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# THE FINITE DIFFERENCE SCHEME FOR MATHEMATICAL MODELING OF WOOD DRYING PROCESS

R. ČIEGIS[1], V. STARIKOVIČIUS[1,2]

[1] *Vilnius Gediminas Technical University*
Saulėtekio al. 11, LT-2040, Vilnius, Lithuania
E-mail: rc@fm.vtu.lt

[2] *Vilnius University, Faculty of Mathematics and Informatics*
Naugarduko 24, LT-2600, Vilnius, Lithuania
E-mail: vs@sc.vtu.lt

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**ABSTRACT**

This work discusses issues on the design of finite difference schemes for modeling the moisture movement process in the wood. A new finite difference scheme is proposed. The stability and convergence in the maximum norm are proved for different types of boundary conditions.

## 1. INTRODUCTION

We consider a system of linear differential equations

$$(1-m)\frac{\partial c}{\partial t} + \nu\frac{\partial c}{\partial x} = \frac{\partial}{\partial x}\left(d(x)\frac{\partial c}{\partial x}\right) - \frac{\partial s}{\partial t}, \quad 0 < x < 1, \tag{1.1}$$

$$\frac{\partial s}{\partial t} = p(lc - s), \quad 0 \le x \le 1, \tag{1.2}$$

with the boundary conditions

$$-d(0)\frac{\partial c(t,0)}{\partial x} + \mu c(t,0) = g_0(t), \tag{1.3}$$

$$d(1)\frac{\partial c(t,1)}{\partial x} + \mu c(t,1) = g_1(t), \tag{1.4}$$

and initial conditions

$$c(0,x) = c_0, \quad s(0,x) = s_0, \quad 0 \le x \le 1. \tag{1.5}$$

It describes the moisture movement process in the wood. Here $c(x,t)$ is the moisture content in the wood cells and $s(x,t)$ is the moisture content in the walls of the wood cells. In the model it is assumed that there is a moisture exchange between two environments and diffusion and convection processes are determining the moisture movement in the wood cells.

Now we state main assumptions about the coefficients in equations (1.1)-(1.2). The diffusion coefficient $d(x)$ satisfies

$$0 < d_0 \le d(x) \le d_1,$$

for many problems $d$ also depends on $c$, the porosity coefficient $m$ satisfies

$$0 < m < 1,$$

the convection coefficient $\nu$ and exchange rate $p$ are nonnegative numbers. The parameter $l$ is also nonnegative and depends on vapor's pressure, relative humidity and chemical potential. The parameter $\mu$ is positive for the third type boundary condition, and $\mu = 0$ for the Neumann boundary condition.

Finally, assuming that side surfaces are isolated and wood sample is very long in comparison with the other dimensions, we get an one dimensional model.

Such mathematical model for the moisture movement in the wood is proposed and used in [1; 6]. We note that in many applications the model is restricted to a simple diffusion equation ( see, [2; 3] ).

The mathematical model of this paper, i.e. the system of differential equations (1.1)-(1.2), describes a broad class of real world problems. For example the transport of soluble substances in rivers is described by the same processes and a very similarly looking system of differential equations [4].

The rest of the paper is organized as follows. In Section 2 we formulate a finite difference scheme. In Section 3 we prove the stability and convergence of this scheme in the maximum norm. Different types of boundary conditions are investigated. Some concluding remarks are made in Section 4.

## 2. FINITE DIFFERENCE SCHEME

We define a family of discrete grids

$$\Omega_{\tau\times h} = \{(t^n, x_i) : \, t^n = n\tau, \, x_i = ih, \, i = 0, \dots, N, \, n = 0, \dots, K\},$$

where $Nh = 1$ and $K\tau = T$. Common notation is used in our paper (see [5])

$$y_t = \frac{y_i^{n+1} - y_i^n}{\tau}, \quad y_{\bar{x}}^n = \frac{y_i^n - y_{i-1}^n}{h},$$
$$y_x^n = \frac{y_{i+1}^n - y_i^n}{h}, \quad a_i = \frac{d(x_i) + d(x_{i-1})}{2}.$$

