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EDAS METHOD FOR MULTIPLE ATTRIBUTE GROUP DECISION MAKING WITH PROBABILISTIC DUAL HESITANT FUZZY INFORMATION AND ITS APPLICATION TO SUPPLIERS SELECTION

Baoquan NING[1,2], Rui LIN[3], Guiwu WEI[2,4], Xudong CHEN[5*]

[1]*School of Mathematics and Statistics, Liupanshui Normal University, 553004, Liupanshui, P.R. China*
[2]*School of Mathematical Sciences, Sichuan Normal University, Chengdu, P.R. China*
[3]*School of Economics and Management, Chongqing University of Arts and Sciences, Chongqing, China*
[4]*School of Business, Sichuan Normal University, Chengdu, P.R. China*
[5]*School of Accounting, Southwestern University of Finance and Economics, Chengdu, P.R. China*

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Abstract. Probabilistic dual hesitant fuzzy set (PDHFS) is a more powerful and important tool to describe uncertain information regarded as generalization of hesitant fuzzy set (HFS) and dual HFS (DHFS), not only reflects the hesitant attitude of decision-makers (DMs), but also reflects the probability information of DMs. Score function of fuzzy number and weighting method are very important in multi-attribute group decision-making (MAGDM) issues. In many fuzzy environments, the score function and entropy measure have been proposed one after another. Firstly, based on the detailed analysis of the existed score function of PDHF element (PDHFE) and with the help of previous references, we build a novel score function for PDHFE. Secondly, a combined weighting method is built based on the minimum identification information principle by fusing PDHF entropy and Criteria Importance Through Intercriteria Correlation (CRITIC) method. Thirdly, a novel PDHF MAGDM approach (PDHF-EDAS) is built by extending evaluation based on distance from average solution (EDAS) approach to the PDHF environment to solve the issue that the decision attribute information is PDHFE. Finally, the practicability and effectiveness of the PDHF MAGDM technique is verified by suppliers selection (SS) and comparing analysis with existing methods.

Keywords: multi-attribute group decision-making, probabilistic dual hesitant fuzzy set, EDAS method, entropy measure, CRITIC method, suppliers selection.

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*Corresponding author. E-mail: *xutung99@126.com*

Introduction

In the past few decades, some MAGDM issues generally use exact numbers to express decision information. However, with the increasing complexity of real MAGDM issues, the use of accurate numbers to express decision information has been far from real MAGDM, so it is more and more common to use fuzzy numbers to express decision information (Herrera & Martinez, 2000; Lu et al., 2021; M. Zhao et al., 2021b). Zadeh (1965) proposed the famous fuzzy set (FS) in 1965. Since it was proposed, FS and its extensions were applied to many uncertain MADM fields (William-West & Ciucci, 2021; H. Y. Zhang et al., 2022c; M. Zhao et al., 2022). Due to the needs of some special MAGDM issues, extended forms of FS have been put forward. Because FS can only express approval, and can only use one value to express the DM's approval attitude, it cannot express the DM's approval in multiple dimensions, so it has defects in dealing with some MAGDM problems. Some multi-dimensional FSs were built and extensively utilized in many fields (Liang et al., 2013; Lima et al., 2021; Pramanik et al., 2021). Although the FSs that have been built can well describe the situation that DMs approve, it is powerless to describe disapproval. Therefore, the intuitionistic FS (IFS) (Atanassov, 1986) and interval IFS (IVIFS) (Atanassov & Gargov, 1989) were built. M. Zhao et al. (2021) merged MABAC approach with IFS as well as CPT and a new MAGDM approach was built, finally, it was utilized to practical issues in IF environment. S. Zhang et al. (2021b) developed a MAGDM approach under IF environment though merging GRA approach and CPT and it was a MAGDM issue in IF environment. In the process of DMs dealing with some real MAGDM issues, DMs often hesitate between several exact values when expressing decision information. In order to better describe this situation, the famous hesitant FS (HFS) (Torra, 2010) was built, obviously, HFS is more practical in describing the decision information of DMs, some successful cases also prove this understanding (Narayanamoorthy et al., 2021). Although HFS has great advantages in dealing with MAGDM issues, we often think about a very practical problem, that is, the possibility of these values, Z. S. Xu and Zhou (2017) built the probabilistic HFS (PHFS). Liu et al. (2021) developed two models for determining probability of PHFE and the probability of risk status respectively. Krishankumar et al. (2021) extended the COPRAS technique to PHF setting. Liao et al. (2021) developed a MAGDM approach by merging MABAC and CPT to PHF setting and applied it to practical issue. HFS can describe the DM's hesitation between multiple values, but it cannot describe the DM's disapproval attitude, hence, the dual HFS (DHFS) was developed by Zhu, Xu, and Xia (2012). DHFS can better describe DM's hesitant attitude in several exact values, DHFS can clearly describe the DM's support and nonsupport attitude towards the decision-making issue, but after the DM gives the assessment information, the value of these assessment elements is still a problem worth discussing. Hao et al. (2017) built the PDHFS and PDHFE which has more unique advantages than existing FSs. Garg and Kaur (2021) extended the CODAS to the PDHF setting. Q. Zhao et al. (2020) developed the PROMETHEE-II approach with PDHF information. Garg and Kaur (2020b) defined the PDHF correlation coefficient for MAGDM issue. Ren et al. (2017) merged the TODIM approach with PDHFS. C. Zhang et al. (2021a) merged the MULTIMOORA approach with PDHFS. Garg and Kaur (2020a) merged the MSM operator with PDHFS. Ning et al. (2022) defined the PDHFEPGMSM and PDHFWEPGMSM operators under PDHFS.

Similar to other decision making method (Jiang et al., 2022b; D. Zhang et al., 2022; H. Zhang et al., 2022b), EDAS method is a novel decision-making method proposed by Keshavarz Ghorabaee et al. (2015). It changes the evaluation criterion from the extreme ideal solution (TOPSIS method (Hwang & Yoon, 1981; H. Y. Zhang et al., 2022d) and VIKOR method (Opricovic & Tzeng, 2004)) of advantages and disadvantages to the average solution with more practical significance. In the case of differences in multi sectoral objectives, the compromise idea is obviously more consistent with actual interests of the decision-making collective. Because of its remarkable practical value and a large number of applications have been implemented, and integrated into many fuzzy settings and played a very important role. Y. Huang et al. (2021) defined the improved EDAS method with PT. Yahya et al. (2021) proposed the frank aggregation operator-based EDAS approach for IF rough set, and applied it to evaluation of the small hydropower plant. H. M. Zhang (2020) proposed an extended EDAS Method for multivalued neutrosophic sets, the MADM method was applied to choose appropriate investment project for an investment company. Jiang et al. (2022a) built the EDAS approach with CPT for settling picture fuzzy MAGDM issue. Fan et al. (2020) gave a novel EDAS MCGDM model for SVTNS, and it was applied to the investment projects selection. Darko and Liang (2020) developed a modified EDAS approach and Hamacher operators for the q-rung orthopair FS, and the method was utilized to elect a MPPS. Li et al. (2019) proposed the EDAS MAGDM approach for picture fuzzy environment, and the method was applied to select an optimal emergency alternative for an emergency management center (EMC). Lei et al. (2022) merged the EDAS with PDHLS for MAGDM in PDHL environment. Su et al. (2022) merged the EDAS approach with PULS and a MAGDM approach was built. There are many applications of the EDAS method, which will not be listed here.

Although EDAS approach was successfully utilized in various fields and extended to many fuzzy environments, it fails to weight decision attributes, but the weighting method for attributes is crucial issue in MAGDM issue. Therefore, this study selects the widely used critical method and entropy weight method to weight decision attributes. CRITIC weighting method is proposed by Diakoulaki et al. (1995). It can better describe the relationship among all attributes and weight for attributes objectively. After years of development, it was widely used in the weighting of decision attributes. For example, Peng and Garg (2022) utilized the CRITIC to give the weight to intuitionistic fuzzy soft element, and applied it to the CCN cache placement strategy election. Haktanir and Kahraman (2021) utilized the CRITIC to give the weight to evaluation attributes, and applied it to blood testing for COVID-19. Wang et al. (2022) built the GRP and CRITIC method for PUL-MAGDM. Saraji et al. (2021) combined the CRITIC method and COPRAS Method and CRITIC-COPRAS method was applied to MADM. Shi et al. (2021) applied the CRITIC weighting method to comprehensive power quality evaluation method of microgrid. Zafar et al. (2021) applied entropy-CRITIC weight method to an effective blockchain evaluation system. There are many applications of the CRITIC method, which will not be listed here. Entropy plays a key role in describing the information of a fuzzy element. It utilizes the volatility of data to reflect the importance of data, the greater the volatility, the higher the information content, so as to give greater objective weight to the data, it is a very useful objective weighting method. Various kinds of fuzzy entropy have been put forward one after another, intuitionistic fuzzy entropy measures

(Rahimi et al., 2021), Pythagorean fuzzy entropy measures (Thao & Smarandache, 2019; T. T. Xu et al., 2020), hesitant fuzzy entropy measure (Anees et al., 2020), dual hesitant fuzzy entropy measure (H. M. Zhang, 2020) in the corresponding fuzzy environment, DHF entropy measure (Hao et al., 2017).

On the premise of literatures review, we found that only Hao et al. (2017) defined a entropy measure in PDHF environment and which was given with the help of auxiliary function, but the complex calculation process will be hindered in practical application. In order to compare two PDHFEs, the score function of two PDHFEs is also studied in literature (Hao et al., 2017), but we find that the proposed score function does not take into account the hesitant degree of PDHFE and completely ignores the role of hesitant degree, but in fact, hesitant degree will play a very key role in the score function, The research on these two issues will be discussed and studied in detail in Section 3. Meanwhile, considering the advantages of EDAS approach and a large number of successful applications, we think it is necessary to popularize and use it in the PDHF environment, and enrich the decision-making approach for PDHFS.

Finally, the important task of the study is to construct a novel MAGDM technique for by merging EDAS approach to PDHF setting. Firstly, aiming at the disadvantages of score function of PDHFE, we develop a novel score function on the premise of considering the importance of hesitant degree. Then we present a PDHF EDAS (PDHF-EDAS) approach for settling MAGDM issues. Meanwhile, we merge CRITIC approach and entropy weight approach to PDHF-EDAS method to get the weight of decision attributes reasonably. In addition, we apply the proposed PDHF MAGDM technique to SS to testify the practicability of PDHF-EDAS technique. Finally, the parameter and comparison analyses testify the adaptability and availability of PDHF-EDAS approach.

Some main motivations are shown as: (1) In more and more complex decision-making issues, how to effectively obtain the decision-making information in decision-making issues is a crucial issue. PDHFS can even more fully describe the assessment information of DMs. (2) Compared with other evaluation methods, EDAS approach has its unique superiorities, but the application of EDAS approach in PFHF environment is not available at present. (3) CRITIC method and entropy weight method are popular objective weighting methods for decision attribute weighting, which can fully reflect the correlation and volatility of decision attributes. (4) As an important MAGDM problem, scientific and reasonable SS is a very important topic, which is very important for the high-quality development of the company. In order to solve this problem, this study develops PDHF-EDAS method for MAGDM, and integrates critical method and entropy weight method into PDHF-EDAS approach to obtain the objective weight of decision attributes. (5) Finally, the parameter and comparison analyses testify the adaptability and availability of PDHF-EDAS approach.

The main contributions are shown as: (1) The CRITIC and entropy weight approaches are employed to capture the weights of attributes in PDHF MAGDM issue. (2) The PDHF-EDAS approach is built to solve MAGDM problem. (3) The newly constructed PDHF-EDAS approach in the PDHF environment is used to a practice case of SS to illustrate the applicability of PDHF-EDAS approach. (4) The parameter and comparison analyses testify the adaptability and availability of PDHF-EDAS approach. (5) The PDHF-EDAS approach built in this paper not only provides more decision-making methods for solving MAGDM issues in PFHF environment, but also provides more references for solving MAGDM issues in other fuzzy environments.

