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ON THE SOLVABILITY OF NONLINEAR PROBLEM OF MAGNETIZATION

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ABSTRACT

It is investigated the system of kinetic equations describing the magnetization of a medium consisting of single-domain particles. The system includes the nonlinear Landay-Lifschitz equation. The local existence of solution and its uniqueness in spaces $C^k(0,T;X)$, X denotes the Sobolev space, is proved.

1. INTRODUCTION AND STATEMENT OF MAIN RESULT

The dynamic Landay–Lifschitz equations [4] are often applied in the problems of magnetizations. Vl. Skakauskas in 1985 presented a new dynamic system of simultaneous equations describing the magnetization of a medium composed of single–domain particles [5]. There the nonlinear Landay–Lifschitz equation defines the motion of an individual single–domain particle in the magnetic field. There have been some solutions of the simple cases this problem [6]. Later the particular cases were studied in the various functional spaces [7; 3]. In [2] there was investigated the difference scheme applied to solve the nonlinear system of equations.

The purpose of this paper is to prove the existence and uniqueness of solution in a small time interval for the system nonlinear simultaneous equations in the general case.

We consider the boundary value problem of the system of equations

$$\frac{\partial u}{\partial t} = a^1 u \times (u \times v + v), \tag{1.1}$$

$$u = u_0, \quad \text{for } t = 0, \tag{1.2}$$

$$v = a^2 u + a^3 w + a^4 \frac{\partial z}{\partial x}, \tag{1.3}$$

$$w = \int_I a^5 u\,dy, \tag{1.4}$$

$$Lz = \sum_{i=1}^{3}(\sum_{j=1}^{2} b_{ij}\frac{\partial w_i}{\partial x_j} + b_i w_i), \tag{1.5}$$

$$z = \varphi, \quad \text{for } x \in \partial\Omega, \tag{1.6}$$

where

$$Lz =: \sum_{i,j=1}^{2} \alpha_{ij}\frac{\partial^2 z}{\partial x_i \partial x_j} + \sum_{i=1}^{2}\alpha_i \frac{\partial z}{\partial x_i} + \alpha z$$

denotes an elliptic operator.

The motion of an individual single-domain particle in the magnetic field is described by the Landay–Lifschitz equation (1.1), u_0 defines the initial position of the particle. The equation (1.4) gives the magnetization of a medium. The last two equations determine the interaction between magnetic field of the medium and external magnetic field.

Here $\Omega \subset \mathbb{R}^2$ is the bounded domain with boundary $\partial\Omega$, $x = (x_1, x_2)$ is a point of $\Omega \cup \partial\Omega$. y is a point of the bounded domain $I \subset \mathbb{R}$ and let $Q = \Omega \times I$. $u(t,x,y)$, $v(t,x,y)$, $w(t,x)$, $z(t,x)$ are unknown functions. Note that $u = (u_1, u_2, u_3)$, $v = (v_1, v_2, v_3)$, $w = (w_1, w_2, w_3)$ are three-dimensional vectors, while z is a scalar function. $a^1(t,x,y)$ is a given vector $a^1 = (a_1^1, a_2^1, a_3^1)$ and $a^k(t,x,y), k = \overline{2,5}$ are given matrixes, namely, $a^k = \{a_{ij}^k\}_{i,j=1}^3$ for $k = 2,3,5$ and $a^4 = \{a_{ij}^k\}_{i,j=1}^{3,2}$. Scalar functions $b_{ij}(t,x,y)$, $b_i(t,x,y)$, $i = 1,2,3, j = 1,2$ and $\varphi(t,x)$, $\alpha(t,x)$, $\alpha_i(t,x)$, $\alpha_{ij}(t,x)$, $i,j = 1,2$ and a vector $u_0 = u(0,x,y)$ are given data. The symbol $\times$ denotes the vector product of two vectors. All functions in this paper are real.