The finite difference approximation of (1.1)-(1.5) is defined as follows:

$$(1-m)u_t + \nu u_{\bar{x}}^{n+1} = (a u_{\bar{x}}^{n+1})_x - p(lu^{n+1} - v^n), \; i = 1, \dots, N-1, \tag{2.1}$$

$$\frac{h}{2}(1-m)u_{t,0} + \frac{h}{2}\frac{\nu}{d(0)}(\mu u_0^{n+1} - g_0^{n+1})$$
$$= a_1 u_{x,0}^{n+1} - \mu u_0^{n+1} + g_0^{n+1} - \frac{h}{2} p(l u_0^{n+1} - v_0^n), \tag{2.2}$$

$$\frac{h}{2}(1-m)u_{t,N} + \frac{h}{2}\nu u_{\bar{x},N}^{n+1} = -a_N u_{\bar{x},N}^{n+1}$$
$$- \mu u_N^{n+1} + g_1^{n+1} - \frac{h}{2} p(l u_N^{n+1} - v_N^n), \tag{2.3}$$

$$v_t = p(lu^{n+1} - v^{n+1}), \qquad i = 0, \dots, N, \tag{2.4}$$

$$u_i^0 = c_0, \quad v_i^0 = s_0, \quad i = 0, \dots, N. \tag{2.5}$$

Approximating boundary conditions we assume that equation (1.1) is also satisfied on the boundary of the computational region.

Next we describe the algorithm for finding a solution of this finite difference scheme. First from system of equations (2.1), (2.2), (2.3) we find $u^{n+1}$. The matrix of this system is three-diagonal, hence the factorization method can be used to solve it efficiently. Then we can compute $v^{n+1}$ explicitly from equation (2.4).

Let's denote truncation errors of equations (2.1)–(2.4) by $\Psi_{1,i}^{n+1}$, $\Psi_{1,0}^{n+1}$, $\Psi_{1,N}^{n+1}$ and $\Psi_{2,i}^{n+1}$, respectively.

**Lemma 2.1.** *If solutions $c(x,t)$ and $s(x,t)$ of system (1.1)-(1.5) have bounded derivatives $\frac{\partial^2 c}{\partial t^2}$, $\frac{\partial^2 s}{\partial t^2}$, $\frac{\partial^4 c}{\partial x^4}$, $d''(x)$, then*

$$\begin{aligned}
|\Psi_{1,i}^{n+1}| &\leq C(\tau + h), \quad i = 1, \dots, N-1, \\
|\Psi_{1,0}^{n+1}| &\leq C(\tau h + h^2), \\
|\Psi_{1,N}^{n+1}| &\leq C(\tau h + h^2), \\
|\Psi_{2,i}^{n+1}| &\leq C\tau, \quad i = 0, \dots, N.
\end{aligned}$$

The proof of lemma follows from [5].

## 3. ANALYSIS OF STABILITY

In this section we investigate the stability of the finite difference scheme in the maximum norm.

Let

$$\tilde{u}_i^n = c(t^n, x_i) - u_i^n, \quad \tilde{v}_i^n = s(t^n, x_i) - v_i^n$$

be the errors of the discrete solutions. These functions satisfy the following finite difference scheme:

$$(1-m)\tilde{u}_t + \nu \tilde{u}_{\bar{x}}^{n+1} = (a\tilde{u}_{\bar{x}}^{n+1})_x - p(l\tilde{u}^{n+1} - \tilde{v}^n) + \Psi_{1,i}^{n+1}, \qquad i = 1, \dots, N-1, \tag{3.1}$$

$$\frac{h}{2}(1-m)\tilde{u}_{t,0} + \mu\left(\frac{\nu h}{2d(0)} + 1\right)\tilde{u}_0^{n+1} = a_1 \tilde{u}_{x,0}^{n+1} - \frac{hp}{2}(l\tilde{u}_0^{n+1} - \tilde{v}_0^n) + \Psi_{1,0}^{n+1}, \tag{3.2}$$