This article consists of the below sections: the Section 1 mainly reviews the PDHFS; Section 2 puts forward new score function and a method for comparing PDHFE, and an entropy measure for PDHFS; In Section 3, the PDHF entropy and CRITIC is used to obtain the weight; In Section 4, EDAS method and PDHFS are fused, and a MAGDM technique is proposed; In Section 5, the proposed MAGDM technique is used to suppliers selection, finally, the built technique is used to compare with the existing MAGDM approaches, and the superiorities of the built method are put forward though the sensitivity analysis of parameters; the last section summarizes this paper.

1. Preliminaries

Some basic conceptions and aggregation operator of PDHFS are shown in such section.

Definition 1 (Hao et al., 2017)**.** A PDHFS on a fixed set X is record as following form:

$$\Im = \left\{ \left\langle \ell, \hbar(\ell) \middle| \tau(\ell), \lambda(\ell) \middle| \upsilon(\ell) \right\rangle, \ell \in X \right\}, \tag{1}$$

where $\hbar(\ell)$ is MD and $\lambda(\ell)$ is NMD, the components $\hbar(\ell)|\tau(\ell)$ and $\lambda(\ell)|\upsilon(\ell)$ represent those elements in $\hbar(\ell)$ and $\lambda(\ell)$, $\tau(\ell)$ is probability set of $\hbar(\ell)$ and $\upsilon(\ell)$,

$$0 \le \gamma, \eta \le 1,\ 0 \le \gamma^+ + \eta^+ \le 1 \tag{2}$$

and

$$\tau_i \in [0,1],\ \upsilon_j \in [0,1],\ \sum_{i=1}^{\#} \tau_i = 1,\ \sum_{j=1}^{\#\lambda} \upsilon_j = 1, \tag{3}$$

where $\gamma \in \hbar(\ell)$, $\eta \in \lambda(\ell)$, $\gamma^+ \in \hbar^+(\ell) = \cup_{\gamma \in \hbar(x)} \max\{\gamma\}$, $\eta^+ \in \lambda^+(\ell) = \cup_{\eta \in \lambda(x)} \max\{\eta\}$, $\tau_i \in \tau(\ell)$ and $\upsilon_i \in \upsilon(\ell)$. $\#\hbar$ and $\#\lambda$ are the number of elements in $\hbar(\ell)|\tau(\ell)$ and $\lambda(\ell)|\upsilon(\ell)$, respectively. $\Im = \left\langle \hbar(\ell) \middle| \tau(\ell), \lambda(\ell) \middle| \upsilon(\ell) \right\rangle$ is named as PDHFE, record as $\Im = \langle \hbar | \tau, \lambda | \upsilon \rangle$ (Hao et al., 2017).

Under conditions $\sum_{i=1}^{\#\hbar} \tau_i < 1$ and $\sum_{j=1}^{\#\lambda} \upsilon_j < 1$, we normalize the PDHFS by Eq. (4):

$$\overline{\Im} = \left\{ \left\langle \ell, \hbar(\ell) \middle| \tau(\ell), \lambda(\ell) \middle| \upsilon(\ell) \right\rangle, \ell \in X \right\}, \tag{4}$$

where $\tau(\ell) = \tau_i \Big/ \sum_{i=1}^{\#\hbar} \tau_i$, $\upsilon(\ell) = \upsilon_j \Big/ \sum_{j=1}^{\#\lambda} \upsilon_j$.

Next, some operations of PDHFEs should be reviewed (Hao et al., 2017).

Let $\Im$, $\Im_1$ and $\Im_2$ be three PDHFEs, $\Im = \langle \hbar | \tau, \lambda | \upsilon \rangle$, $\Im_1 = \left\langle \hbar_1 \middle| \tau_{\hbar_1}, \lambda_1 \middle| \upsilon_{\lambda_1} \right\rangle$ and $\Im_2 = \left\langle \hbar_2 \middle| \tau_{\hbar_2}, \lambda_2 \middle| \upsilon_{\lambda_2} \right\rangle$, then some operations of PDHFEs are shown as (Hao et al., 2017):

(1) $\Im_1 \oplus \Im_2 = \cup_{\gamma_1 \in \hbar_1, \eta_1 \in \lambda_1, \gamma_2 \in \hbar_2, \eta_2 \in \lambda_2} \left\{ \left\{ \gamma_1 + \gamma_2 - \gamma_1\gamma_2 \middle| p_{\gamma_1} p_{\gamma_2} \right\}, \left\{ \eta_1\eta_2 \middle| q_{\eta_1} q_{\eta_2} \right\} \right\}$;

(2) $\Im_1 \otimes \Im_2 = \cup_{\gamma_1 \in \hbar_1, \eta_1 \in \lambda_1, \gamma_2 \in \hbar_2, \eta_2 \in \lambda_2} \left\{ \left\{ \gamma_1\gamma_2 \middle| p_{\gamma_1} p_{\gamma_2} \right\}, \left\{ \eta_1 + \eta_2 - \eta_1\eta_2 \middle| q_{\eta_1} q_{\eta_2} \right\} \right\}$;

(3) $\lambda\Im = \cup_{\gamma\in\hbar,\eta\in\lambda}\left\{\left\{\left(1-(1-\gamma)^{\lambda}\right)\middle|p_{\gamma}\right\},\left\{\eta^{\lambda}\middle|q_{\eta}\right\}\right\}$;

(4) $\Im^{\lambda} = \cup_{\gamma\in\hbar,\eta\in\lambda}\left\{\left\{\gamma^{\lambda}\middle|p_{\gamma}\right\},\left\{\left(1-(1-\eta)^{\lambda}\right)\middle|q_{\eta}\right\}\right\}$;

(5) $$\Im^{c} = \begin{cases} \bigcup_{\gamma_j\in\hbar_\Im,\eta_k\in\lambda_\Im}\left\langle\left\{\eta_k\middle|\upsilon_k\right\},\left\{\gamma_j\middle|\tau_j\right\}\right\rangle, & \text{if } \hbar_\Im\neq\phi \text{ and } \lambda_\Im\neq\phi\\ \bigcup_{\gamma_j\in\hbar_\Im}\left\langle\left\{1-\gamma_j\middle|\tau_j\right\},\{\phi\}\right\rangle, & \text{if } \hbar_\Im\neq\phi \text{ and } \lambda_\Im=\phi\\ \bigcup_{\eta_k\in\lambda_\Im}\left\langle\{\phi\},\left\{1-\eta_k\middle|\upsilon_k\right\}\right\rangle, & \text{if } \hbar_\Im=\phi \text{ and } \lambda_\Im\neq\phi \end{cases}.$$

In order to compare two PDHFEs, Hao et al. (2017) and Z. S. Xu and Zhou (2017) developed the score calculation formula and accuracy function of PDHFEs.

Definition 2 (Hao et al., 2017). $\Im = \left\langle\hbar|\tau,\lambda|\upsilon\right\rangle$ is a PDHFE, then the score function is:

$$s(\Im) = \sum_{i=1\,\gamma\in\hbar}^{\#\hbar}\hbar_i\cdot\tau_i - \sum_{j=1\,\gamma\in\lambda}^{\#\lambda}\lambda_j\cdot\upsilon_j. \tag{5}$$

Definition 3 (Z. S. Xu & Zhou, 2017). $\Im = \left\langle\hbar|\tau,\lambda|\upsilon\right\rangle$ is a PDHFE, the accuracy function is:

$$h(\Im) = \sum_{i=1\,\gamma\in\hbar}^{\#\hbar}\hbar_i\cdot\tau_i + \sum_{j=1\,\gamma\in\lambda}^{\#\lambda}\lambda_j\cdot\upsilon_j. \tag{6}$$

Some comparison approaches of PDHFEs are given as follows (Z. S. Xu & Zhou, 2017):

If $s(\Im_1) > s(\Im_2)$, then $\Im_1 > \Im_2$; On the contrary, there is $\Im_1 < \Im_2$. If $s(\Im_1) = s(\Im_2)$, then (1) If $h(\Im_1) < h(\Im_2)$, then $\Im_1 < \Im_2$; (2) If $h(\Im_1) = h(\Im_2)$, then $\Im_1 = \Im_2$.

The score function calculation formula in Definition 2 is expressed by the deviation between the mean values of MD and NMD, without considering the influence of hesitant degree. If the two PDHFEs $\Im_1$ and $\Im_2$ have different deviation of mean of MD and NMD, but the hesitant degree is very different, which may lead to great problems, because the hesitant degree largely shows the high uncertainty and risk level of DMs. In the next section, we will propose a novel score function based on D_α operator (Atanassov, 1989) and parameter determination method (M. J. Huang & Li, 2013), it fully considers the hesitation of DMs, and compared with the existing methods for comparing two PDHFEs, it can better compare two PDHFEs.

Hao et al. (2017) defined the PDHF weighted averaging (PDHFWA) operator.

Definition 4 (Hao et al., 2017). Let $\Im_i = \left\langle\hbar_i|\tau_{h_i},\lambda_i|\upsilon_{g_i}\right\rangle (i=1,2,\cdots,n)$ be n PDHFEs and their weight be $\omega = (\omega_1,\omega_2,\cdots,\omega_n)$ with $\omega_j\in[0,1]$ and $\sum_{j=1}^{n}\omega_j = 1$. Then the PDHFWA operator is defined as:

$$PDHFWA(\Im_1,\Im_2,\cdots,\Im_n,) = \mathop{\oplus}_{j=1}^{n}\omega_j\Im_j = \bigcup_{\substack{\gamma_1\in\hbar_1,\gamma_2\in\hbar_2,\cdots,\gamma_n\in\hbar_n\\ \eta_1\in\lambda_1,\eta_2\in\lambda_2,\cdots,\eta_n\in\lambda_n}}\left\{\left\{\left(1-\prod_{j=1}^{n}(1-\gamma_j)^{\omega_j}\right)\middle|\prod_{j=1}^{n}p_{\gamma_j}\right\},\left\{\left(\prod_{j=1}^{n}\eta_j^{\omega_j}\right)\middle|\prod_{j=1}^{n}q_{\eta_j}\right\}\right\}. \tag{7}$$

2. The novel score function and entropy measure for PDHFE

In the section, we propose a fresh score function depend on D_α operator (Atanassov, 1989) and parameter determination method (M. J. Huang & Li, 2013), it totally thinks over the hesitant degree of DMs, and compared with the existing methods for comparing two PDHFEs, it can better compare two PDHFEs.

In Section 2, we fully analyze the influence of hesitant degree on the score function. In order to better reflect the importance of hesitation, we built a novel score function of PDHFE, which fully reflects the influence of hesitant degree on the novel score function. Next, we will analyze the new score function proposed in this study in combination with D_α operator (Atanassov, 1989) and parameter determination method (M. J. Huang & Li, 2013).

On the one hand, we should pay more attention to negative information in some decision-making issues. Such as, when we buy a product, when we find some negative information about the product, even if there is a lot of good information about the product, we will treat the product carefully. Therefore, we should pay more attention to negative information in decision-making. In such study, we take the NMD of PDHFS as negative information and give it a more crucial position in PDHFEs.

On the other hand, in Section 2, we have analyzed the important role of hesitant degree in the calculation of score function for PDHFE. In the above example, the score function of PDHFE $\Im_2$ is the largest. Among the three PDHFEs $\Im_1$, $\Im_2$ and $\Im_3$, $\Im_2$ is better than the other two. Obviously, the importance of hesitant degree is not considered in the comparison process of the three PDHFEs. For the sake of reflecting the importance of hesitant degree, because the PDHFE is composed of MD and NMD, we consider assigning one part of hesitant degree to MD and the other part to NMD.

Atanassov (1989) gave an improved method of hesitant degree in intuitionistic fuzzy set and proposed D_α operator.