It is assumed that the operator L is regular elliptic. It means that there exist two numbers $\mu_1, \mu_2 > 0$ such that the inequality

$$\mu_1(\xi_1^2 + \xi_2^2) \le \sum_{i,j=1}^{2} \alpha_{ij}(t,x)\xi_i\xi_j \le \mu_2(\xi_1^2 + \xi_2^2)$$

holds for all $\xi_1, \xi_2 \in \mathbb{R}$ and for all $x \in \Omega$, $0 \le t < T$.

Let $\| \|_{2,\Omega}^{(l)}$, $l = 0,1,2,\dots$ denote the usual norm of the Sobolev space $W_2^l(\Omega)$ and $W_{2,\infty}^l(Q)$ is a Banach space with norm

$$\|u\|_{2,\infty,Q}^{(l)} = \sup_{y\in I}\|u(y)\|_{2,\Omega}^{(l)}.$$

We define a Banach space $C^k(0,T;X)$ of k times continuously differentiable functions in $[0;T]$ with values in a Banach space X and with finite norm

$$\sum_{i=1}^{k} \sup_{t\in[0,T]} \|u^{(i)}(t)\|_X,$$

where $\| \|_X$ denotes the norm in the space X.

As the main result we prove the following theorem.

Theorem 1.1. *Let $\partial\Omega \subset C^3$ and $\varphi \in C(0,T;W_2^{5/2}(\partial\Omega))$, $u_0 \in W_{2,\infty}^{2,0}(Q)$. If $a^k \in C(0,T;W_{2,\infty}^{2,0}(Q))$, $k = \overline{1,5}$; $\alpha_{ij}, \alpha_i, \alpha \in C(0,T;C^1(\Omega))$, $i,j = 1,2$; b_{ij}, $b_i \in C(0,T;W_2^2(\Omega))$, $i = 1,2,3, j = 1,2$ and the problem (1.5), (1.6) has a unique solution in the space $W_2^3(\Omega)$ for each fixed $t \leq T$, then there exists $T_0 > 0$ that the problem (1.1)–(1.6) for $T < T_0$ has a unique solution*

$$u \in C^1(0,T;W_{2,\infty}^{2,0}(Q)), v \in C(0,T;W_{2,\infty}^{2,0}(Q)),$$

$$w \in C^1(0,T;W_2^2(\Omega)), z \in C(0,T;W_2^3(\Omega)).$$

2. AUXILIARIES

Lemma 2.1. *Suppose that the conditions of smoothness of given data are satisfied. If $u \in C(0,T;W_{2,\infty}^{2,0}(Q))$ and the problem (1.5), (1.6) for each $0 \leq t < T$ has a unique solution in the space $W_2^3(\Omega)$, then*

$$v \in C(0,T;W_{2,\infty}^{2,0}(Q)), w \in C^1(0,T;W_2^2(\Omega)), z \in C(0,T;W_2^3(\Omega))$$

and for all $0 \leq t < T$

$$\|v(t)\|_{2,\infty,Q}^{(2)}, \|w(t)\|_{2,\Omega}^{(2)}, \|z(t)\|_{2,\Omega}^{(3)} \leq c_1 \|u(t)\|_{2,\infty,Q}^{(2)}. \tag{2.1}$$

Here a constant c_1 is independent of functions u, v, w, z.

Proof. We will assume there is a number μ such that

$$\|a_i^1\|_{2,\Omega}^{(2)}, \|a_{ij}^k\|_{2,\Omega}^{(2)}, \|b_i\|_{2,\Omega}^{(2)}, \|b_{ij}\|_{2,\Omega}^{(2)} \leq \mu \tag{2.2}$$

for $k = \overline{2,5}$, $i,j = 1,2,3$, for all $0 \leq t < T$ and for almost all $y \in I$.

