\sum_{j=1}^{n}w_j=1$. Furthermore,

$$\begin{aligned}&\mathrm{IITFOWG}_{w}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)\\&=\prod_{j=1}^{n}\left(\tilde{a}_{\sigma(j)}\right)^{w_j}\\&=\left(\left[\prod_{j=1}^{n}\left(a_{\sigma(j)}\right)^{w_j},\prod_{j=1}^{n}\left(b_{\sigma(j)}\right)^{w_j},\prod_{j=1}^{n}\left(c_{\sigma(j)}\right)^{w_j},\prod_{j=1}^{n}\left(d_{\sigma(j)}\right)^{w_j}\right];\right.\\&\left.\left[\prod_{j=1}^{n}\underline{\mu}_{\sigma(j)}^{\;w_j},\prod_{j=1}^{n}\overline{\mu}_{\sigma(j)}^{\;w_j}\right],\left[1-\prod_{j=1}^{n}\left(1-\underline{\nu}_{\sigma(j)}\right)^{w_j},1-\prod_{j=1}^{n}\left(1-\overline{\nu}_{\sigma(j)}\right)^{w_j}\right]\right),\end{aligned}\tag{6}$$

where $\left(\sigma(1),\sigma(2),\cdots,\sigma(n)\right)$ is a permutation of $\left(1,2,\cdots,n\right)$, such that $\tilde{\alpha}_{\sigma(j-1)}\ge\tilde{\alpha}_{\sigma(j)}$ for all $j=2,\cdots,n$.

The IITFOWG operator has the following properties.

Theorem 1. (Commutativity).

$$\mathrm{IITFOWG}_{w}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)=\mathrm{IITFOWG}_{w}\left(\tilde{a}_1^*,\tilde{a}_2^*,\cdots,\tilde{a}_n^*\right),$$

where $\tilde{a}_j^*\left(j=1,2,\cdots,n\right)$ is any permutation of $\tilde{a}_j\left(j=1,2,\cdots,n\right)$.

Theorem 2. (Idempotency) If $\tilde{a}_j\left(j=1,2,\cdots,n\right)=\tilde{a}$ for all j, then

$$\mathrm{IITFOWG}_{w}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)=\tilde{a}.$$

From Definitions 6 and 7, we know that the IITFWG operator weights the interval intuitionistic trapezoidal fuzzy arguments while the IITFOWG operator weights the ordered positions of the interval intuitionistic trapezoidal fuzzy arguments instead of weighting the arguments themselves. Therefore, weights represent different aspects in both the IITFWG and IITFOWG operators. However, both the operators consider only one of them. To solve this drawback, in the following we shall propose an interval intuitionistic trapezoidal fuzzy hybrid geometric (IITFHG) operator.

Definition 8. An interval intuitionistic trapezoidal fuzzy hybrid geometric (IITFHG) operator of dimension n is a mapping IITFHG: $Q^n \to Q$, that has an associated vector $w=\left(w_1,w_2,\cdots,w_n\right)^T$ such that $w_j>0$ and $\sum_{j=1}^{n} w_j=1$. Furthermore,

$$\begin{aligned}&\text{IITFHG}_{\omega,w}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)\\&=\prod_{j=1}^{n}\left(\dot{\tilde{a}}_{\sigma(j)}\right)^{w_j}\\&=\left(\left[\prod_{j=1}^{n}\left(\dot{a}_{\sigma(j)}\right)^{w_j},\prod_{j=1}^{n}\left(\dot{b}_{\sigma(j)}\right)^{w_j},\prod_{j=1}^{n}\left(\dot{c}_{\sigma(j)}\right)^{w_j},\prod_{j=1}^{n}\left(\dot{d}_{\sigma(j)}\right)^{w_j}\right];\right.\\&\left.\left[\prod_{j=1}^{n}\dot{\underline{\mu}}_{\sigma(j)}^{w_j},\prod_{j=1}^{n}\dot{\bar{\mu}}_{\sigma(j)}^{w_j}\right],\left[1-\prod_{j=1}^{n}\left(1-\dot{\underline{v}}_{\sigma(j)}\right)^{w_j},1-\prod_{j=1}^{n}\left(1-\dot{\bar{v}}_{\sigma(j)}\right)^{w_j}\right]\right),\end{aligned} \tag{7}$$

where $\dot{\tilde{a}}_{\sigma(j)}$ is the j th largest of the weighted interval intuitionistic trapezoidal fuzzy numbers $\dot{\tilde{a}}_j\left(\dot{\tilde{a}}_j=\tilde{a}_j^{n\omega_j},\ j=1,2,\cdots,n\right)$, $\omega=\left(\omega_1,\omega_2,\cdots,\omega_n\right)^T$ be the weight vector of $\tilde{a}_j\left(j=1,2,\cdots,n\right)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$, and n is the balancing coefficient.