$$\frac{h}{2}(1-m)\tilde{u}_{t,N} + \mu \tilde{u}_N^{n+1} = -\left(a_N + \frac{h}{2}\nu\right)\tilde{u}_{\bar{x},N}^{n+1} - \frac{hp}{2}(l\tilde{u}_N^{n+1} - \tilde{v}_N^n) + \Psi_{1,N}^{n+1}, \tag{3.3}$$

$$\tilde{v}_t = p(l\tilde{u}^{n+1} - \tilde{v}^{n+1}) + \Psi_{2,i}^{n+1}, \quad i = 0, \dots, N, \tag{3.4}$$

$$\tilde{u}_i^0 = 0, \ \ \tilde{v}_i^0 = 0, \ \ i = 0, \dots, N. \tag{3.5}$$

Our goal is to prove unconditional stability and convergence of scheme (2.1)-(2.5) for the Neumann and third type boundary conditions.

### 3.1. Third type boundary conditions

In this section we assume that $\mu > 0$. Let define the following norms

$$\|\Psi_2^k\|_C = \max_{0 \le i \le N} |\Psi_{2,i}^k|,$$

$$\|\tilde{\Psi}\|_1 = \frac{T}{1-m}\left(\max_{1\le k\le K}\left(\max_{0<i<N}|\Psi_{1,i}^k|\right) + \max_{1\le k\le K}\|\Psi_2^k\|_C\right) + \frac{1}{\mu}\left(\frac{h}{2}\max_{1\le k\le K}\|\Psi_2^k\|_C + \max_{1\le k\le K}|\Psi_{1,0}^k| + \max_{1\le k\le K}|\Psi_{1,N}^k|\right).$$

Then we have

**Theorem 3.1.** *The finite difference scheme* (2.1)-(2.5) *with* $\mu > 0$ *is unconditionally stable, and the errors of discrete solutions satisfy the estimations*

$$\begin{aligned} &\|\tilde{u}^{n+1}\|_C \le \|\tilde{\Psi}\|_1 \,, \\ &\|\tilde{v}^{n+1}\|_C \le l\|\tilde{\Psi}\|_1 + \frac{1}{p} \max_{1\le k\le K} \|\Psi_2^k\|_C \,. \end{aligned} \tag{3.6}$$

*Proof.* Let assume that

$$\max_{1\le k\le K} \|\tilde{u}^k\|_C = |\tilde{u}_i^{n+1}|\,.$$

We consider three different cases.

*Case* 1: $i = 0$. From equation (3.2) we obtain

$$\left(\mu\left(\frac{\nu h}{2d(0)} + 1\right) + \frac{hpl}{2}\right)|\tilde{u}_0^{n+1}| \le \frac{hp}{2}|\tilde{v}_0^n| + |\Psi_{1,0}^{n+1}|.$$

Similarly we consider

$$\max_{1\le k\le n} \|\tilde{v}^k\|_C = |\tilde{v}_j^q|\,.$$

From equation (3.4) we get

$$\begin{aligned} |\tilde{v}_j^q| &\le l|\tilde{u}_j^q| + \frac{1}{p}|\Psi_{2,j}^q| \\ &\le l\|\tilde{u}^{n+1}\|_C + \frac{1}{p}\|\Psi_2^q\|_C. \end{aligned} \tag{3.7}$$

Therefore we have proved the inequality

$$|\tilde{u}_0^{n+1}| \le \frac{1}{\mu}\left(\frac{h}{2}\|\Psi_2^q\|_C + |\Psi_{1,0}^{n+1}|\right).$$

*Case* 2: $i = N$. Applying the same argument for equation (3.3) and using (3.7) we prove that

$$\begin{aligned} \left(\mu + \frac{hpl}{2}\right)|\tilde{u}_N^{n+1}| &\le \frac{hp}{2}|\tilde{v}_N^n| + |\Psi_{1,N}^{n+1}|, \\ |\tilde{u}_N^{n+1}| &\le \frac{1}{\mu}\left(\frac{h}{2}\|\Psi_2^q\|_C + |\Psi_{1,N}^{n+1}|\right). \end{aligned}$$