By the Minkowski and Cauchy inequalities [8] from (1.4) it follows that

$$\|w\|_{2,\Omega}^2 \leq \Big\{ \int_I \Big(\sum_{i,j=1}^{3} (\max_\Omega |a_{ij}^5|)^2 \|u\|_{2,\Omega}^2 \Big)^{1/2} dy \Big\}^2,$$

because $|a^5 u| \leq \sum_{i,j=1}^{3} |a_{ij}^5|^2 |u|^2$. Since $\Omega \subset \mathbb{R}^2$, by Sobolev's imbedding theorem,

$$\max_\Omega |u| \leq c_2 \|u\|_{2,\Omega}^{(2)} \tag{2.3}$$

for $u \in W_2^2(\Omega)$ and

$$\|u\|_{4,\Omega} \leq c_3 \|u\|_{2,\Omega}^{(1)} \tag{2.4}$$

for $u \in W_2^1(\Omega)$, where the constants c_1, c_2 do not depend on u [8].

Taking into account (2.2), (2.3) we get the estimate

$$\|w\|_{2,\Omega} \leq 3\mu c_2 mesI \|u\|_{2,\Omega}.$$

Next, we have

$$\|w_x\|_{2,\Omega}^2 \leq \sum_{k=1}^{2} \Big\{ \int_I \big[(\sum_{i,j=1}^{3} (\max_\Omega |a_{ij}^5|)^2 \|u_{x_k}\|_{2,\Omega}^2)^{1/2}$$

$$+ (\sum_{i,j=1}^{3} \|a_{ijx_k}^5\|_{2,\Omega}^2 (\max_\Omega |u|)^2)^{1/2} \big] \, dy \Big\}^2.$$

This inequality combined with (2.2)–(2.4) gives us

$$\|w_x\|_{2,\Omega} \leq 3\mu c_2 mesI (2\|u\|_{2,\Omega}^{(2)} + \sqrt{2}\|u_x\|_{2,\Omega}).$$

Applying the Minkovski and Cauchy inequalities and using (2.2)–(2.4) we evaluate

$$\|w_{xx}\|_{2,\Omega} \leq \{27\mu(c_2 + 2c_3)\|u\|_{2,\Omega}^{(2)} + 9c_2\mu\|u_{xx}\|_{2,\Omega}\} mesI.$$

Now, estimates of w, w_x and w_{xx} imply the bound

$$\|w(t)\|_{2,\Omega}^{(2)} \leq c_4 \|u(t)\|_{2,\Omega}^{(2)}, \tag{2.5}$$

which is valid for all $t \in [0,T]$ and for almost $y \in I$. Here c_4 is a constant independent of u and w.

Let F denote the right-hand side of (1.5). We now will obtain the estimate of F. Using the Cauchy inequality and (2.2)–(2.4) we bound

$$(\|b_i w_i\|_{2,\Omega}^{(1)})^2 \leq c_2^2 \mu \big\{ \mu \|w_i\|_{2,\Omega}^2 + 2 \sum_{k=1}^{2} \big[\mu c_3^2 (\|w_i\|_{2,\Omega}^{(1)})^2 + \|w_{ix_k}\|_{2,\Omega}^2 \big] \big\}, i = 1,2,3. \tag{2.6}$$

The other term of F is bounded above as follows

$$(\|b_{ij} w_{ix_j}\|_2^{(1)})^2 \leq (\max_\Omega |b_{ij}|)^2 \|w_{ix_j}\|_{2,\Omega}^2$$

$$+2\sum_{k=1}^{2}\{(\max_{\Omega}|b_{ij}|)^2\|w_{ix_jx_k}\|^2_{2,\Omega}+\|b_{ijx_k}\|^2_{4,\Omega}\|w_{ix_j}\|^2_{4,\Omega}\}, i=1,2,3, j=1,2.$$

Because of (2.3), (2.4) the right-hand side is less than

$$\mu^2\{c_2\|w_{ix_j}\|^2_{2,\Omega}+2\sum_{k=1}^{2}(c_2^2\|w_{ix_jx_k}\|^2_{2,\Omega}+c_3^4(\|w_{ix_j}\|^{(1)}_{2,\Omega})^2)\}, i=1,2,3, j=1,2. \tag{2.7}$$