Theorem 3. The IITFWG operator is a special case of the IITFHG operator.
Proof. Let $w=\left(1/n,1/n,\cdots,1/n\right)$, then

$$\begin{aligned}&\text{IITFHG}_{\omega,w}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)=\prod_{j=1}^{n}\left(\dot{\tilde{a}}_{\sigma(j)}\right)^{w_j}=\prod_{j=1}^{n}\left(\dot{\tilde{a}}_{\sigma(j)}\right)^{\frac{1}{n}}\\&=\prod_{j=1}^{n}\left(\tilde{a}_{\sigma(j)}\right)^{\omega_j}=\text{IITFWG}_{\omega}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right).\end{aligned}$$

Which completes the proof of Theorem 3.

Theorem 4. The IITFOWG operator is a special case of the IITFHG operator.
Proof. Let $\omega=\left(1/n,1/n,\cdots,1/n\right)$, then $\dot{\tilde{a}}_j=\tilde{a}_j$, $i=1,2,\cdots,n$.

$$\text{IITFHG}_{\omega,w}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)=\prod_{j=1}^{n}\left(\dot{\tilde{a}}_{\sigma(j)}\right)^{w_j}$$

$$=\prod_{j=1}^{n}\left(\tilde{a}_{\sigma(j)}\right)^{w_j}=\text{IITFOWG}_w\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right).$$

This completes the proof of Theorem 4.

So we know that the IITFHG operator generalizes both the IITFWG and IITFOWG operators, and reflects the importance degrees of both the given arguments and their ordered positions.

4. An approach to multiple attribute group decision making with interval intuitionistic trapezoidal fuzzy information

In this section, we shall investigate the multiple attribute group decision making (MAGDM) problems based on the IITFWG and IITFHG operator in which both the attribute weights and the expert weights takes the form of real numbers, attribute values takes the form of interval intuitionistic trapezoidal fuzzy numbers.

Let $A=\{A_1,A_2,\cdots,A_m\}$ be a discrete set of alternatives, and $G=\{G_1,G_2,\cdots,G_n\}$ be the set of attributes, $\omega=\left(\omega_1,\omega_2,\cdots,\omega_n\right)$ is the weighting vector of the attributes $G_j\left(j=1,2,\cdots,n\right)$, where $\omega_j\in\left[0,1\right]$, $\sum_{j=1}^{n}\omega_j=1$. Let $D=\{D_1,D_2,\cdots,D_t\}$ be the set of decision makers, $\nu=\left(\nu_1,\nu_2,\cdots,\nu_n\right)$ be the weighting vector of decision makers, with $\nu_k\in\left[0,1\right]$, $\sum_{k=1}^{t}\nu_k=1$. Suppose that $\tilde{R}_k=\left(\tilde{r}_{ij}^{(k)}\right)_{m\times n}=\left(\left[a_{ij}^{(k)},b_{ij}^{(k)},c_{ij}^{(k)},d_{ij}^{(k)}\right];\tilde{\mu}_{ij}^{(k)},\tilde{\nu}_{ij}^{(k)}\right)_{m\times n}$ is the intuitionistic trapezoidal fuzzy decision matrix, where $\tilde{\mu}_{ij}^{(k)}$ indicates the degree that the alternative A_i satisfies the attribute G_j given by the decision maker D_k, $\tilde{\nu}_{ij}^{(k)}$ indicates the degree that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker D_k, $\tilde{\mu}_{ij}^{(k)}\subseteq\left[0,1\right]$, $\tilde{\nu}_{ij}^{(k)}\subseteq\left[0,1\right]$, $0\le\sup\tilde{\mu}_{ij}^{(k)}+\sup\tilde{\nu}_{ij}^{(k)}\le 1$, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$, $k=1,2,\cdots,t$.