Definition 5 (Atanassov, 1989)**.** Let $\alpha \in [0,1]$ be a fixed number, and X be a fixed set. For an IFS $\mathrm{H} = \left\{ \left\langle \ell, \mu_{\mathrm{H}}(\ell), \nu_{\mathrm{H}}(\ell) \right\rangle \middle| \ell \in X \right\}$, the D_α operator is shown as: $D_\alpha(\mathrm{H}) = \left\{ \left\langle \ell, \mu_{\mathrm{H}}(\ell) + \alpha \cdot \pi_{\mathrm{H}}(\ell), \nu_{\mathrm{H}}(\ell) + (1-\alpha) \cdot \pi_{\mathrm{H}}(\ell) \right\rangle \middle| \ell \in X \right\}$, where $\pi_{\mathrm{H}}(\ell)$ is the hesitant degree of $\ell \in X$ to the set H.

D_α operator divides the hesitant degree into two parts. One part of the degree hesitant is allocated to the MD and the other part is allocated to the NMD. In D_α operator, α is a very important parameter, which can determine how many hesitant degrees are allocated to the MD and how many to the NMD.

M. J. Huang and Li (2013) developed a formula for determining α on the basis of reference (Atanassov, 1989) as following:

$$\alpha = \frac{1}{2} + \frac{\mu_{\mathrm{H}}(\ell) - \nu_{\mathrm{H}}(\ell)}{2} + \frac{\mu_{\mathrm{H}}(\ell) - \nu_{\mathrm{H}}(\ell)}{2} \pi_{\mathrm{H}}(\ell). \tag{8}$$

From the form of the above formula, we can see that when the number of votes supported is more than the number of votes against, that is, the greater the $\mu_{\mathrm{H}}(\ell) - \nu_{\mathrm{H}}(\ell) > 0$, the greater the hesitation part assigned to the mean of MD, and the greater the α; When the number of support votes is less than the number of opposition votes, that is, the smaller the $\mu_H(\ell)-\nu_H(\ell)<0$, the smaller the hesitation part assigned to the mean of MD, and the smaller the α.

Let $\Im=\langle \hbar|\tau,\lambda|\upsilon\rangle$ be a PDHFE, $\hbar$ and λ take values in interval [0,1]. We call $\sum_{j=1}^{\#\hbar}\hbar_j\tau_j$ the mean of the MD of the PDHFE $\Im$, $\sum_{j=1}^{\#\lambda}\lambda_j\upsilon_j$ the mean of the NMD of the PDHFE $\Im$, and $1-\sum_{j=1}^{\#\hbar}\hbar_j\tau_j-\sum_{j=1}^{\#\lambda}\lambda_j\upsilon_j$ the mean of the hesitant degree of the PDHFE $\Im$, then the mean of the MD and the mean of the NMD constitute a PDHFE. Therefore, we use the idea of D_α operator (Atanassov, 1989) and the method of determining parameter α in M. J. Huang and Li (2013) to improve the hesitant degree in PDHFS, and put the NMD in α more important position. After the above analysis, we built a novel score function for PDHFE.

Definition 6. Let $\Im=\langle \hbar|\tau,\lambda|\upsilon\rangle$ be a PDHFE, then the novel score function is defined:

$$s_{D_\alpha}(\Im)=\frac{1+s_\hbar(\Im)-s_\lambda(\Im)}{2}, \tag{9}$$

where $s_\hbar(\Im)=\gamma(\Im)+\alpha\pi_\Im$, $s_\lambda(\Im)=\eta(\Im)+(1-\alpha)\pi_\Im$, $\pi_\Im=1-\gamma(\Im)-\eta(\Im)$, $\gamma(\Im)=\sum_{j=1}^{\#\hbar}\hbar_j\tau_j$, $\eta(\Im)=\sum_{j=1}^{\#\lambda}\lambda_j\upsilon_j$ and $\alpha=\frac{1}{2}+\frac{\gamma(\Im)-\eta(\Im)}{2}+\frac{\gamma(\Im)-\eta(\Im)}{2}\pi_\Im$.

From the $s_{D_\alpha}(\Im)$, the value range of $s_{D_\alpha}(\Im)$ is $[0,1]$. For two PDHFEs $\Im_1$ and $\Im_2$, if $s_{D_\alpha}(\Im_1)>s_{D_\alpha}(\Im_2)$, then $\Im_1$ is superior to $\Im_2$ and is recorded as $\Im_1\succ\Im_2$.

From the above Definition 6, the mean pf the hesitant $\pi_\Im$ of PDHFE $\Im$ is divided into two parts through parameter α, one part is assigned to the mean of the MD of the PDHFE $\Im$, and the other part is assigned to the mean of the NMD of the PDHFE $\Im$. According to Definition 2, $s(\Im)=\sum_{i=1\,\gamma\in\hbar}^{\#\hbar}\hbar_i\cdot\tau_i-\sum_{j=1\,\gamma\in\lambda}^{\#\lambda}\lambda_j\cdot\upsilon_j$ is a score function of PDHFE $\Im$. So $\alpha=\frac{1}{2}+\frac{s(\Im)}{2}+\frac{s(\Im)}{2}\pi_\Im$ The value of α varies not only with the means of MD and NMD, but also with the hesitant degree $\pi_\Im$. If $s(\Im)>0$, it increases, and if $s(\Im)<0$, it decreases.

Therefore, the novel score function of PDHFE fully reflects the hesitant degree of PDHFE. In addition, we find that with the increasing of the NMD of PDHFE, the value of novel score function of the PDHFE will become smaller. Thus, the novel score function for PDHFE can better reflect the value of NMD. Next, let's make a detailed analysis.

Example 1. Let $\Im_1=\langle\{0.3|0.5,0.4|0.5\},\{0.1|0.5,0.2|0.5\}\rangle$, $\Im_2=\langle\{0.6|1\},\{0.3|0.5,0.4|0.5\}\rangle$ and $\Im_3=\langle\{0.5|0.5,0.6|0.5\},\{0.3|0.5,0.4|0.5\}\rangle$ be three PDHFEs, then score function values of $\Im_1$, $\Im_2$ and $\Im_3$ by the novel score function can be calculated as:

From the Definition 6, we can calculate the mean of hesitant degree of $\Im_1$ is $\pi_{\Im_1}=0.5$, then $\alpha_{\Im_1}=0.65$. Thus, $s_\hbar(\Im_1)=0.675$ and $s_\lambda(\Im_1)=0.325$, therefore, $s_{D_\alpha}(\Im_1)=0.675$.

In like manner, we can get $\pi_{\Im_2}=0.05$, then $\alpha_{\Im_2}=0.6313$. Thus, $s_\hbar(\Im_2)=0.6316$ and $s_\lambda(\Im_2)=0.3684$, therefore, $s_{D_\alpha}(\Im_2)=0.6316$;

$\pi_{\Im_3}=0.1$, then $\alpha_{\Im_3}=0.61$. Thus, $s_\hbar(\Im_3)=0.611$ and $s_\lambda(\Im_3)=0.389$, therefore, $s_{D_\alpha}(\Im_3)=0.611$. Obviously, $s_{D_\alpha}(\Im_1)>s_{D_\alpha}(\Im_2)>s_{D_\alpha}(\Im_3)$, we get the rank of three PDHFEs is $\Im_1\succ\Im_2\succ\Im_3$.

The rank is different from Example 1, let's make a detailed analysis.

(1) Since $\eta(\Im_2)=\eta(\Im_3)=0.35$, we can find the mean values of NMD of $\Im_2$ and $\Im_3$ are same. But $\gamma(\Im_2)=0.6$ and $\gamma(\Im_3)=0.55$, so $\gamma(\Im_2)>\gamma(\Im_3)$, therefore $\Im_2 > \Im_3$ in Examples 1 and 2. The result is in line with people's cognition.

(2) We can get the $\Im_1 \succ \Im_3$ by the novel score function, while the result is $\Im_3 \succ \Im_1$ by the compare method in the Definition 2. We can see that in Definition 2, if we only use compare $\Im_1$ and $\Im_3$ by the score function, then $\Im_1 = \Im_3$, so we need to consider the values of accuracy function of $\Im_1$ and $\Im_3$ to compare the size of $\Im_1$ and $\Im_3$. This is because the role of hesitant degree in the comparison process is not taken into account when comparing $\Im_1$ and $\Im_3$ according to Definition 2. We compared with the comparing method proposed for two PDHFEs in Definition 2, the method built in this study fully considers the role of hesitant degree in the comparison of two PDHFEs, and assigns the mean of hesitant degree to the mean value of MD and NMD through α. The biggest superiority of the comparison method built in this study is that it can directly compare two PDHFEs by using the novel score function, which is simpler and more direct than the comparison method defined 2.

In the two comparison methods of Definitions 2 and 6, we can see from the results of Examples 1 and 2 that the sequencing of $\Im_1$ and $\Im_3$ is different. Although the mean deviation of MD and NMD of $\Im_1$ and $\Im_3$ is the same, according to the calculation process, we can see that the mean of MD of $\Im_1$ is assigned a larger mean value of hesitation, while $\Im_3$ is assigned a smaller mean value of hesitation, so the score function of a is larger, sort higher.

The changes of the new score function with the change of the means of MD and NMD of PDHFE $\Im$ will be analyzed in more depth.

In the light of the $s_{D_\alpha}(\Im)$ in Definition 6, we can simplify the novel score function as follows:

$$s_{D_\alpha}(\Im)=\frac{1+s_{\hbar}(\Im)-s_{\lambda}(\Im)}{2}=\frac{1+(2-2\alpha)\gamma(\Im)-2\alpha\eta(\Im)+2\alpha-1}{2}.$$

Since $\alpha=\frac{1}{2}+\frac{s(\Im)}{2}+\frac{s(\Im)}{2}\pi_{\Im}$, so $0\le\alpha\le1$.

From the simplified form of the new scoring function, we can get the following results: (1) $s_{D_\alpha}(\Im)$ decreases with the increase of $\gamma(\Im)$, therefore, $s_{D_\alpha}(\Im)$ is a monotonically decreasing function of $\gamma(\Im)$. (2) $s_{D_\alpha}(\Im)$ increases with the increase of $\eta(\Im)$, therefore, $s_{D_\alpha}(\Im)$ is a monotonically increasing function of $\eta(\Im)$.

From the above analysis, the novel score function is more in line with people's cognition:

When the MD of a PDHFE is larger than the NMD, it is obvious that its score function should be larger; Conversely, it shows that MD is smaller than NMD, its score function is also smaller.

For $s_{D_\alpha}(\Im)$, we observe that when $\alpha=\frac{1}{2}$, the novel score function of PDHFE is the same as that in Definition 2. Therefore, the $s_{D_\alpha}(\Im)$ is a expansion form of the already existing that in Definition 2, which is more available and direct in comparing two PDHFEs.

Entropy measure plays an important role in decision attribute weighting in uncertain MADM. We have introduced the fuzzy entropy in each fuzzy environment in detail in the introduction. In this section, we defined a fresh PDHF entropy with the aid of the score function built in Section 3.1.

In a MADM problem, it is assumed that there are m alternatives $X=\{X_1,X_2,\cdots,X_m\}$ with n decision attributes $C=\{C_1,C_2,\cdots,C_n\}$, which construct a decision matrix $\mathrm{M}=\left(\Im_{ij}\right)_{m\times n}$, where $\Im_{ij}=\left\langle \hbar_{ij}\middle|\tau_{ij},\lambda_{ij}\middle|\upsilon_{ij}\right\rangle$. Then the PDHF entropy measure is built:

$$E_j=-\frac{1}{\ln m}\sum_{i=1}^{m}s\left(\Im_{ij}\right)\ln s\left(\Im_{ij}\right), \tag{10}$$

where $s\left(\Im_{ij}\right)=\begin{cases}\dfrac{s_{D_\alpha}\left(\Im_{ij}\right)-\min\limits_{1\le i\le m}\left(s_{D_\alpha}\left(\Im_{ij}\right)\right)}{\max\limits_{1\le i\le m}\left(s_{D_\alpha}\left(\Im_{ij}\right)\right)-\min\limits_{1\le i\le m}\left(s_{D_\alpha}\left(\Im_{ij}\right)\right)} & s_{D_\alpha}\left(\Im_{ij}\right)\in B\\ \dfrac{\max\limits_{1\le i\le m}\left(s_{D_\alpha}\left(\Im_{ij}\right)\right)-s_{D_\alpha}\left(\Im_{ij}\right)}{\max\limits_{1\le i\le m}\left(s_{D_\alpha}\left(\Im_{ij}\right)\right)-\min\limits_{1\le i\le m}\left(s_{D_\alpha}\left(\Im_{ij}\right)\right)} & s_{D_\alpha}\left(\Im_{ij}\right)\in C\end{cases}$

and $s\left(\Im_{ij}\right)=0$, then $s\left(\Im_{ij}\right)\ln s\left(\Im_{ij}\right)=0$.