Finally, combining (2.6), (2.7) we find that

$$(\|F\|^{(1)}_{2,\Omega})^2\le 9\mu^2\sum_{i=1}^{3}\sum_{j=1}^{2}\{c_2^2(\|w_{ix_j}\|^2_{2,\Omega}+\|w_i\|^2_{2,\Omega})$$

$$+2c_2^2(\sum_{k=1}^{2}\|w_{ix_jx_k}\|^2_{2,\Omega}+\|w_{ix_j}\|^2_{2,\Omega})+2c_2^2c_3^2(\|w_i\|^{(1)}_{2,\Omega})^2+c_3^4(\|w_{ix_j}\|^{(1)}_{2,\Omega})^2\}.$$

Therefore

$$\|F\|^{(1)}_{2,\Omega}\le 3\mu(3c_2^2+4c_2^2c_3^2+c_3^4)^{1/2}\|w\|^{(2)}_{2,\Omega}.$$

This inequality combined with (2.5) for all $0\le t<T$ gives us

$$\|F(t)\|^{(1)}_{2,\Omega}\le c_5\|u(t)\|^{(2)}_{2,\Omega}, \tag{2.8}$$

where c_5 does not depend on u.

Hence for each fixed t from $[0,T]$ the right-hand side of (1.5) is a function from the space $W_2^1(\Omega)$. Therefore by assumptions of Theorem the problem (1.5), (1.6) has a unique solution in the space $W_2^3(\Omega)$ and for each $0\le t<T$ the following inequality holds [1]

$$\|z(t)\|^{(3)}_{2,\Omega}\le c_6\|F(t)\|^{(1)}_{2,\Omega}+\|\varphi(t)\|^{(5/2)}_{2,\partial\Omega},$$

where the constant c_6 is independent of z.
Further, applying (2.8) we have

$$\|z(t)\|^{(3)}_{2,\Omega}\le c_7\|u(t)\|^{(2)}_{2,\Omega}+c_6\|\varphi(t)\|^{(5/2)}_{2,\partial\Omega} \tag{2.9}$$

for all $t\in[0,T)$.

Our next step is to estimate the function v. By the Cauchy inequality and (2.2)–(2.4) we obtain

$$\|a^2u\|_{2,\Omega}\le\{\sum_{i,j=1}^{3}(\max_{\Omega}|a_{ij}|)^2\|u\|^2_{2,\Omega}\}^{1/2}\le 3c_2\mu\|u\|_{2,\Omega}.$$

Similarly

$$\|(a^2u)_{x_k}\|_{2,\Omega} \le c_2\mu(\|u\|^{(2)}_{2,\Omega} + \|u_{x_k}\|_{2,\Omega}),\ k=1,2$$

and

$$\|(a^2u)_{x_kx_l}\|_{2,\Omega} \le 27\mu(c_1+2c_2^2)\|u\|^{(2)}_{2,\Omega} + 9c_1\mu\|u_{xx}\|_{2,\Omega},\ k,l=1,2.$$

From the last three inequalities we conclude that

$$\|a^2u\|^{(2)}_{2,\Omega} \le 3\mu\{(11+3\sqrt{3})c_2 + 18c_3^2\}\|u\|^{(2)}_{2,\Omega}.$$

The same inequality is true for the second term of right-hand side of (1.3) a^3w. It easy to verify that the third term of right-hand side of (1.3) is bounded above as follows

$$\|(a^4z_x)\|^{(2)}_{2,\Omega} \le 2\mu\{(3\sqrt{3}+4\sqrt{6})c_2 + 6\sqrt{6}c_3^2\}\|z\|^{(3)}_{2,\Omega}.$$

Now, from these estimates using (2.5), (2.9) we have

$$\|v(t)\|^{(2)}_{2,\Omega} \le c_8\|u(t)\|^{(2)}_{2,\Omega} \tag{2.10}$$

for all $0 \le t < T$ and for almost all $y \in I$. A constant c_8 depends only on given data and the numbers μ, c_2, c_3.