In the following, we apply the IITFWG and IITFHG operators to multiple attribute group decision making with interval intuitionistic trapezoidal fuzzy information. The method involves the following steps:

Step 1. Utilizing the decision information given in the interval intuitionistic trapezoidal fuzzy decision matrix $\tilde{R}_k$, and the IITFWG operator

$$\tilde{r}_i^{(k)}=\left(\left[a_i^{(k)},b_i^{(k)},c_i^{(k)},d_i^{(k)}\right];\tilde{\mu}_i^{(k)},\tilde{\nu}_i^{(k)}\right)$$

$$=\text{IITFWG}_{\omega}\left(\tilde{r}_{i1}^{(k)},\tilde{r}_{i2}^{(k)},\cdots,\tilde{r}_{in}^{(k)}\right),i=1,2,\cdots,m,k=1,2,\cdots,t$$

to derive the individual overall interval intuitionistic trapezoidal fuzzy numbers $\tilde{r}_i^{(k)}$ of the alternative A_i.

Step 2. Utilizing the IITFHG operator:

$$\begin{aligned}\tilde{r}_i &= \left(\left[a_i, b_i, c_i, d_i\right]; \mu_i, v_i\right) \\ &= \mathrm{IITFHG}_{v,w}\left(\tilde{r}_i^{(1)}, \tilde{r}_i^{(2)}, \cdots, \tilde{r}_i^{(t)}\right), i = 1, 2, \cdots, m\end{aligned}$$

to derive the collective overall interval intuitionistic trapezoidal fuzzy numbers $\tilde{r}_i\ (i = 1, 2, \cdots, m)$ of the alternative A_i, where $v = (v_1, v_2, \cdots, v_n)$ is the weighting vector of decision makers, with $v_k \in [0,1]$, $\sum_{k=1}^{t} v_k = 1$; $w = (w_1, w_2, \cdots, w_n)$ is the associated weighting vector of the ITFHG operator, with $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$.

Step 3. Calculate the scores $S(\tilde{r}_i)\ (i = 1, 2, \cdots, m)$ of the collective overall interval intuitionistic trapezoidal fuzzy numbers $\tilde{r}_i\ (i = 1, 2, \cdots, m)$ to rank all the alternatives $A_i\ (i = 1, 2, \cdots, m)$ and then to select the best one(s) (if there is no difference between two scores $S(\tilde{r}_i)$ and $S(\tilde{r}_j)$, then we need to calculate the accuracy degrees $H(\tilde{r}_i)$ and $H(\tilde{r}_j)$ of the overall interval intuitionistic trapezoidal fuzzy numbers $\tilde{r}_i$ and $\tilde{r}_j$, respectively, and then rank the alternatives A_i and A_j in accordance with the accuracy degrees $H(\tilde{r}_i)$ and $H(\tilde{r}_j)$.

Step 4. Rank all the alternatives $A_i\ (i = 1, 2, \cdots, m)$ and select the best one(s) in accordance with $S(\tilde{r}_i)$ and $H(\tilde{r}_i)\ (i = 1, 2, \cdots, m)$.

Step 5. End.

5. Numerical example

Thus, in this section we shall present a numerical example to show potential evaluation of emerging technology commercialization with uncertain linguistic information in order to illustrate the method proposed in this paper. There is a panel with four possible emerging technology enterprises $A_i\ (i = 1, 2, 3, 4, 5)$ to select. The experts selects four attribute to evaluate the five possible emerging technology enterprises: ①G_1 is the technical advancement; ②G_2 is the potential market and market risk; ③G_3 is the industrialization infrastructure, human resources and financial conditions; ④G_4 is the employment creation and the development of science and technology. The five possible alternatives $A_i\ (i = 1, 2, \cdots, 5)$ are to be evaluated using the interval intuitionistic trapezoidal fuzzy numbers by the three decision makers (whose weighting vector $v = (0.35, 0.40, 0.25)^T$)) under the above four attributes (whose weighting vector $\omega = (0.2, 0.1, 0.3, 0.4)^T$), and three decision matrices are to be constricted as listed in the following matrices $\tilde{R}_k = \left(\tilde{r}_{ij}^{(k)}\right)_{5\times4}\ (k = 1, 2, 3)$, respectively:

$$\tilde{R}_1=\begin{bmatrix}
([0.5,0.6,0.7,0.8];[0.4,0.5],[0.3,0.4]) & ([0.1,0.2,0.3,0.4];[0.4,0.6],[0.2,0.4]) & ([0.5,0.6,0.8,0.9];[0.1,0.3],[0.5,0.6]) & ([0.4,0.5,0.6,0.7];[0.3,0.4],[0.3,0.5]) \\
([0.6,0.7,0.8,0.9];[0.6,0.7],[0.2,0.3]) & ([0.5,0.6,0.7,0.8];[0.6,0.7],[0.2,0.3]) & ([0.4,0.5,0.7,0.8];[0.4,0.7],[0.1,0.2]) & ([0.5,0.6,0.7,0.9];[0.5,0.6],[0.1,0.3]) \\
([0.1,0.2,0.4,0.5];[0.6,0.7],[0.1,0.2]) & ([0.2,0.3,0.5,0.6];[0.5,0.6],[0.3,0.4]) & ([0.5,0.6,0.7,0.8];[0.5,0.6],[0.1,0.3]) & ([0.3,0.5,0.7,0.9];[0.4,0.5],[0.2,0.4]) \\
([0.3,0.4,0.5,0.6];[0.2,0.3],[0.2,0.3]) & ([0.1,0.3,0.4,0.5];[0.6,0.7],[0.1,0.3]) & ([0.1,0.3,0.5,0.7];[0.3,0.4],[0.1,0.2]) & ([0.6,0.7,0.8,0.9];[0.3,0.7],[0.1,0.2]) \\
([0.2,0.3,0.4,0.5];[0.7,0.8],[0.1,0.2]) & ([0.3,0.4,0.5,0.6];[0.3,0.5],[0.1,0.3]) & ([0.2,0.3,0.4,0.5];[0.5,0.6],[0.2,0.3]) & ([0.5,0.6,0.7,0.8];[0.3,0.4],[0.5,0.6])
\end{bmatrix}$$

$$\tilde{R}_2=\begin{bmatrix}
([0.4,0.5,0.6,0.7];[0.3,0.4],[0.4,0.5]) & ([0.1,0.2,0.3,0.4];[0.5,0.6],[0.1,0.3]) & ([0.4,0.5,0.7,0.8];[0.4,0.5],[0.3,0.4]) & ([0.3,0.4,0.5,0.6];[0.4,0.6],[0.2,0.4]) \\
([0.5,0.6,0.7,0.8];[0.3,0.6],[0.3,0.4]) & ([0.4,0.5,0.6,0.7];[0.4,0.7],[0.1,0.2]) & ([0.3,0.4,0.6,0.7];[0.5,0.6],[0.2,0.3]) & ([0.4,0.5,0.6,0.8];[0.6,0.7],[0.2,0.3]) \\
([0.1,0.2,0.3,0.4];[0.6,0.8],[0.1,0.2]) & ([0.1,0.2,0.4,0.5];[0.5,0.6],[0.1,0.2]) & ([0.4,0.5,0.6,0.7];[0.5,0.7],[0.2,0.3]) & ([0.2,0.4,0.6,0.8];[0.1,0.3],[0.5,0.6]) \\
([0.2,0.3,0.4,0.5];[0.4,0.5],[0.3,0.5]) & ([0.1,0.2,0.3,0.5];[0.5,0.8],[0.1,0.2]) & ([0.1,0.2,0.4,0.6];[0.2,0.5],[0.3,0.4]) & ([0.5,0.6,0.7,0.8];[0.4,0.7],[0.1,0.2]) \\
([0.1,0.2,0.3,0.4];[0.6,0.7],[0.2,0.3]) & ([0.2,0.3,0.4,0.5];[0.6,0.7],[0.1,0.2]) & ([0.1,0.2,0.3,0.4];[0.5,0.7],[0.2,0.3]) & ([0.4,0.5,0.6,0.7];[0.6,0.7],[0.1,0.3])
\end{bmatrix}$$