3. A approach to obtain combined weight

3.1. Determine objective weight based on CRITIC and entropy weight methods

Diakoulaki et al. (1995) proposed CRITIC method for computing objective weight. It is mainly determined by two factors, one is standard deviation, which reflects the variation degree of the stated attribute; the other is correlation coefficient, if there is strong positive correlation between two stated attributes, it indicates two indexes have low conflict, if there is a strong negative correlation, it means that the two indicators have high conflict. In the below section, we merge the CRITIC method to PDHF environment and proposed the entropy weight method for PDHFE.

Let $\Im_{ij}$ $\left(i=1,2,\cdots,m;j=1,2,\cdots,n\right)$ be PDHF assessment value of ith alternative under jth attribute, η_j represents the entropy weight of jth attribute and ω_j represents the objective weight of jth attribute obtained by CRITIC method, C represents the set of all cost attributes and B represents the set of all benefit attributes. Next, the calculation process of PDHF objective weight ω_j and η_j of each attribute.

3.1.1. Calculating steps of the CRITIC method

Step 1. Calculate $S=\left(s_{D_\alpha}\left(\Im_{ij}\right)\right)_{m\times n}$ of each PDHFE by Eq. (11):

$$s_{D_\alpha}\left(\Im_{ij}\right)=\frac{1+s_{\hbar}\left(\Im_{ij}\right)-s_{\lambda}\left(\Im_{ij}\right)}{2}. \tag{11}$$

Step 2. Convert the score function decision matrix S into the normalized score function decision matrix $Q=\left(s_{ij}\right)_{m\times n}$ by Eq. (12):

$$s_{ij} = \begin{cases} \dfrac{s_{D_\alpha}\left(\Im_{ij}\right) - \min\limits_{1\le i\le m}\left(s_{D_\alpha}\left(\Im_{ij}\right)\right)}{\max\limits_{1\le i\le m}\left(s_{D_\alpha}\left(\Im_{ij}\right)\right) - \min\limits_{1\le i\le m}\left(s_{D_\alpha}\left(\Im_{ij}\right)\right)} & s_{D_\alpha}\left(\Im_{ij}\right) \in B \\ \dfrac{\max\limits_{1\le i\le m}\left(s_{D_\alpha}\left(\Im_{ij}\right)\right) - s_{D_\alpha}\left(\Im_{ij}\right)}{\max\limits_{1\le i\le m}\left(s_{D_\alpha}\left(\Im_{ij}\right)\right) - \min\limits_{1\le i\le m}\left(s_{D_\alpha}\left(\Im_{ij}\right)\right)} & s_{D_\alpha}\left(\Im_{ij}\right) \in C \end{cases}, \tag{12}$$

where B and C indicates the set of beneficial and cost attributes, respectively.

Step 3. Compute the standard deviation of jth attribute by Eq. (13):

$$\sigma_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(s_{ij} - \overline{s_j}\right)^2}, \left(j = 1,2,\cdots,n\right), \tag{13}$$

where $\overline{s_j} = \frac{1}{m}\sum_{i=1}^{m} s_{ij}\left(j = 1,2,\cdots,n\right)$.

Step 4. Obtain the correlation coefficient between jth attribute and kth attribute by Eq. (14):

$$\rho_{jk} = \frac{\sum_{i=1}^{m}\left(s_{ij} - \overline{s_j}\right)\left(s_{ik} - \overline{s_k}\right)}{\sqrt{\sum_{i=1}^{m}\left(s_{ij} - \overline{s_j}\right)^2 \sum_{i=1}^{m}\left(s_{ik} - \overline{s_k}\right)^2}}. \tag{14}$$

Step 5. Calculate the value of influence degree for each attribute as follows:

$$c_j = \sigma_j \sum_{k=1}^{n}\left(1 - \rho_{jk}\right), j = 1,2,\cdots,n. \tag{15}$$

Step 6. Determine ω_j by the following equation:

$$\omega_j = \frac{c_j}{\sum_{j=1}^{n} c_j}. \tag{16}$$

3.1.2. Calculating steps of the entropy weight method

Step 1. Compute the score function values $s_{D_\alpha}\left(\Im_{ij}\right)$ of every element $\Im_{ij}$ by Eq. (11).

Step 2. Compute the total entropy for each attribute by Eq. (17):

$$E_j = -\frac{1}{\ln m}\sum_{i=1}^{m} s_{ij}\ln s_{ij} \tag{17}$$

and if $s_{ij} = 0$, then $s_{ij}\ln s_{ij} = 0$.

Step 3. Compute η_j by Eq. (18):

$$\eta_j = \frac{1 - E_j}{\sum_{j=1}^{n}\left(1 - E_j\right)}. \tag{18}$$

3.2. Determine combined weights

Assume $w=\left(w_1,w_2,\cdots,w_n\right)$, where $\sum_{j=1}^{n} w_j=1$, $0\le w_j\le 1$. The objective weights $\omega=\left\{\omega_1,\omega_2,\cdots,\omega_n\right\}$ and $\eta=\left\{\eta_1,\eta_2,\cdots,\eta_n\right\}$ are calculated by Eqs (16)–(18), where $\sum_{j=1}^{n}\omega_j=1$, $0\le\omega_j\le 1$ and $\sum_{j=1}^{n}\eta_j=1$, $0\le\eta_j\le 1$. To let the combined weight reflects each weighting method, $\varpi=\left(\varpi_1,\varpi_2,\cdots,\varpi_n\right)$, $\omega=\left\{\omega_1,\omega_2,\cdots,\omega_n\right\}$, $\eta=\left\{\eta_1,\eta_2,\cdots,\eta_n\right\}$ and $w=\left(w_1,w_2,\cdots,w_n\right)$ should be as close as possible, we can get:

$$\begin{cases}\min F=\sum_{j=1}^{n}\varpi_j\ln\frac{\varpi_j}{\omega_j}+\sum_{j=1}^{n}\varpi_j\ln\frac{\varpi_j}{\eta_j}+\sum_{j=1}^{n}\varpi_j\ln\frac{\varpi_j}{w_j}\\ s.t.\ \ \sum_{j=1}^{n}\varpi_j=1,\varpi_j\ge 0\end{cases}. \tag{19}$$

According to the Lagrange multiplier method, ϖ_j is computed by Eq. (20):

$$\varpi_j=\frac{\sqrt{\omega_j\eta_j w_j}}{\sum_{j=1}^{n}\sqrt{\omega_j\eta_j w_j}}, \tag{20}$$

which can reflect both subjective and objective information.

4. EDAS technique for PDHF-MAGDM issues

The following is a basic description of a PDHF-MAGDM problem. Let $X=\left\{X_1,X_2,\cdots,X_m\right\}$ be a group of alternatives, and $C=\left\{C_1,C_2,\cdots,C_n\right\}$ be decision attributes with the weight $\varpi=\left\{\varpi_1,\varpi_2,\cdots,\varpi_n\right\}$, where $\varpi_j\in\left[0,1\right], j=1,2,\cdots,n$, $\sum_{j=1}^{n}\varpi_j=1$, and $e=\left\{e_1,e_2,\cdots,e_p\right\}$ be a group of experts, whose weight is $\theta=\left\{\theta_1,\theta_2,\cdots,\theta_p\right\}$, where $\theta_k\in\left[0,1\right], k=1,2,\cdots,p$, $\sum_{k=1}^{p}\theta_k=1$. Suppose a MAGDM problem has *n* attributes $C=\left\{C_1,C_2,\cdots,C_n\right\}$, furthermore, each expert gives the evaluation value as PDHFs $\Im_{ij}^k$, and Figure 1 shows the flowchart of the proposed PDHF-MAGDM technique. Next, we will apply the built MAGDM technique named as PDHF-EDAS to MAGDM problem with PDHF information, the MAGDM technique includes the below steps:

Step 1. Obtain decision matrix $\Im=\left(\Im_{ij}^k\right)_{m\times n}\left(k=1,2,\cdots,p\right)$ depend on *p* experts.

Step 2. Obtain the final collective matrix $D=\left(d_{ij}\right)_{m\times n}$ by the Eq. (21):

$$d_{ij}=\mathop{\oplus}_{k=1}^{p}\theta_k\Im_{ij}^k=\bigcup_{\gamma_{ij}^k\in\hbar_{ij}^k,\eta_{ij}^k\in\lambda_{ij}^k}\left\{\left\{\left(1-\prod_{k=1}^{p}\left(1-\gamma_{ij}^k\right)^{\theta_k}\middle|\prod_{k=1}^{p}p_{\gamma_{ij}^k}\right)\right\},\left\{\left(\prod_{k=1}^{p}\left(\eta_{ij}^k\right)^{\theta_k}\middle|\prod_{k=1}^{p}q_{\eta_{ij}^k}\right)\right\}\right\}. \tag{21}$$

Step 3. Convert the PDHF matrix into the normalized PDHF decision-making matrix $N=\left(n_{ij}\right)_{m\times n}=\left(d_{ij}^C\right)_{m\times n}$ by the complement operation.

Step 4. Compute the score matrix $S=\left(s_{D_\alpha}\left(n_{ij}\right)\right)_{m\times n}$ by Eq. (11).

Step 5. Transform the matrix S into the normalized score matrix $Q=\left(s_{ij}\right)_{m\times n}$ by Eq. (22):

$$s_{ij}=\begin{cases}\dfrac{s_{D_\alpha}\left(n_{ij}\right)-\min\limits_{1\le i\le m}\left(s_{D_\alpha}\left(n_{ij}\right)\right)}{\max\limits_{1\le i\le m}\left(s_{D_\alpha}\left(n_{ij}\right)\right)-\min\limits_{1\le i\le m}\left(s_{D_\alpha}\left(n_{ij}\right)\right)} & s_{D_\alpha}\left(n_{ij}\right)\in B\\ \dfrac{\max\limits_{1\le i\le m}\left(s_{D_\alpha}\left(n_{ij}\right)\right)-s_{D_\alpha}\left(n_{ij}\right)}{\max\limits_{1\le i\le m}\left(s_{D_\alpha}\left(n_{ij}\right)\right)-\min\limits_{1\le i\le m}\left(s_{D_\alpha}\left(n_{ij}\right)\right)} & s_{D_\alpha}\left(n_{ij}\right)\in C\end{cases}. \tag{22}$$

Step 6. Compute the combined weight ϖ by Eq. (20).

Step 7. Determine the average solution *PDHFAV* in the light of all attributes by Eq. (23):

$$PDHFAV=\left[PDHFAV_j\right]_{1\times n}, \tag{23}$$

where $PDHFAV_j=\dfrac{\sum_{i=1}^{m}s_{ij}}{m}$.