*Case* 3: $0 < i < N$. From equation (3.1) using (3.7) we obtain

$$\begin{aligned} \left(1 + \frac{pl\tau}{1-m}\right)|\tilde{u}_i^{n+1}| &\le |\tilde{u}_i^n| + \frac{p\tau}{1-m}|\tilde{v}_i^n| + \frac{\tau}{1-m}|\Psi_{1,i}^{n+1}|, \\ |\tilde{u}_i^{n+1}| &\le \|\tilde{u}^n\|_C + \frac{\tau}{1-m}\left(\max_{0<i<N}|\Psi_{1,i}^{n+1}| + \|\Psi_2^q\|_C\right). \end{aligned}$$

Proceeding by induction we repeat the same procedure for

$$\max_{1\le k\le n} \|\tilde{u}^k\|_C$$

until we get the boundary point or reach the initial time layer. ■

*Corollary 3.1.* The global error of the difference solution can be estimated as

$$\|\tilde{u}^n\|_C \le C(\tau + h), \quad \|\tilde{v}^n\|_C \le C(\tau + h).$$

The proof of the corollary follows from Lemma 2.1 and Theorem 3.1.

*Remark 3.1.* For the Dirichlet boundary conditions we can obtain similar results using the same investigation technique. But this approach doesn't work for the Neumann boundary conditions, when $\mu = 0$.

### 3.2. The Neumann boundary conditions

In this section we prove unconditional stability and convergence of scheme (2.1)-(2.5) for the Neumann boundary conditions. In the scheme (3.1)-(3.5) now we have the following boundary conditions

$$\frac{h}{2}(1-m)\tilde{u}_{t,0} = a_1 \tilde{u}_{x,0}^{n+1} - \frac{hp}{2}(l\tilde{u}_0^{n+1} - \tilde{v}_0^n) + \Psi_{1,0}^{n+1}, \tag{3.8}$$

$$\frac{h}{2}(1-m)\tilde{u}_{t,N} = -a_N \tilde{u}_{\bar{x},N}^{n+1} - \frac{hp}{2}(l\tilde{u}_N^{n+1} - \tilde{v}_N^n) + \Psi_{1,N}^{n+1}, \tag{3.9}$$

Let define the following norms

$$\begin{aligned}
\|\tilde{w}^n\|_2 &= \|\tilde{u}^n\|_C + \frac{1+p\tau}{1-m}\ |\tilde{v}^n\|_C, \\
\|\Psi_1^{n+1}\|_3 &= \max\left\{\max_{0<i<N} |\Psi_{1,i}^{n+1}|, \frac{2}{h}|\Psi_{1,0}^{n+1}|, \frac{2}{h}|\Psi_{1,N}^{n+1}|\right\}, \\
\|\Psi^n\|_4 &= \|\Psi_1^n\|_3 + \|\Psi_2^n\|_C.
\end{aligned}$$

**Theorem 3.2.** *Finite difference schema* (2.1)-(2.5) *with* $\mu = 0$ *is unconditionally stable, and the global error satisfies the following stability inequality:*

$$\|\tilde{w}^{n+1}\|_2 \le \|\tilde{w}^n\|_2 + \frac{\tau}{1-m}\|\Psi^n\|_4. \tag{3.10}$$

*Proof.* First we multiply equation (3.1) by $\tau/(1-m)$. Using the maximum principle we get

$$\Big(1+\frac{\tau}{1-m}\Big(\frac{\nu}{h}+\frac{a_{i+1}+a_i}{h^2}+pl\Big)\Big)|\tilde u_i^{n+1}| \le \|\tilde u^n\|_C$$
$$+\frac{\tau}{1-m}\left(\frac{a_{i+1}+a_i}{h^2}+\frac{\nu}{h}\right)\ |\tilde u^{n+1}\|_C+\frac{p\tau}{1-m}\|\tilde v^n\|_C$$
$$+\frac{\tau}{1-m}\max_{0<i<N}|\Psi_{1,i}^{n+1}|,\ i=1,\dots,N-1. \tag{3.11}$$