$$\tilde{R}_3=\begin{bmatrix}
([0.6,0.7,0.8,0.9];[0.2,0.5],[0.3,0.4]) & ([0.2,0.3,0.4,0.5];[0.4,0.5],[0.1,0.2]) & ([0.6,0.7,0.9,1.0];[0.3,0.6],[0.2,0.3]) & ([0.5,0.6,0.7,0.8];[0.3,0.7],[0.1,0.3]) \\
([0.7,0.8,0.9,1.0];[0.2,0.7],[0.2,0.3]) & ([0.6,0.7,0.8,0.9];[0.3,0.6],[0.2,0.4]) & ([0.5,0.6,0.8,0.9];[0.4,0.7],[0.1,0.2]) & ([0.6,0.7,0.8,1.0];[0.5,0.8],[0.1,0.2]) \\
([0.2,0.3,0.5,0.6];[0.5,0.6],[0.3,0.4]) & ([0.3,0.4,0.6,0.7];[0.7,0.8],[0.1,0.2]) & ([0.6,0.7,0.8,0.9];[0.5,0.6],[0.2,0.3]) & ([0.4,0.6,0.8,1.0];[0.4,0.5],[0.3,0.4]) \\
([0.4,0.5,0.6,0.7];[0.3,0.6],[0.2,0.4]) & ([0.2,0.4,0.5,0.6];[0.4,0.6],[0.2,0.3]) & ([0.2,0.4,0.6,0.8];[0.1,0.4],[0.3,0.6]) & ([0.7,0.8,0.9,1.0];[0.3,0.7],[0.1,0.2]) \\
([0.3,0.4,0.5,0.6];[0.6,0.7],[0.1,0.3]) & ([0.4,0.5,0.6,0.7];[0.5,0.6],[0.3,0.4]) & ([0.3,0.4,0.5,0.6];[0.5,0.6],[0.2,0.3]) & ([0.6,0.7,0.8,0.9];[0.7,0.8],[0.1,0.2])
\end{bmatrix}$$

we utilize the proposed procedure to get the most desirable alternative(s).

Step 1. Utilize the decision information given in the interval intuitionistic trapezoidal fuzzy decision matrix $\tilde{R}_k$, and the IITFWG operator to derive the individual overall interval intuitionistic trapezoidal fuzzy values $\tilde{r}_i^{(k)}$ of the alternative A_i.

$$\tilde{r}_1^{(1)} = \left(\left[0.3893, 0.4993, 0.6294, 0.7331\right]; \left[0.2352, 0.3995\right], \left[0.3587, 0.5061\right]\right)$$

$$\tilde{r}_2^{(1)} = \left(\left[0.4850, 0.5858, 0.7189, 0.8586\right]; \left[0.4939, 0.6581\right], \left[0.1312, 0.2714\right]\right)$$

$$\tilde{r}_3^{(1)} = \left(\left[0.2696, 0.4178, 0.6052, 0.7417\right]; \left[0.4743, 0.5753\right], \left[0.1627, 0.3344\right]\right)$$

$$\tilde{r}_4^{(1)} = \left(\left[0.2551, 0.4460, 0.5901, 0.7257\right]; \left[0.3215, 0.5292\right], \left[0.1210, 0.2314\right]\right)$$

$$\tilde{r}_5^{(1)} = \left(\left[0.3005, 0.4074, 0.5116, 0.6145\right]; \left[0.4143, 0.5306\right], \left[0.3133, 0.4252\right]\right)$$

$$\tilde{r}_1^{(2)} = \left(\left[0.3104, 0.4173, 0.5451, 0.6478\right]; \left[0.3862, 0.5238\right], \left[0.2658, 0.4125\right]\right)$$

$$\tilde{r}_2^{(2)} = \left(\left[0.3837, 0.4850, 0.6188, 0.7584\right]; \left[0.4749, 0.6481\right], \left[0.2119, 0.3121\right]\right)$$

$$\tilde{r}_3^{(2)} = \left(\left[0.2000, 0.3474, 0.5016, 0.6384\right]; \left[0.2724, 0.5044\right], \left[0.3133, 0.4175\right]\right)$$

$$\tilde{r}_4^{(2)} = \left(\left[0.2187, 0.3366, 0.4862, 0.6373\right]; \left[0.3322, 0.5996\right], \left[0.2063, 0.3320\right]\right)$$

$$\tilde{r}_5^{(2)} = \left(\left[0.1866, 0.3005, 0.4074, 0.5116\right]; \left[0.5681, 0.7000\right], \left[0.1515, 0.2906\right]\right)$$

$$\tilde{r}_1^{(3)} = \left(\left[0.4998, 0.6047, 0.7331, 0.8356\right]; \left[0.2847, 0.6042\right], \left[0.1738, 0.3121\right]\right)$$

$$\tilde{r}_2^{(3)} = \left(\left[0.5858, 0.6865, 0.8191, 0.9587\right]; \left[0.3699, 0.7271\right], \left[0.1312, 0.2432\right]\right)$$

$$\tilde{r}_3^{(3)} = \left(\left[0.3821, 0.5253, 0.7076, 0.8441\right]; \left[0.4730, 0.5741\right], \left[0.2528, 0.3533\right]\right)$$

$$\tilde{r}_4^{(3)} = \left(\left[0.3792, 0.5519, 0.6929, 0.8275\right]; \left[0.2221, 0.5651\right], \left[0.1943, 0.3947\right]\right)$$

$$\tilde{r}_5^{(3)} = \left(\left[0.4074, 0.5116, 0.6145, 0.7166\right]; \left[0.5933, 0.6943\right], \left[0.1528, 0.2729\right]\right)$$