From (2.5), (2.9), (2.10) we get the estimates of Lemma 2.1. The continuity of functions v, w, z with respect to t follows from (2.1) and the properties of u. ■

Lemma 2.2. *Let $u_0 \in W^{2,0}_{2,\infty}(Q)$ and $a^1, v \in C(0,T;W^{2,0}_{2,\infty}(Q))$. Then $u \in C^1(0,T;W^{2,0}_{2,\infty}(Q))$ and for all $0 \le t < T$*

$$\|u(t)\|^{(2)}_{2,\infty,Q} \le \|u_0\|^{(2)}_{2,\infty,Q} + c_9\int_0^t \big(\|u(\tau)\|^{(2)}_{2,\infty,Q}\big)^2\big(1+\|u(\tau)\|^{(2)}_{2,\infty,Q}\big)\,d\tau. \tag{2.11}$$

Here a constant c_9 does not depend on u.

Proof. In the space $C(0,T;W^{2,0}_{2,\infty}(Q))$ we consider the function $A(t,u) = a^1u\times(u\times v+v)$. It easy to prove that $A(u,t) \in C(0,T;W^{2,0}_{2,\infty}(Q))$. Therefore the problem (1.1), (1.2) is equivalent to the equation

$$u(t) = u_0 + \int_0^t A(\tau)\,d\tau.$$

By Minkowski's inequality

$$\|u(t)\|^{(2)}_{2,\infty,Q} \le \|u_0(t)\|^{(2)}_{2,\infty,Q} + \int_0^t \big[\|A(\tau)\|_{2,\infty,Q}$$

$$+ \sum_{k=1}^{2} \|A_{x_k}(\tau)\|_{2,\infty,Q} + \sum_{k,l=1}^{2} \|A_{x_k x_l}(\tau)\|_{2,\infty,Q}\big]\, d\tau. \tag{2.12}$$

We will estimate each norm separately.

Using the Cauchy inequality and (2.2)–(2.4) we bound

$$\|A\|_{2,\Omega} \le \sqrt{3}c_2^2\mu(1 + c_1\|u\|^{(2)}_{2,\Omega})\|u\|^{(2)}_{2,\Omega}\|v\|_{2,\Omega}.$$

For the norm of the first derivative we obtain

$$\|A_{x_k}\|_{2,\Omega} \le \sqrt{3}c_2^2\mu\big\{\big(1 + c_2\|u\|^{(2)}_{2,\Omega}\big)\|u\|^{(2)}_{2,\Omega}\|v\|^{(2)}_{2,\Omega}$$

$$+(1 + 2c_2\|u\|^{(2)}_{2,\Omega})\|v\|^{(2)}_{2,\Omega}\|u_{x_k}\|_{2,\Omega} + (1 + c_2\|u\|^{(2)}_{2,\Omega})\|u\|^{(2)}_{2,\Omega}\|v_{x_k}\|_{2,\Omega}\big\},\ k = 1, 2.$$

The norm of the second derivative is bounded above as follows

$$\|A_{x_k x_l}\|_{2,\Omega} \le \sqrt{3}c_2\mu\big\{c_2(c_2 + 12c_3^2)(\|u\|^{(2)}_{2,\Omega})^2\|v\|^{(2)}_{2,\Omega} + (c_2 + 6c_3^2)\|u\|^{(2)}_{2,\Omega}\|v\|^{(2)}_{2,\Omega}$$

$$+c_2(1 + c_2\|u\|^{(2)}_{2,\Omega})\|v\|^{(2)}_{2,\Omega}\|u_{x_k x_l}\|_{2,\Omega} + c_2\|u\|^{(2)}_{2,\Omega}\big[c_2\|u_{x_k x_l}\|_{2,\Omega}\|v\|^{(2)}_{2,\Omega}$$

$$+(1 + c_2\|u\|^{(2)}_{2,\Omega})\|v_{x_k x_l}\|_{2,\Omega}\big]\big\},\ k, l = 1, 2.$$

Now, the estimates of A, A_{x_k} and $A_{x_k x_l}$ imply that the integrand expression in (2.12) for all $0 \le t < T$ and almost all $y \in I$ is less than

$$c_{10}\|u(t)\|^{(2)}_{2,\Omega}\|v(t)\|^{(2)}_{2,\Omega}\big(1 + \|u(t)\|^{(2)}_{2,\Omega}\big),$$

where c_{10} does not depend on u and v.