Step 8. Obtain the positive and negative distances from average (*PDHFPDA*) and (*PDHFNDA*) matrixes in the light of the type of attribute and shown as:

$$\begin{cases}PDHFPDA=\left[PDHFPDA_{ij}\right]_{m\times n}\\ PDHFNDA=\left[PDHFNDA_{ij}\right]_{m\times n}\end{cases} \tag{24}$$

if jth attribute is benefit,

$$\begin{cases}PDHFPDA_{ij}=\dfrac{\max\left(0,\left(s_{ij}-PDHFAV_j\right)\right)}{PDHFAV_j}\\ PDHFNDA_{ij}=\dfrac{\max\left(0,\left(PDHFAV_j-s_{ij}\right)\right)}{PDHFAV_j}\end{cases} \tag{25}$$

and if jth attribute is cost,

$$\begin{cases}PDHFPDA_{ij}=\dfrac{\max\left(0,\left(PDHFAV_j-s_{ij}\right)\right)}{PDHFAV_j}\\ PDHFNDA_{ij}=\dfrac{\max\left(0,\left(s_{ij}-PDHFAV_j\right)\right)}{PDHFAV_j}\end{cases}. \tag{26}$$

Step 9. Determine the weighted sum of $PDHFSP_i$ and $PDHFSN_i$ by Eq. (27):

$$\begin{cases}PDHFSP_i=\sum_{j=1}^{n}\varpi_j PDHFPDA_{ij}\\ PDHFSN_i=\sum_{j=1}^{n}\varpi_j PDHFNDA_{ij}\end{cases}, \tag{27}$$

where ϖ_j is jth attribute' weight.

Step 10. Compute the normalize values of $PDHFNSP_i$ and $PDHFNSN_i$ as follows:

$$\begin{cases} PDHFNSP_i = \dfrac{PDHFSP_i}{\max\left(PDHFSP_i\right)} \\ PDHFNSN_i = 1 - \dfrac{PDHFSN_i}{\max\left(PDHFSN_i\right)} \end{cases}. \quad (28)$$

Step 11. Compute the score ($PDHFAS_i$) by the Eq. (29):

$$PDHFAS_i = \frac{1}{2}\left(PDHFNSP_i + PDHFNSN_i\right), \quad (29)$$

where $PDHFAS_i \in \left[0,1\right]$.

Step 12. Sort all alternatives, the alternative with biggest score value is optimal one.

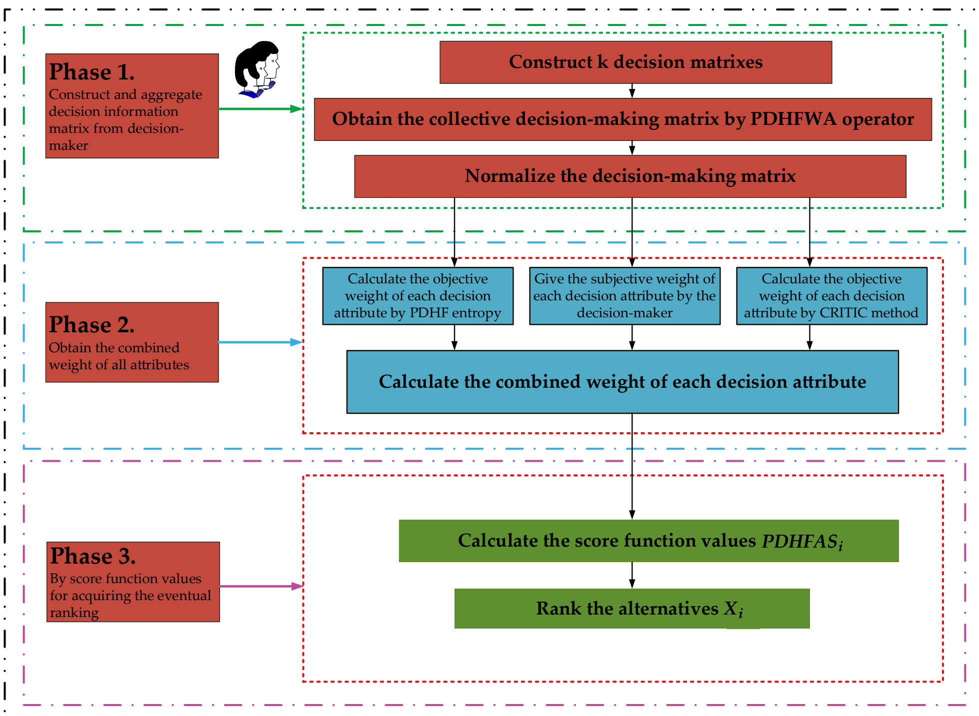

Figure 1. Flowchart of the developed PDHF MAGDM approach

5. A numerical instance and comparison analysis

5.1. A numerical instance

With the deepening of economic globalization, supply chain management (SCM) has become an important factor to improve the international competitiveness of enterprises in the highly competitive global economy. One of the crucial problems in SCM information system is SS. Finding the optimal supplier from alternatives depend on the criteria of cost, service and risk is a complex MADM problem. In the illustrative example (adapted from Hao et al., 2017), assume that the core enterprise company of a supply chain intends to select the best supplier from six alternative suppliers $X=\{X_1,X_2,X_3,X_4,X_5,X_6\}$, and according to the following four attributes $C=\{C_1,C_2,C_3,C_4\}$. The four attributes are C_1: supplier basic information, which refers to the minimum index requirements set to meet the products and services required by the enterprise's production, including quality, order fulfillment rate, on-time delivery rate, flexibility and cost; C_2: supplier's knowledge and technology capability, mainly including supplier's technical level and supplier's innovation capability; C_3: the integration of the supplier's corporate culture and strategy, and the compatibility in corporate culture and strategy can promote both parties to establish a stronger strategic alliance and reduce risks; C_4: the ability of information communication can be measured by the information technology level of both parties, the time for suppliers to respond effectively to the requirements of the SC and the information transparency of suppliers. In the evaluation process, there three experts $e_i\,(i=1,2,3)$ to choose the most excellent supplier, whose weight vector is $\theta=\{0.2,0.3,0.5\}$, the three experts give the original preference values of X_1, X_2, X_3, X_4, X_5 and X_6 under C_1, C_2, C_3 and C_4. All assessments values are given in Tables 1, 2 and 3.

Table 1. The PDHF decision information matrix obtained by e_1

Alternatives	C_1	C_2	C_3	C_4
X_1	$\langle\{0.7\|0.2,0.6\|0.2,0.5\|0.6\},\{0.2\|1\}\rangle$	$\langle\{0.7\|1\},\{0.25\|1\}\rangle$	$\langle\{0.2\|1\},\{0.2\|1\}\rangle$	$\langle\{0.7\|0.5,0.6\|0.5\},\{0.3\|1\}\rangle$
X_2	$\langle\{0.1\|1\},\{0.4\|1\}\rangle$	$\langle\{0.3\|1\},\{0.7\|1\}\rangle$	$\langle\{0.7\|1\},\{0.3\|0.5,0.2\|0.5\}\rangle$	$\langle\{0.3\|1\},\{0.3\|1\}\rangle$
X_3	$\langle\{0.6\|1\},\{0.35\|1\}\rangle$	$\langle\{0.56\|1\},\{0.2\|1\}\rangle$	$\langle\{0.1\|1\},\{0.7\|1\}\rangle$	$\langle\{0.2\|0.6,0.4\|0.4\},\{0.4\|1\}\rangle$
X_4	$\langle\{0.05\|0.7,0.2\|0.3\},\{0.5\|1\}\rangle$	$\langle\{0.3\|0.5,0.2\|0.5\},\{0.6\|0.5,0.5\|0.5\}\rangle$	$\langle\{0.8\|1\},\{0.15\|1\}\rangle$	$\langle\{0.2\|1\},\{0.6\|1\}\rangle$
X_5	$\langle\{0.15\|1\},\{0.8\|1\}\rangle$	$\langle\{0.5\|1\},\{0.5\|1\}\rangle$	$\langle\{0.8\|0.6,0.6\|0.4\},\{0.15\|1\}\rangle$	$\langle\{0.12\|1\},\{0.7\|0.9,0.6\|0.1\}\rangle$
X_6	$\langle\{0.08\|1\},\{0.6\|1\}\rangle$	$\langle\{0.1\|0.6,0.3\|0.4\},\{0.7\|1\}\rangle$	$\langle\{0.3\|1\},\{0.65\|1\}\rangle$	$\langle\{0.5\|1\},\{0.2\|0.3,0.4\|0.7\}\rangle$

Table 2. The PDHF decision information matrix obtained by e_2

Alternatives	C_1	C_2	C_3	C_4
X_1	$\langle\{0.5\|1\},\{0.5\|1\}\rangle$	$\langle\{0.2\|1\},\{0.4\|0.8,0.6\|0.2\}\rangle$	$\langle\{0.7\|0.4,0.4\|0.6\},\{0.3\|0.7,0.2\|0.3\}\rangle$	$\langle\{0.6\|0.7,0.7\|0.3\},\{0.25\|1\}\rangle$
X_2	$\langle\{0.3\|0.5,0.5\|0.5\},\{0.4\|1\}\rangle$	$\langle\{0.1\|1\},\{0.6\|0.6,0.8\|0.4\}\rangle$	$\langle\{0.4\|0.8,0.3\|0.2\},\{0.5\|0.3,0.4\|0.7\}\rangle$	$\langle\{0.2\|0.3,0.3\|0.7\},\{0.6\|1\}\rangle$
X_3	$\langle\{0.1\|0.1,0.2\|0.9\},\{0.5\|1\}\rangle$	$\langle\{0.2\|0.5,0.3\|0.5\},\{0.3\|0.5,0.2\|0.5\}\rangle$	$\langle\{0.2\|1\},\{0.7\|0.6,0.5\|0.4\}\rangle$	$\langle\{0.5\|1\},\{0.4\|1\}\rangle$
X_4	$\langle\{0.2\|1\},\{0.6\|0.9,0.7\|0.1\}\rangle$	$\langle\{0.1\|1\},\{0.7\|1\}\rangle$	$\langle\{0.2\|1\},\{0.6\|1\}\rangle$	$\langle\{0.1\|0.2,0.2\|0.8\},\{0.2\|0.6,0.3\|0.4\}\rangle$
X_5	$\langle\{0.2\|1\},\{0.7\|1\}\rangle$	$\langle\{0.45\|1\},\{0.5\|1\}\rangle$	$\langle\{0.8\|0.9,0.6\|0.1\},\{0.11\|1\}\rangle$	$\langle\{0.3\|1\},\{0.2\|1\}\rangle$
X_6	$\langle\{0.4\|0.4,0.5\|0.6\},\{0.5\|1\}\rangle$	$\langle\{0.3\|0.4,0.4\|0.6\},\{0.5\|1\}\rangle$	$\langle\{0.3\|1\},\{0.6\|1\}\rangle$	$\langle\{0.2\|1\},\{0.6\|1\}\rangle$

Table 3. The PDHF decision information matrix obtained by e_3

Alternatives	C_1	C_2	C_3	C_4
X_1	$\langle\{0.4\|1\},\{0.5\|1\}\rangle$	$\langle\{0.9\|1\},\{0.1\|1\}\rangle$	$\langle\{0.3\|1\},\{0.5\|0.4,0.6\|0.6\}\rangle$	$\langle\{0.6\|1\},\{0.3\|1\}\rangle$
X_2	$\langle\{0.75\|1\},\{0.2\|1\}\rangle$	$\langle\{0.4\|1\},\{0.6\|1\}\rangle$	$\langle\{0.2\|0.7,0.4\|0.3\},\{0.2\|1\}\rangle$	$\langle\{0.3\|1\},\{0.6\|1\}\rangle$
X_3	$\langle\{0.6\|0.6,0.8\|0.4\},\{0.1\|1\}\rangle$	$\langle\{0.5\|1\},\{0.2\|1\}\rangle$	$\langle\{0.1\|1\},\{0.8\|1\}\rangle$	$\langle\{0.2\|0.7,0.4\|0.3\},\{0.6\|1\}\rangle$
X_4	$\langle\{0.2\|1\},\{0.7\|1\}\rangle$	$\langle\{0.5\|0.6,0.7\|0.4\},\{0.1\|1\}\rangle$	$\langle\{0.3\|0.3,0.5\|0.7\},\{0.2\|0.5,0.5\|0.5\}\rangle$	$\langle\{0.1\|0.6,0.3\|0.4\},\{0.6\|1\}\rangle$
X_5	$\langle\{0.3\|0.7,0.4\|0.3\},\{0.4\|0.6,0.5\|0.4\}\rangle$	$\langle\{0.6\|1\},\{0.1\|0.5,0.2\|0.5\}\rangle$	$\langle\{0.7\|1\},\{0.2\|1\}\rangle$	$\langle\{0.1\|0.45,0.3\|0.55\},\{0.5\|0.5,0.65\|0.5\}\rangle$
X_6	$\langle\{0.2\|0.2,0.1\|0.8\},\{0.7\|1\}\rangle$	$\langle\{0.2\|1\},\{0.8\|1\}\rangle$	$\langle\{0.2\|0.8,0.3\|0.2\},\{0.6\|1\}\rangle$	$\langle\{0.35\|1\},\{0.5\|0.5,0.6\|0.5\}\rangle$

In the below sections, the calculation process of PDHF-EDAS for SS is shown as follows.