Analogously from boundary conditions (3.8)-(3.9) we get

$$\Big(1+\frac{\tau}{1-m}\Big(\frac{2a_1}{h^2}+pl\Big)\Big)|\tilde u_0^{n+1}| \le \|\tilde u^n\|_C$$
$$+\frac{\tau}{1-m}\left(\frac{2a_1}{h^2}\|\tilde u^{n+1}\|_C+p\|\tilde v^n\|_C+\frac{2}{h}|\Psi_{1,0}^{n+1}|\right), \tag{3.12}$$
$$\Big(1+\frac{\tau}{1-m}\Big(\frac{2a_N}{h^2}+pl\Big)\Big)|\tilde u_N^{n+1}| \le \|\tilde u^n\|_C$$
$$+\frac{\tau}{1-m}\left(\frac{2a_N}{h^2}\|\tilde u^{n+1}\|_C+p\|\tilde v^n\|_C+\frac{2}{h}|\Psi_{1,N}^{n+1}|\right). \tag{3.13}$$

Using inequalities (3.11)-(3.13) we get

$$\|\tilde u^{n+1}\|_C \le \left(1+\frac{pl\tau}{1-m}\right)^{-1}\Big(\|\tilde u^n\|_C+\frac{p\tau}{1-m}\|\tilde v^n\|_C$$
$$+\frac{\tau}{1-m}\|\Psi_1^{n+1}\|_3\Big). \tag{3.14}$$

Then making use of (3.4) with (3.14) gives

$$(1+p\tau)\|\tilde v^{n+1}\|_C \le \left(1+\frac{pl\tau}{1-m}\right)^{-1}\Big(\|\tilde v^n\|_C+\frac{pl\tau}{1-m} \tag{3.15}$$
$$+\Big((1-m)\|\tilde u^n\|_C+(1+p\tau)\|\tilde v^n\|_C+\tau\|\Psi_1^{n+1}\|_3\Big)\Big)$$
$$+\tau\|\Psi_2^{n+1}\|_C\,.$$

Adding inequalities (3.14) and (3.15) we get the statement of the theorem. ■

*Corollary 3.2.* The global error of the difference solution can be estimated as

$$\|\tilde u^n\|_C \le C(\tau+h),\quad \|\tilde v^n\|_C \le C(\tau+h)\,. \tag{3.16}$$

**3.3. Pure diffusion problem**

If the convection term is not presented in equation (1.1), i.e. $\nu = 0$, then we have the following truncation errors

$$|\Psi_{1,i}^{n+1}| \le C(\tau + h^2), \quad i = 1, \dots, N-1.$$

In the case of the Dirichlet boundary conditions it follows from Theorem 3.1, Corollary 3.1 and Remark 3.1 that the global error of the difference solution can be estimated as

$$\|\tilde{u}^n\|_C \le C(\tau + h^2), \quad \|\tilde{v}^n\|_C \le C(\tau + h^2).$$

Hence the difference solution has the second order accuracy with respect the space grid step.

However for the Neumann boundary conditions we still have error estimations (3.16).

Let consider the problem with the third type boundary condition on one boundary and with the Neumann condition on the remaining boundary. Then we again can improve the main error estimation (3.16). We will use a modified approach of error analysis.

Let's look for the solution of the finite difference scheme (3.1), (3.2), (3.9), (3.4), (3.5) given in a form

$$(\tilde{u}^n, \tilde{v}^n)^T = (\tilde{U}^n, \tilde{v}^n)^T + (z^n, 0)^T.$$

We define the function $z_i^n$ as a solution of the auxiliary finite difference scheme

$$\begin{aligned} (1-m)z_t &= (a z_{\bar{x}}^{n+1})_x - plz_i^{n+1}, \qquad (3.17)\\ \frac{h}{2}(1-m)z_{t,0} + \mu z_0^{n+1} &= a_1 z_{x,0}^{n+1} - \frac{hpl}{2} z_0^{n+1} + \Psi_{1,0}^{n+1}, \\ \frac{h}{2}(1-m)z_{t,N} &= -a_N z_{\bar{x},N}^{n+1} - \frac{hpl}{2} z_N^{n+1} + \Psi_{1,N}^{n+1}, \\ z_i^0 &= 0, \quad i = 0, \dots, N. \end{aligned}$$