Step 2. Utilize the IITFHG operator to derive the collective overall interval intuitionistic trapezoidal fuzzy values $\tilde{r}_i \left(i = 1, 2, \cdots, m\right)$ of the alternative A_i (Let $w = \left(0.20, 0.50, 0.30\right)^T$).

$$\tilde{r}_1 = \left(\left[0.3318, 0.4414, 0.5732, 0.6802\right]; \left[0.2966, 0.4712\right], \left[0.2981, 0.4498\right]\right)$$

$$\tilde{r}_2 = \left(\left[0.4471, 0.5501, 0.6866, 0.8303\right]; \left[0.4549, 0.6547\right], \left[0.1653, 0.2902\right]\right)$$

$$\tilde{r}_3 = \left(\left[0.2437, 0.3924, 0.5689, 0.7090\right]; \left[0.3783, 0.5380\right], \left[0.2383, 0.3773\right]\right)$$

$$\tilde{r}_4 = \left(\left[0.2442, 0.4045, 0.5535, 0.6984\right]; \left[0.2958, 0.5467\right], \left[0.1674, 0.3014\right]\right)$$

$$\tilde{r}_5 = \left(\left[0.2186, 0.3313, 0.4392, 0.5458\right]; \left[0.4990, 0.6260\right], \left[0.2148, 0.3483\right]\right)$$

Step 3. Calculate the scores $S\left(\tilde{r}_i\right) \left(i = 1, 2, \cdots, 5\right)$ of the overall interval intuitionistic trapezoidal fuzzy numbers $\tilde{r}_i \left(i = 1, 2, \cdots, 5\right)$

$$S\left(\tilde{r}_1\right) = 0.0050, S\left(\tilde{r}_2\right) = 0.2056, S\left(\tilde{r}_3\right) = 0.0719, S\left(\tilde{r}_4\right) = 0.0887, S\left(\tilde{r}_5\right) = 0.1078 .$$

Step 4. Rank all the alternatives $A_i \left(i = 1, 2, 3, 4, 5\right)$ in accordance with the scores $S\left(\tilde{r}_i\right)$ $\left(i = 1, 2, \cdots, 5\right)$ of the interval intuitionistic trapezoidal fuzzy numbers $\tilde{r}_i \left(i = 1, 2, \cdots, 5\right)$: $A_2 \succ A_5 \succ A_4 \succ A_3 \succ A_1$, and thus the most desirable alternative is A_5.

6. Conclusion

In this paper, we investigate the multiple attribute group decision making (MAGDM) problems in which both the attribute weights and the expert weights take the form of real numbers, attribute values take the form of interval intuitionistic trapezoidal fuzzy numbers. Firstly, some operational laws of interval intuitionistic trapezoidal fuzzy numbers are introduced. Then, we have developed the interval intuitionistic trapezoidal fuzzy ordered weighted geometric (IITFOWG) operator and interval intuitionistic trapezoidal fuzzy hybrid geometric (IITFHG) operator. The IITFHG operator firstly weights the given arguments, and reorders the weighted arguments in descending order and weights these ordered arguments by the IITFHG weights, and finally aggregates all the weighted arguments into a collective one. Obviously, the IITFHG operator generalizes both the IITFWG and IITFOWG operators, and reflects the importance degrees of both the given argument and the ordered position of the argument. Furthermore, the IITFHG operator can relieve the influence of unfair arguments on the decision results by using the IITFHG weights to assign low weights to those "false" or "biased" ones. We have studied some desirable properties of these operators and applied the IITFWG and IITFHG operators to multiple attribute group decision making with interval intuitionistic trapezoidal fuzzy information. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness. In future research, our work will focus on the application of interval intuitionistic trapezoidal fuzzy multiple attribute group decision making in the fields such as investment, personnel examination, medical diagnosis, and military system efficiency evaluation.

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