Step 1. Since all attributes belong to B, hence, normalized process is omitted.

Step 2. The collective decision matrix for six alternative suppliers under four attributes is obtained by polymerizing the three experts' opinion by Eq. (21) and recorded in Table 4.

Table 4. The final collective decision-making information matrix

Alternatives	C_1	C_2
X_1	$\left\langle \{0.45\|0.6, 0.48\|0.2, 0.51\|0.2\}, \{0.42\|1\} \right\rangle$	$\left\langle \{0.77\|1\}, \{0.18\|0.8, 0.21\|0.2\} \right\rangle$
X_2	$\left\langle \{0.56\|0.5, 0.6\|0.5\}, \{0.28\|1\} \right\rangle$	$\left\langle \{0.3\|1\}, \{0.62\|0.6, 0.67\|0.4\} \right\rangle$
X_3	$\left\langle \{0.49\|0.06, 0.64\|0.04, 0.51\|0.54, 0.65\|0.36\}, \{0.21\|1\} \right\rangle$	$\left\langle \{0.44\|0.5, 0.46\|0.5\}, \{0.2\|0.5, 0.23\|0.5\} \right\rangle$
X_4	$\left\langle \{0.17\|0.7, 0.2\|0.3\}, \{0.62\|0.9, 0.65\|0.1\} \right\rangle$	$\left\langle \{0.34\|0.3, 0.49\|0.2, 0.36\|0.3, 0.51\|0.2\}, \{0.25\|0.5, 0.26\|0.5\} \right\rangle$
X_5	$\left\langle \{0.24\|0.7, 0.3\|0.3\}, \{0.54\|0.6, 0.61\|0.4\} \right\rangle$	$\left\langle \{0.54\|1\}, \{0.22\|0.5, 0.32\|0.5\} \right\rangle$
X_6	$\left\langle \{0.2\|0.32, 0.25\|0.08, 0.24\|0.48, 0.29\|0.12\}, \{0.61\|1\} \right\rangle$	$\left\langle \{0.21\|0.24, 0.25\|0.36, 0.25\|0.16, 0.29\|0.24\}, \{0.68\|1\} \right\rangle$

Alternatives	C_3	C_4
X_1	$\left\langle \{0.31\|0.6, 0.44\|0.4\}, \{0.32\|0.12, 0.35\|0.18, 0.36\|0.28, 0.39\|0.42\} \right\rangle$	$\left\langle \{0.6\|0.35, 0.63\|0.15, 0.62\|0.35, 0.65\|0.15\}, \{0.28\|1\} \right\rangle$
X_2	$\left\langle \{0.37\|0.14, 0.45\|0.06, 0.4\|0.56, 0.48\|0.24\}, \{0.25\|0.35, 0.25\|0.15, 0.27\|0.35, 0.29\|0.15\} \right\rangle$	$\left\langle \{0.27\|0.3, 0.3\|0.7\}, \{0.52\|1\} \right\rangle$
X_3	$\left\langle \{0.13\|1\}, \{0.68\|0.4, 0.75\|0.6\} \right\rangle$	$\left\langle \{0.31\|0.42, 0.4\|0.18, 0.34\|0.28, 0.43\|0.12\}, \{0.49\|1\} \right\rangle$
X_4	$\left\langle \{0.43\|0.3, 0.52\|0.7\}, \{0.26\|0.5, 0.42\|0.5\} \right\rangle$	$\left\langle \{0.12\|0.12, 0.22\|0.08, 0.15\|0.48, 0.25\|0.32\}, \{0.43\|0.6, 0.49\|0.4\} \right\rangle$
X_5	$\left\langle \{0.65\|0.04, 0.72\|0.36, 0.7\|0.06, 0.76\|0.54\}, \{0.16\|1\} \right\rangle$	$\left\langle \{0.17\|0.45, 0.27\|0.55\}, \{0.39\|0.05, 0.45\|0.05, 0.41\|0.45, 0.46\|0.45\} \right\rangle$
X_6	$\left\langle \{0.25\|0.8, 0.3\|0.2\}, \{0.61\|1\} \right\rangle$	$\left\langle \{0.34\|1\}, \{0.44\|0.15, 0.48\|0.15, 0.51\|0.35, 0.55\|0.35\} \right\rangle$

Step 3. Compute the score function decision information matrix $S=\left(s_{D_\alpha}\left(n_{ij}\right)\right)_{m\times n}$ though using Eq. (11), all computing results are recorded in Table 5.

Table 5. The score function decision-making matrix S

Alternatives	C_1	C_2	C_3	C_4
X_1	0.529	0.8043	0.5003	0.6859
X_2	0.6722	0.3197	0.6117	0.359
X_3	0.7275	0.6719	0.1554	0.4157
X_4	0.2252	0.6158	0.5929	0.3002
X_5	0.314	0.6655	0.8227	0.3463
X_6	0.2769	0.2695	0.3005	0.4035

Step 4. Convert the score function decision matrix S into the normalized score function decision-making matrix $Q=\left(s_{ij}\right)_{m\times n}$ by Eq. (22), all computing results are recorded in Table 6.

Table 6. The normalized score function decision-making matrix Q

Alternatives	C_1	C_2	C_3	C_4
X_1	0.6049	1	0.5169	1
X_2	0.8899	0.0938	0.6838	0.1525
X_3	1	0.7524	0	0.2995
X_4	0	0.6476	0.6556	0
X_5	0.1769	0.7405	1	0.1194
X_6	0.1029	0	0.2174	0.2678

Step 5. Obtain the attribute weight by Eq. (20) which are recorded in Table 7.

Table 7. The combined weight

Attributes	C_1	C_2	C_3	C_4
ω_j	0.23	0.2149	0.347	0.208
η_j	0.2843	0.2871	0.1962	0.1811
w_j	0.1	0.3	0.5	0.1
ϖ_j	0.1747	0.294	0.3987	0.1326

Step 6. Calculate the average solution by Eq. (23), the computing results are given in Table 8.

Table 8. The *PDHFAV*

Attributes	C_1	C_2	C_3	C_4
PDHFAV	0.4624	0.5391	0.5123	0.3065

Step 7. Because the all attributes in the study are benefit, we calculate the *PDHFPDA* and *PDHFNDA* matrixes by Eq. (25), *PDHFPDA* and *PDHFNDA* in Tables 9–10.

Table 9. The *PDHFPDA* matrix

Alternatives	C_1	C_2	C_3	C_4
X_1	0.0869	0.7087	0	1.5556
X_2	0.5991	0	0.0521	0
X_3	0.7969	0.2857	0	0
X_4	0	0.1065	0.0086	0
X_5	0	0.2654	0.5385	0
X_6	0	0	0	0

Table 10. The *PDHFNDA* matrix

Alternatives	C_1	C_2	C_3	C_4
X_1	0	0	0.2048	0
X_2	0	0.8397	0	0.6103
X_3	0	0	1	0.2347
X_4	1	0	0	1
X_5	0.6822	0	0	0.6948
X_6	0.8151	1.0000	0.6656	0.3157

Step 8. Calculate the weighted sum $PDHFSP_i$ and $PDHFSN_i$ by using Eq. (27), all computing results are recorded in Table 11.

Table 11. The weighted sum $PDHFSP_i$ and $PDHFSN_i$

Alternatives	X_1	X_2	X_3	X_4	X_5	X_6
$PDHFSP_i$	0.4298	0.1254	0.2232	0.0348	0.2927	0.0000
$PDHFSN_i$	0.0816	0.3278	0.4298	0.3073	0.2113	0.7436

Step 9. Calculate the $PDHFNSP_i$ and $PDHFNSN_i$ by Eq. (28) (See Table 12).

Table 12. The $PDHFNSP_i$ and $PDHFNSN_i$

Alternatives	X_1	X_2	X_3	X_4	X_5	X_6
$PDHFNSP_i$	0.8902	0.5592	0.4220	0.5867	0.7158	0
$PDHFNSN_i$	0.8902	0.5592	0.4220	0.5867	0.7158	0

Step 10. Calculate the $PDHFAS_i\left(i=1,2,\cdots,6\right)$ by Eq. (29) (See Table 13).

Table 13. The $PDHFAS_i$ value

Alternatives	X_1	X_2	X_3	X_4	X_5	X_6
$PDHFAS_i$	0.9451	0.4255	0.4706	0.3338	0.6984	0

From the values of $PDHFAS_i\left(i=1,2,\cdots,6\right)$ in Table 13, the sequencing for six alternatives is $X_1 \succ X_5 \succ X_2 \succ X_4 \succ X_3 \succ X_6$, the optimal alternative is X_1.

5.2. Compare analysis

5.2.1. Compare with BASD based PROMETHEE-II method in Q. Zhao et al. (2020)

Here, we compare the method proposed with BASD based PROMETHEE-II method proposed by Q. Zhao et al. (2020), let's bring the data into the BASD based PROMETHEE-II model. Calculate the overall BASD degrees $\Lambda^+(X_i)(i=1,2,\cdots,6)$, $\Lambda^-(X\ )(i=1,2,\cdots,6)$ and $\Lambda(X_i)(i=1,2,\cdots,6)$ for $X_i\,(i=1,2,\cdots,6)$, the overall BASD degrees are given in Table 14.

Table 14. The overall BASD degrees $H^+(X_i)$ and $H^-(X_i)$

$\Lambda^+(X_1)$	$\Lambda^+(X_2)$	$\Lambda^+(X_3)$	$\Lambda^+(X_4)$	$\Lambda^+(X_5)$	$\Lambda^+(X_6)$
2.0368	0.9834	1.1349	0.3328	1.4581	0.0748
$\Lambda^-(X_1)$	$\Lambda^-(X_2)$	$\Lambda^-(X_3)$	$\Lambda^-(X_4)$	$\Lambda^-(X_5)$	$\Lambda^-(X_6)$
0.2984	0.7957	1.1184	1.2642	0.8262	1.7179
$\Lambda(X_1)$	$\Lambda(X_2)$	$\Lambda(X_3)$	$\Lambda(X_4)$	$\Lambda(X_5)$	$\Lambda(X_6)$
1.7384	0.1877	0.0165	–0.9314	0.6319	–1.6432

Thus, the order is $X_1 \succ X_5 \succ X_2 \succ X_3 \succ X_4 \succ X_6$ and the best supplier is X_1.

5.2.2. Comparison with decision-making model in Hao et al. (2017)

Here, we compare the method proposed in this study with the visualization model depend on the PDHF entropy proposed by Hao et al. (2017), let's bring the data into the model. The sequencing is $X_1 \succ X_4 \succ X_5 \succ X_2 \succ X_3 \succ X_6$ for all suppliers and the best supplier is X_1.

From the comparison with the two methods, although the sequencing has a little different, the best supplier is X_1, which testifies that the MAGDM technique is effective, but the following analysis can also see the advantages of our proposed method.

5.2.3. Compare with the two operators in Garg and Kaur (2018)

The data in Table 2 and $\omega=(0.1747,0.294,0.3987,0.1326)^T$ are substituted into Eqs (30) and (31) and all calculation results are shown in Tables 15 and 16, the order is $X_1 \succ X_5 \succ X_2 \succ X_4 \succ X_6 \succ X_3$, and the best supplier is X_1.