Let's define constants

$$\Gamma_0 = \max_{1 \le n \le K} |\Psi_1(t^n, x_0)| = O(\tau + h^2)\,,$$
$$\Gamma_N = \max_{1 \le n \le K} |\Psi_1(t^n, x_N)| = O(\tau + h^2)\,.$$

It follows from the maximum principle that (see, [5])

$$|z_i^n| \le Z_i, \, n = 0, \dots, K,$$

where $Z_i$ is the solution of the finite difference scheme

$$
\begin{aligned}
-(aZ_{\bar{x}})_x &= 0, \quad i = 1, \dots, N-1, \qquad (3.18)\\
-a_1 Z_{x,0} + \left(\mu + \frac{hpl}{2}\right) Z_0 &= \Gamma_0,\\
a_N Z_{\bar{x},N} + \frac{hpl}{2} Z_N &= \Gamma_N.
\end{aligned}
$$

Next we multiply equations (3.18) by $Z_i h$ and add the obtained equalities. After simple transformations we get

$$(aZ_{\bar{x}}, Z_{\bar{x}}] + \frac{hpl}{2} Z_N^2 + \left(\mu + \frac{hpl}{2}\right) Z_0^2 = \Gamma_0 Z_0 + \Gamma_N Z_N\,,$$

where

$$(y, v] = \sum_{i=1}^{N} y_i v_i h\,.$$

Using the embedding theorem [5]

$$\|Z\|_C^2 \le 2\left(\|Z_{\bar{x}}]|^2 + Z_N^2\right)\,,$$

we prove that

$$\|Z\|_C \le C \max\{\Gamma_0, \Gamma_N\} = O(\tau + h^2)\,.$$

The vector $(\tilde{U}_i^n, \tilde{v}_i^n)^T$ is the solution of finite difference scheme (3.1), (3.4), with $\nu = 0$, homogeneous boundary conditions (3.2), (3.9), and the truncation error

$$\tilde{\Psi}_2(t^n, x_i) = \Psi_2(t^n, x_i) + plz_i^{n+1}, \quad i = 0, \dots, N.$$

It follows from Theorem 3.2 that

$$\|\tilde{U}^n\|_C \le C(\tau + h^2), \quad \|\tilde{v}^n\|_C \le C(\tau + h^2).$$

So we have proved that for $\nu = 0$ and mixed boundary conditions the following estimations of the global error

$$\|\tilde{u}^n\|_C \le C(\tau + h^2), \quad \|\tilde{v}^n\|_C \le C(\tau + h^2).$$

are valid.

## 4. CONCLUSIONS

For the system of two differential equations we have examined the 3-point finite difference scheme. It is proved that this scheme is monotone. Stability analysis is done in the maximum norm. Unconditional stability is proved for all basic cases of boundary conditions, i.e. the Dirichlet, Neumann and third type boundary conditions. The modified approach is used in the case of Neumann and mixed type boundary conditions.

A father step would be to analyze in detail the nonlinear problem.

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## MEDIENOS DŽIOVINIMO UŽDAVINIO SPRENDIMAS BAIGTINIŲ SKIRTUMŲ METODU

R. Čiegis, V. Starikovičius

Šiame darbe sprendžiama dviejų diferencialinių lygčių sistema, kuri aprašo įvairius procesus, svarbius pramonėje, ekologijoje, technikoje. Tokių uždavinių pavyzdžiai yra medienos džiovinimo procesas, upių taršos koncentracijos modeliavimas ir kiti.
Darbe sudaryta baigtinių skirtumų schema, kurios sprendinys yra monotoniška funkcija, o realizacijos algoritmas – ekonomiškas. Ištirtas tokios baigtinių skirtumų schemos stabilumas maksimumo normoje ir įrodyta, kad schema yra nesąlygiškai stabili pirmojo, antrojo ir trečiojo tipo kraštinių sąlygų atvejais. Pateikta konvergavimo įrodymo metodikos modifikacija, leidžianti pagrįsti didesnį diskrečiojo sprendinio tikslumą, kai matematiniame modelyje nėra konvekcijos nario.