Finally, taking into account (2.1), we get (2.11). ■

Lemma 2.3. *Under the assumptions of Theorem 1.1 there exists $T_0 > 0$ that for all $T < T_0$*

$$\max_{t\in[0,T]}\big\{\|u(t)\|^{(2)}_{2,\infty,Q}, \|v(t)\|^{(2)}_{2,\infty,Q}, \|w(t)\|^{(2)}_{2,\Omega}, \|z(t)\|^{(2)}_{2,\Omega}\big\} \le c_{11}. \tag{2.13}$$

The constant c_{11} is independent of u, v, w, z.

Proof. Let

$$\beta(t)=\|u_0\|^{(2)}_{2,\infty,Q}+c_9\int\limits_0^t\big(\|u(\tau)\|^{(2)}_{2,\infty,Q}\big)^2\big(1+\|u(\tau)\|^{(2)}_{2,\infty,Q}\big)\,d\tau.$$

This implies

$$\frac{d\beta}{dt}=c_9\big(\|u(t)\|^{(2)}_{2,\infty,Q}\big)^2\big(1+\|u(t)\|^{(2)}_{2,\infty,Q}\big).$$

From the definition of β and (2.11) it follows that

$$\|u(t)\|^{(2)}_{2,\infty,Q}\le\beta(t). \tag{2.14}$$

Therefore

$$\frac{d\beta}{dt}\le c_9\beta^2(1+\beta) \tag{2.15}$$

and in addition

$$\beta_0=\beta(0)=\|u_0\|^{(2)}_{2,\infty,Q}. \tag{2.16}$$

The solution of the differential problem

$$\frac{d\beta}{dt}=c_9\beta^2(1+\beta),$$

$$\beta_0=\beta(0)$$

is an increasing function which majorize the solution of the problem (2.15), (2.16). Therefore there exists a number $T_0>0$ such that the function $\beta(t)$ is bounded in the interval $[0,T]$ for all $T<T_0$.

Now the statement of Lemma 2.3 follows from (2.14) and (2.1). ■

3. PROOF OF THEOREM

In the space $C(0,T;W^{2,0}_{2,\infty}(Q))$ we define the operator

$$B(u(t))=u_0+\int\limits_0^t a^1u(\tau)\times\big(u(\tau)\times v(\tau)+v(\tau)\big)\,d\tau.$$

We will prove that B is a contractive operator. Let u^1,u^2 be two functions of $C(0,T;W^{2,0}_{2,\infty}(Q))$, while v^1,v^2 are the corresponding solutions of (2.3)–(2.6). Let $\eta=u^2-u^1$ and $\beta=v^2-v^1$. Then

$$B(u^2)-B(u^1)=\int\limits_0^t\psi(\tau)\,d\tau, \tag{3.1}$$

where

$$\psi = a^1 u^2 \times (u^2 \times v^2 + v^2) - a^1 u^1 \times (u^1 \times v^1 + v^1)$$

$$= a^1 \eta \times (u^2 \times v^2 + v^2) + a^1 u^1 \times (\eta \times v^2 + u^1 \times \beta + \beta).$$