Table 15. The outcome with the PDHFWEA operator

Alternatives	Aggregation results	Scores
X_1	$\left\langle \begin{Bmatrix} 0.5401\|0.126,\cdots\cdots \\ \cdots\cdots,0.5976\|0.012 \end{Bmatrix}, \begin{Bmatrix} 0.2804\|0.096,\cdots\cdots \\ \cdots\cdots,3592\|0.084 \end{Bmatrix} \right\rangle$	0.2268
X_2	$\left\langle \begin{Bmatrix} 0.3741\|0.021,\cdots\cdots \\ \cdots\cdots,0.4314\|0.084 \end{Bmatrix}, \begin{Bmatrix} 0.3748\|0.21,\cdots\cdots \\ \cdots\cdots,0.4704\|0.06 \end{Bmatrix} \right\rangle$	–0.0175
X_3	$\left\langle \begin{Bmatrix} 0.3159\|0.0126,\cdots\cdots \\ \cdots\cdots,0.3755\|0.0216 \end{Bmatrix}, \begin{Bmatrix} 0.3849\|0.2,\cdots\cdots \\ \cdots\cdots,0.4192\|0.3 \end{Bmatrix} \right\rangle$	–0.0625
X_4	$\left\langle \begin{Bmatrix} 0.2832\|0.0076,\cdots\cdots \\ \cdots\cdots,0.3777\|0.0134 \end{Bmatrix}, \begin{Bmatrix} 0.3245\|0.135,\cdots\cdots \\ \cdots\cdots,0.4008\|0.01 \end{Bmatrix} \right\rangle$	–0.026
X_5	$\left\langle \begin{Bmatrix} 0.2973\|0.0126,\cdots\cdots \\ \cdots\cdots,0.3543\|0.0891 \end{Bmatrix}, \begin{Bmatrix} 0.2805\|0.015,\cdots\cdots \\ \cdots\cdots,0.3338\|0.09 \end{Bmatrix} \right\rangle$	0.0281
X_6	$\left\langle \begin{Bmatrix} 0.1628\|0.0614,\cdots\cdots \\ \cdots\cdots,0.2025\|0.0058 \end{Bmatrix}, \begin{Bmatrix} 0.6001\|0.15,\cdots\cdots \\ \cdots\cdots,0.6167\|0.35 \end{Bmatrix} \right\rangle$	–0.4328

Table 16. The outcome with the PDHFWEG operator

Alternatives	Aggregation results	Scores
X_1	$\left\langle \begin{Bmatrix} 0.5519\|0.126,\cdots\cdots \\ \cdots\cdots,06226\|0.012 \end{Bmatrix}, \begin{Bmatrix} 0.2932\|0.096,\cdots\cdots \\ \cdots\cdots,0.3303\|0.084 \end{Bmatrix} \right\rangle$	0.2648
X_2	$\left\langle \begin{Bmatrix} 0.4094\|0.021,\cdots\cdots \\ \cdots\cdots,0.4535\|0.084 \end{Bmatrix}, \begin{Bmatrix} 0.415\|0.21,\cdots\cdots \\ \cdots\cdots,0.4495\|0.06 \end{Bmatrix} \right\rangle$	0.0027
X_3	$\left\langle \begin{Bmatrix} 0.3423\|0.0126,\cdots\cdots \\ \cdots\cdots,0.3836\|0.0216 \end{Bmatrix}, \begin{Bmatrix} 0.4609\|0.2,\cdots\cdots \\ \cdots\cdots,0.5117\|0.3 \end{Bmatrix} \right\rangle$	–0.1314
X_4	$\left\langle \begin{Bmatrix} 0.3285\|0.0076,\cdots\cdots \\ \cdots\cdots,0.4404\|0.0134 \end{Bmatrix}, \begin{Bmatrix} 0.353\|0.135,\cdots\cdots \\ \cdots\cdots,0.4305\|0.01 \end{Bmatrix} \right\rangle$	–0.0069
X_5	$\left\langle \begin{Bmatrix} 0.4735\|0.0126,\cdots\cdots \\ \cdots\cdots,0.5435\|0.0891 \end{Bmatrix}, \begin{Bmatrix} 0.2824\|0.015,\cdots\cdots \\ \cdots\cdots,0.3378\|0.09 \end{Bmatrix} \right\rangle$	0.2041
X_6	$\left\langle \begin{Bmatrix} 0.2846\|0.0614,\cdots\cdots \\ \cdots\cdots,0.3496\|0.0058 \end{Bmatrix}, \begin{Bmatrix} 0.6125\|0.15,\cdots\cdots \\ \cdots\cdots,0.6244\|0.35 \end{Bmatrix} \right\rangle$	–0.3102

5.3. Sensitivity analysis of parameters

(1) From the comparison and analysis of the two methods, the research result given by the MAGDM technique built in the study is basically consistent with the that obtained by Q. Zhao et al. (2020) and Hao et al. (2017). However, this study considers the advantages of subjective weighting and objective weighting, which not only overcomes the disadvantages of giving decision attribute weight artificially and ignoring the importance of objective data, but also overcomes the disadvantages of paying attention to objective data and ignoring people's subjective initiative, which is more in line with the reality and gives DMs more choices;

(2) Based on the detailed analysis of the disadvantages of the existing score function, combined with the research contents of literatures (Atanassov, 1989; M. J. Huang & Li, 2013), a novel score function for PDHFE is built, the novel score function of PDHFE is more tally with people's cognition, and a comparison method of two PDHFEs is given. This method can better and more directly compare with two PDHFEs.

(3) The concept of PDHF entropy is proposed. It does not need to use other auxiliary functions, retains its most original information, and enriches the concept of PDHF entropy;

(4) We can observe the change of the order of the evaluated alternatives with the change of subjective weight w. It can give DMs more choices, which shows the superiority and effectiveness.

Next, we analyze the change of the sequencing of the evaluated alternatives with the change of subjective weight w, all computing results are recorded in Table 17.

Table 17. The sequencing for different values of subjective weight w

w_1	w_2	w_3	w_4	Sequencing
0.1	0.3	0.5	0.1	$X_1 \succ X_5 \succ X_2 \succ X_4 \succ X_3 \succ X_6$
0.2	0.1	0.6	0.1	$X_1 \succ X_5 \succ X_2 \succ X_3 \succ X_4 \succ X_6$
0.15	0.1	0.7	0.05	$X_1 \succ X_5 \succ X_2 \succ X_3 \succ X_4 \succ X_6$
0.3	0.2	0.4	0.1	$X_1 \succ X_5 \succ X_3 \succ X_2 \succ X_4 \succ X_6$
0.25	0.25	0.25	0.25	$X_1 \succ X_5 \succ X_3 \succ X_2 \succ X_4 \succ X_6$
0.05	0.35	0.5	0.1	$X_1 \succ X_5 \succ X_3 \succ X_2 \succ X_4 \succ X_6$

Conclusions

In the study, by combing and studying the relevant decision-making methods of PDHFS in the early stage, we found that there is no research on the use of EDAS in PDHF environment. At the same time, through systematic analysis, we found that the existing score function of PDHFE is not very reasonable, there is also a lack of research on PDHF entropy. In view of the above problems, the following are the main superiorities of PDHF-EDAS approach.

(1) Aiming at the disadvantage that the existing score function of PDHFE does not consider the hesitant degree, we define a new score function. Through case analysis, it is more consistent with the actual situation and enriches the relevant research of PDHFS. (2) With the help of the new score function, we define the PDHF entropy and CRITIC weighting method for decision attribute weighting. At the same time, combined with the subjective weighting method, we use the minimum identification information principle to weight the decision attribute, such a weighting method not only overcomes the disadvantages of artificially determining the weight, but also overcomes the disadvantages of only relying on objective data to obtain the weight of decision attributes and ignoring people's subjective initiative. This research enriches the weighting method of decision attributes whose decision attribute information is probability dual hesitation fuzzy element. (3) We integrate the widely used EDAS method with PDHF environment, and propose a new MAGDM method in PDHF environment, which enriches the MAGDM method in PDHF environment. (4) We apply the new MAGDM technique to supplier optimization. From the comparative analysis with other methods, the new method is effective, which also enriches the supplier optimization method in the supply chain.

In the next research, we will pay more attention to the fusion of other decision-making approaches and PDHFS, as well as the development of some new aggregation operators. For example, we will study the fusion of BWM method, ARAS method, WASPAS method and PDHFS, so as to propose some new decision-making approaches in PDHF environment, as well as development and research of Bonferroni mean (BM) operator, Hamy mean (HM) and power average (PA) operators in PDHF environment. Finally, we will apply the built new decision-making methods to supplier selection, locations selection and other decision-making issues.

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References

Anees, J., Zhang, H. C., Baig, S., Lougou, B. G., & Bona, T. G. R. (2020). Hesitant fuzzy entropy-based opportunistic clustering and data fusion algorithm for heterogeneous wireless sensor networks. *Sensors*, *20*(3), 913. https://doi.org/10.3390/s20030913

Atanassov, K., & Gargov, G. (1989). Interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *31*(3), 343–349. https://doi.org/10.1016/0165-0114(89)90205-4

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3

Atanassov, K. T. (1989). More on intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *33*(1), 37–45. https://doi.org/10.1016/0165-0114(89)90215-7

Darko, A. P., & Liang, D. C. (2020). Some q-rung orthopair fuzzy Hamacher aggregation operators and their application to multiple attribute group decision making with modified EDAS method. *Engineering Applications of Artificial Intelligence*, *87*, 103259. https://doi.org/10.1016/j.engappai.2019.103259

Diakoulaki, D., Mavrotas, G., & Papayannakis, L. (1995). Determining objective weights in multiple criteria problems: The critic method. *Computers & Operations Research*, *22*(7), 763–770. https://doi.org/10.1016/0305-0548(94)00059-H

Fan, J. P., Jia, X. F., & Wu, M. Q. (2020). A new multi-criteria group decision model based on Single-valued triangular Neutrosophic sets and EDAS method. *Journal of Intelligent & Fuzzy Systems*, *38*(2), 2089–2102. https://doi.org/10.3233/JIFS-190811

Garg, H., & Kaur, G. (2018). Algorithm for probabilistic dual hesitant fuzzy multi-criteria decision-making based on aggregation operators with new distance measures. *Mathematics*, *6*(12), 280. https://doi.org/10.3390/math6120280

Garg, H., & Kaur, G. (2020a). Quantifying gesture information in brain hemorrhage patients using probabilistic dual hesitant fuzzy sets with unknown probability information. *Computers & Industrial Engineering*, *140*, 106211. https://doi.org/10.1016/j.cie.2019.106211

Garg, H., & Kaur, G. (2020b). A robust correlation coefficient for probabilistic dual hesitant fuzzy sets and its applications. *Neural Computing & Applications*, *32*, 8847–8866. https://doi.org/10.1007/s00521-019-04362-y

Garg, H., & Kaur, G. (2021). Algorithms for screening travelers during Covid-19 outbreak using probabilistic dual hesitant values based on bipartite graph theory. *Applied and Computational Mathematics*, *20*(1), 22–48.

Haktanir, E., & Kahraman, C. (2021). A Novel CRITIC based weighted FMEA method: Application to COVID-19 blood testing process. *Journal of Multiple-Valued Logic and Soft Computing*, *37*(3–4), 247–275.

Hao, Z. N., Xu, Z. S., Zhao, H., & Su, Z. (2017). Probabilistic dual hesitant fuzzy set and its application in risk evaluation. *Knowledge-Based Systems*, *127*, 16–28. https://doi.org/10.1016/j.knosys.2017.02.033

Herrera, F., & Martinez, L. (2000). A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems*, *8*(6), 746–752. https://doi.org/10.1109/91.890332

Huang, M. J., & Li, K. W. (2013). A novel approach to characterizing hesitations in intuitionistic fuzzy numbers. *Journal of Systems Science and Systems Engineering*, *22*, 283–294. https://doi.org/10.1007/s11518-013-5213-x

Huang, Y., Lin, R., & Chen, X. (2021). An enhancement EDAS method based on prospect theory. *Technological and Economic Development of Economy*, *27*(5), 1019–1038. https://doi.org/10.3846/tede.2021.15038

Hwang, C. L., & Yoon, K. P. (1981). *Lecture notes in economics and mathematical systems: Vol. 186. Multiple attribute decision making: Methods and applications a state-of-the-art survey*. Springer. https://doi.org/10.1007/978-3-642-48318-9

Jiang, Z., Wei, G., & Chen, X. (2022a). EDAS method based on cumulative prospect theory for multiple attribute group decision-making under picture fuzzy environment. *Journal of Intelligent & Fuzzy Systems*, *42*(3), 1723–1735. https://doi.org/10.3233/JIFS-211171

Jiang, Z., Wei, G., & Guo, Y. (2022b). Picture fuzzy MABAC method based on prospect theory for multiple attribute group decision making and its application to suppliers selection. *Journal of Intelligent & Fuzzy Systems*, *42*(4), 3405–3415. https://doi.org/10.3233/JIFS-211359

Keshavarz Ghorabaee, M., Zavadskas, E. K., Olfat, L., & Turskis, Z. (2015). Multi-criteria inventory classification using a new method of Evaluation Based on Distance from Average Solution (EDAS). *Informatica*, *26*(3), 435–451. https://doi.org/10.15388/Informatica.2015.57

Krishankumar, R., Garg, H., Arun, K., Saha, A., Ravichandran, K. S., & Kar, S. (2021). An integrated decision-making COPRAS approach to probabilistic hesitant fuzzy set information. *Complex & Intelligent Systems*, *7*, 2281–2298. https://doi.org/10.1007/s40747-021-00387-w

Lei, F., Wei, G., Shen, W., & Guo, Y. (2022). PDHL-EDAS method for multiple attribute group decision making and its application to 3D printer selection. *Technological and Economic Development of Economy*, *28*(1), 179–200. https://doi.org/10.3846/tede.2021.15884

Li, X., Ju, Y. B., Ju, D. W., Zhang, W. K., Dong, P. W., & Wang, A. H. (2019). Multi-attribute group decision making method based on EDAS under picture fuzzy environment. *IEEE Access*, *7*, 141179–141192. https://doi.org/10.1109/ACCESS.2019.2943348

Liang, Z. W., Liu, X. C., Ye, B. Y., & Wang, Y. J. (2013). Performance investigation of fitting algorithms in surface micro-topography grinding processes based on multi-dimensional fuzzy relation set. *International Journal of Advanced Manufacturing Technology*, *67*, 2779–2798. https://doi.org/10.1007/s00170-012-4692-0

Liao, N., Gao, H., Wei, G., & Chen, X. (2021). CPT-MABAC-based multiple attribute group decision making method with probabilistic hesitant fuzzy information. *Journal of Intelligent & Fuzzy Systems*, *41*(6), 6999–7014. https://doi.org/10.3233/JIFS-210889

Lima, A., Palmeira, E. S., Bedregal, B., & Bustince, H. (2021). Multidimensional fuzzy sets. *IEEE Transactions on Fuzzy Systems*, *29*(8), 2195–2208. https://doi.org/10.1109/TFUZZ.2020.2994997

Liu, X. D., Wang, Z. W., Zhang, S. T., & Garg, H. (2021). An approach to probabilistic hesitant fuzzy risky multiattribute decision making with unknown probability information. *International Journal of Intelligent Systems*, *36*(10), 5714–5740. https://doi.org/10.1002/int.22527

Lu, J., Zhang, S., Wu, J., & Wei, Y. (2021). COPRAS method for multiple attribute group decision making under picture fuzzy environment and their application to green supplier selection. *Technological and Economic Development of Economy*, *27*(2), 369–385. https://doi.org/10.3846/tede.2021.14211

Narayanamoorthy, S., Ramya, L., Kang, D., Baleanu, D., Kureethara, J. V., & Annapoorani, V. (2021). A new extension of hesitant fuzzy set: An application to an offshore wind turbine technology selection process. *IET Renewable Power Generation*, *15*(15), 2340–2355. https://doi.org/10.1049/rpg2.12168

Ning, B., Wei, G., Lin, R., & Guo, Y. (2022). A novel MADM technique based on extended power generalized Maclaurin symmetric mean operators under probabilistic dual hesitant fuzzy setting and its application to sustainable suppliers selection. *Expert Systems with Applications*, *204*, 117419. https://doi.org/10.1016/j.eswa.2022.117419

Opricovic, S., & Tzeng, G. H. (2004). Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research*, *156*(2), 445–455. https://doi.org/10.1016/S0377-2217(03)00020-1

Peng, X. D., & Garg, H. (2022). Intuitionistic fuzzy soft decision making method based on CoCoSo and CRITIC for CCN cache placement strategy selection. *Artificial Intelligence Review*, *55*, 1567–1604. https://doi.org/10.1007/s10462-021-09995-x

Pramanik, R., Baidya, D. K., & Dhang, N. (2021). Reliability assessment of three-dimensional bearing capacity of shallow foundation using fuzzy set theory. *Frontiers of Structural and Civil Engineering*, *15*, 478–489. https://doi.org/10.1007/s11709-021-0698-8

Rahimi, M., Kumar, P., Moomivand, B., & Yari, G. (2021). An intuitionistic fuzzy entropy approach for supplier selection. *Complex & Intelligent Systems*, *7*, 1869–1876. https://doi.org/10.1007/s40747-020-00224-6

Ren, Z. L., Xu, Z. S., & Wang, H. (2017, December). An extended TODIM method under probabilistic dual hesitant fuzzy information and its application on enterprise strategic assessment. In *2017 IEEE International Conference on Industrial Engineering and Engineering Management* (*IEEM*) (pp. 1464–1468). Singapore. IEEE. https://doi.org/10.1109/IEEM.2017.8290136

Saraji, M. K., Streimikiene, D., & Kyriakopoulos, G. L. (2021). Fermatean fuzzy CRITIC-COPRAS method for evaluating the challenges to Industry 4.0 adoption for a sustainable digital transformation. *Sustainability*, *13*(17), 9577. https://doi.org/10.3390/su13179577

Shi, H. T., Li, Y. F., Jiang, Z. N., & Zhang, J. (2021). Comprehensive power quality evaluation method of microgrid with dynamic weighting based on CRITIC. *Measurement* & *Control*, *54*(5–6), 1097–1104. https://doi.org/10.1177/00202940211016092

Su, Y., Zhao, M., Wei, G., Wei, C., & Chen, X. (2022). Probabilistic uncertain linguistic EDAS method based on prospect theory for multiple attribute group decision-making and its application to green finance. *International Journal of Fuzzy Systems*, *24*, 1318–1331. https://doi.org/10.1007/s40815-021-01184-w

Thao, N. X., & Smarandache, F. (2019). A new fuzzy entropy on Pythagorean fuzzy sets. *Journal of Intelligent* & *Fuzzy Systems*, *37*(1), 1065–1074. https://doi.org/10.3233/JIFS-182540

Torra, V. (2010). Hesitant fuzzy sets. *International Journal of Intelligent Systems*, *25*(6), 529–539. https://doi.org/10.1002/int.20418

Wang, S., Wei, G., Lu, J., Wu, J., Wei, C., & Chen, X. (2022). GRP and CRITIC method for probabilistic uncertain linguistic MAGDM and its application to site selection of hospital constructions. *Soft Computing*, *26*, 237–251. https://doi.org/10.1007/s00500-021-06429-2

William-West, T. O., & Ciucci, D. (2021). Decision-theoretic five-way approximation of fuzzy sets. *Information Sciences*, *572*, 200–222. https://doi.org/10.1016/j.ins.2021.04.105

Xu, T. T., Zhang, H., & Li, B. Q. (2020). Pythagorean fuzzy entropy and its application in multiple-criteria decision-making. *International Journal of Fuzzy Systems*, *22*, 1552–1564. https://doi.org/10.1007/s40815-020-00877-y

Xu, Z. S., & Zhou, W. (2017). Consensus building with a group of decision makers under the hesitant probabilistic fuzzy environment. *Fuzzy Optimization and Decision Making*, *16*, 481–503. https://doi.org/10.1007/s10700-016-9257-5

Yahya, M., Naeem, M., Abdullah, S., Qiyas, M., & Aamir, M. (2021). A novel approach on the intuitionistic fuzzy rough frank aggregation operator-based EDAS method for multicriteria group decision-making. *Complexity*, *2021*, 5534381. https://doi.org/10.1155/2021/5534381

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

Zafar, S., Alamgir, Z., & Rehman, M. H. (2021). An effective blockchain evaluation system based on entropy-CRITIC weight method and MCDM techniques. *Peer-to-Peer Networking and Applications*, *14*, 3110–3123. https://doi.org/10.1007/s12083-021-01173-8

Zhang, C., Li, D. Y., Liang, J. Y., & Wang, B. L. (2021a). MAGDM-oriented dual hesitant fuzzy multigranulation probabilistic models based on MULTIMOORA. *International Journal of Machine Learning and Cybernetics*, *12*, 1219–1241. https://doi.org/10.1007/s13042-020-01230-3

Zhang, D., Su, Y., Zhao, M., & Chen, X. (2022a). CPT-TODIM method for interval neutrosophic MAGDM and its application to third-party logistics service providers selection. *Technological and Economic Development of Economy*, *28*(1), 201–219. https://doi.org/10.3846/tede.2021.15758

Zhang, H., Wei, G., & Chen, X. (2022b). SF-GRA method based on cumulative prospect theory for multiple attribute group decision making and its application to emergency supplies supplier selection. *Engineering Applications of Artificial Intelligence*, *110*, 104679. https://doi.org/10.1016/j.engappai.2022.104679

Zhang, H. M. (2020). Distance and entropy measures for dual hesitant fuzzy sets. *Computational* & *Applied Mathematics*, *39*, 91. https://doi.org/10.1007/s40314-020-1111-2

Zhang, H. Y., Wei, G. W., & Chen, X. D. (2022c). Spherical fuzzy Dombi power Heronian mean aggregation operators for multiple attribute group decision-making. *Computational & Applied Mathematics*, *41*, 98. https://doi.org/10.1007/s40314-022-01785-7

Zhang, H. Y., Wei, G. W., & Wei, C. (2022d). TOPSIS method for spherical fuzzy MAGDM based on cumulative prospect theory and combined weights and its application to residential location. *Journal of Intelligent & Fuzzy Systems*, *42*(3), 1367–1380. https://doi.org/10.3233/JIFS-210267

Zhang, S., Gao, H., Wei, G., & Chen, X. (2021b). Grey relational analysis method based on cumulative prospect theory for intuitionistic fuzzy multi-attribute group decision making. *Journal of Intelligent & Fuzzy Systems*, *41*(2), 3783–3795. https://doi.org/10.3233/JIFS-211461

Zhao, M., Gao, H., Wei, G., Wei, C., & Guo, Y. (2022). Model for network security service provider selection with probabilistic uncertain linguistic TODIM method based on prospect theory. *Technological and Economic Development of Economy*, *28*(3), 638–654. https://doi.org/10.3846/tede.2022.16483

Zhao, M., Wei, G., Chen, X., & Wei, Y. (2021a). Intuitionistic fuzzy MABAC method based on cumulative prospect theory for multiple attribute group decision making. *International Journal of Intelligent Systems*, *36*(11), 6337–6359. https://doi.org/10.1002/int.22552

Zhao, M., Wei, G., Guo, Y., & Chen, X. (2021b). CPT-TODIM method for interval-valued bipolar fuzzy multiple attribute group decision making and application to industrial control security service provider selection. *Technological and Economic Development of Economy*, *27*(5), 1186–1206. https://doi.org/10.3846/tede.2021.15044

Zhao, Q., Ju, Y. B., & Pedrycz, W. (2020). A method based on bivariate almost stochastic dominance for multiple criteria group decision making with probabilistic dual hesitant fuzzy information. *IEEE Access*, *8*, 203769–203786. https://doi.org/10.1109/ACCESS.2020.3035906

Zhu, B., Xu., Z., & Xia, M. (2012). Dual hesitant fuzzy sets. *Journal of Applied Mathematics*, *2012*, 2607–2645. https://doi.org/10.1155/2012/879629