We obtain the estimate of ψ similarly to the estimates of lemmas. Using the Minkowski and Cauchy inequalities and applying (2.2)– (2.4) we have

$$\|\psi\|_{2,\Omega} \leq \sqrt{3} c_2^2 \mu \big\{ (1 + c_1 \|u^2\|_{2,\Omega}^{(2)}) \|v^2\|_{2,\Omega}^{(2)} \|\eta\|_{2,\Omega}$$

$$+ \big[c_2 \|v^2\|_{2,\Omega}^{(2)} \|\eta\|_{2,\Omega} + (1 + c_2 \|v^1\|_{2,\Omega}^{(2)}) \|\beta\|_{2,\Omega}) \big] \|v^1\|_{2,\Omega}^{(2)} \big\}.$$

The first derivatives of ψ we bound

$$\|\psi_{x_k}\|_{2,\Omega} \leq \sqrt{3} c_2^2 \mu \big\{ (1 + c_2 \|u^2\|_{2,\Omega}^{(2)}) \|v^2\|_{2,\Omega}^{(2)} (\|\eta\|_{2,\Omega}^{(2)} + \|\beta_{x_k}\|_{2,\Omega})$$

$$+ \big[c_2 \|\eta\|_{2,\Omega}^{(2)} \|v^2\|_{2,\Omega}^{(2)} + (1 + c_2 \|u^1\|_{2,\Omega}^{(2)}) \|\beta\|_{2,\Omega}^{(2)} \big] (\|u^1\|_{2,\Omega}^{(2)} + \|u^1_{x_k}\|_{2,\Omega})$$

$$+ \big[c_2 \|u^2_{x_k}\|_{2,\Omega} \|v^2\|_{2,\Omega}^{(2)} + (1 + c_2 \|u^2\|_{2,\Omega}^{(2)}) \|v^2_{x_k}\|_{2,\Omega} \big] \|\eta\|_{2,\Omega}^{(2)} + c_2 (\|\eta_{x_k}\|_{2,\Omega} \|v^2\|_{2,\Omega}^{(2)}$$

$$+ \|\eta\|_{2,\Omega}^{(2)} [\|v^2_{x_k}\|_{2,\Omega} + \|u^1_{x_k}\|_{2,\Omega} \|\beta\|_{2,\Omega}^{(2)} + (1 + c_2 \|u^1\|_{2,\Omega}^{(2)}) \|\beta_{x_k}\|_{2,\Omega} \big\}, \ k = 1, 2.$$

The second derivatives of ψ are estimated as follows

$$\|\psi_{x_k x_l}\|_{2,\Omega} \leq \sqrt{3} c_2 \mu \big\{ (1 + c_1 \|u^2\|_{2,\Omega}^{(2)}) \|v^2\|_{2,\Omega}^{(2)} \big[(c_2 + 2c_3^2) \|\eta\|_{2,\Omega}^{(2)} + c_1 \|\eta_{x_k x_l}\|_{2,\Omega} \big]$$

$$+ c_2 \big[\|\eta\|_{2,\Omega}^{(2)} \|v^2\|_{2,\Omega}^{(2)} + (1 + \|u^1\|_{2,\Omega}^{(2)}) \|\beta\|_{2,\Omega}^{(2)} \big] \big[(c_2 + 2c_3^2) \|u^1\|_{2,\Omega}^{(2)} + c_2 \|u^1_{x_k x_l}\|_{2,\Omega} \big]$$

$$+ 4c_3^2 \big[(1 + \|u^2\|_{2,\Omega}^{(2)}) \|v^2\|_{2,\Omega}^{(2)} \|\eta\|_{2,\Omega}^{(2)} + (2c_2 \|\eta\|_{2,\Omega}^{(2)} \|v^2\|_{2,\Omega}^{(2)} + (1 + 2c_2 \|u^1\|_{2,\Omega}^{(2)}) \|\beta\|_{2,\Omega}^{(2)})$$

$$\times \|u^1\|_{2,\Omega}^{(2)} \big] + c_2 \|\eta\|_{2,\Omega}^{(2)} \big[(c_2 \|u^2_{x_k x_l}\|_{2,\Omega} + 2c_3 \|u^2\|_{2,\Omega}^{(2)}) \|v^2\|_{2,\Omega}^{(2)} + (1 + c_2 \|u^2\|_{2,\Omega}^{(2)})$$

$$\times \|v^2_{x_k x_l}\|_{2,\Omega} \big] + c_2 \|u^1\|_{2,\Omega}^{(2)} \big[(c_2 \|\eta_{x_k x_l}\|_{2,\Omega} + 2c_3^2 \|\eta\|_{2,\Omega}^{(2)}) \|v^2\|_{2,\Omega}^{(2)} + c_2 \|\eta\|_{2,\Omega}^{(2)} \|v^2_{x_k x_l}\|_{2,\Omega}$$

$$+ (c_2 \|u^1_{x_k x_l}\|_{2,\Omega} + 2c_3^2 \|u^1\|_{2,\Omega}^{(2)}) \|\beta\|_{2,\Omega}^{(2)} + (1 + c_2 \|u^1\|_{2,\Omega}^{(2)}) \|\beta_{x_k x_l}\|_{2,\Omega} \big] \big\}, \ k, l = 1, 2.$$

These estimates are valid for all $0 \leq t < T$ and for almost all $y \in I$. The last two inequalities combined with (2.11) gives us

$$\|\psi(t)\|_{2,\Omega}^{(2)} \leq c_{12} \|\eta(t)\|_{2,\Omega}^{(2)} + c_{13} \|\beta(t)\|_{2,\Omega}^{(2)} \tag{3.2}$$

for all $0 \leq t < T$ and for almost all $y \in I$. Here c_{12}, c_{13} do not depend on η and β. From the equation (2.3) and the inequality (2.10) we obtain

$$\|\beta(t)\|_{2,\Omega}^{(2)} \leq c_{14} \|\eta(t)\|_{2,\Omega}^{(2)}$$

for all $0 \le t < T$ and for almost all $y \in I$. Therefore from here and (3.2) we get the estimate

$$\|\psi(t)\|^{(2)}_{2,\infty,Q} \le c_{15}\|\eta(t)\|^{(2)}_{2,\infty,Q} \tag{3.3}$$

for all $0 \le t < T$ with the constant c_{15} independent of η. Since $\eta = u^2 - u^1$, we conclude from (3.1) and (3.3) that

$$\|B(u^2) - B(u^1)\|^{(2)}_{2,\infty,Q} \le c_{15} \int\limits_0^t \|u^2 - u^1(\tau)\|^{(2)}_{2,\infty,Q}\, d\tau$$

for all $0 \le t < T$ and c_{15} does not depend on u^1, u^2.

Thus for $t < 1/c_{15}$, $B(u(t))$ is the contractive operator in the space $C(0,T;W^{2,0}_{2,\infty}(Q))$. Now the statement of Theorem follows from Lemmas 2.1, 2.2.

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NETIESINIO ĮMAGNETINIMO UŽDAVINIO IŠSPRENDŽIAMUMAS

P. KATAUSKIS

Nagrinėjama netiesinių lygčių sistema, aprašanti medžiagos, sudarytos iš viendomenių dalelių, įmagnetinimą. Matematinis modelis pasiūlytas V. Skakausko 1973 m. Atskiros dalelės judėjimą magnetiniame lauke apibrėžia netiesinė vektorinė Landay-Lifšico lygtis. Medžiagos įmagnetinimas aprašomas Maksvelo lygtimis. Tiriamoji lygčių sistema gauta įvedus vektorinį gradientą. Įrodyta lokalaus pagal laiką sprendinio egzistencija ir vienatis erdvėse $C(0,T^0,X)$, čia X – Sobolevo erdvės.

Teiginys pagrindžiamas parodant, kad tam tikras operatorius erdvėje
$C(0,T^0,X)$ yra suspaudžiantysis, kai laiko intervalas yra trumpas. Įrodymas paremtas aprioriniais įverčiais, taikomos įdėjimo teoremos, todėl gautas rezultatas teisingas, kai nagrinėjama aprėžta sritis plokštumoje.