ON RELIABILITY-BASED OPTIMIZATION IN RIGID-PLASTIC FRAME DESIGN

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1. Introduction

The aim of structural design is to achieve a structural system in an optimal fashion, fulfilling a set of safety requirements and predefined needs. Steel frames, in some cases reinforced concrete frames, can be approximated when close to collapse as rigid-plastic frames [1].

There are two basic ways for reliability analysis of such structures, namely the kinematic and static approaches. The first deals with evaluation of all plastic collapse mechanisms as failure modes and is easy to grasp. If a set of mechanisms to be evaluated is complete, the system failure probability coincides with exact reliability to plastic collapse. But for large structures it becomes intractable, because of the difficulties to evaluate all possible mechanisms. Therefore, the main problem here is determination of the most stochastically important mechanisms in calculating the failure probability. Automatically this yield the lower bound of the system failure probability, P_f. On the other hand, the static approach does not deal with failure modes and yields naturally to the upper bound of P_f. But for small values of P_f we deal with tightness problems [1,2,3].

While the absolute safety evaluation by itself is problematic, the approximate methods, ensuring sufficient accuracy for reliability-based engineering design purposes is of interest. The paper considers two possible solution ways, based on kinematic and static approaches.

2. Formulation of the problem

A discrete model of frame structure, the carrying capacity of which is described by the vector of limit bending moments, $\mathbf{M}_o$ is under consideration. The components of the vector $\mathbf{M}_o$ depend on the cross-section dimensions and the yield limit of material.

Geometry of the structure, external loads application places, position of critical cross-sections are specified in a deterministic manner. Behaviour of material is assumed to be ideal rigid-plastic. External loads and material are random. The only possible collapse mode is plastic collapse. The only stress component, causing the plastic collapse, is the bending moment (no shear and torsion effects are taken into account). External loads are quasistatic (no dynamic effects are evaluated).

Optimization problem can be stated as:

$$min \varphi \left(\mathbf{M}_o \right) \qquad (1)$$

$$P_f = P_r \left[\bigcup_{i=1}^{m} Z_i < 0 \right] \le P_{fd}, \qquad (2)$$

where:

P_r - probability,

Z_i - certain performance function, containing $\mathbf{M}_o$

P_{fd} - prespecified design overall structure

failure probability.

Conservative approximation of P_f according A.Cornell [4] can be expressed by:

$$\max_{i=1}^{m} P_r(Z_i) \le P_f \le \sum_{i=1}^{m} P_r(Z_i). \qquad (3)$$

3. Kinematic approach

Performance function, Z_i for any plastic collapse mode, can be expressed by means of internal and external works as:

$$Z_i = U_i - W_i = \mathbf{M}_o^T \dot{\Theta} - \mathbf{F}^T \dot{\mathbf{u}}, \qquad (4)$$

where:

U_i - internal work;

W_i - external work;

$\dot{\Theta}$ - vector of deformation (deviation) rates,

$\dot{u}$ - vector of displacement rates,

$\mathbf{F}$ - vector of external forces.

Introducing the reliability index [5]:

$$\beta_{Z_i} = \frac{Z_i - \overline{Z}_i}{\sigma_{Z_i}},$$

in the reduced normalized random variables coordinate space, the i-th failure is calculated as:

$$P_{fi} = \Phi\left(\beta_{Z_i}\right), \tag{5}$$

where:

$\Phi(\cdot)$ - the standardized normal distribution function,

σ_{Z_i} - the standard deviation of Z_i,

$\overline{Z}_i$ - the mean value of Z_i.

When the variables to be considered are not normal, the Rosenblatt transformation [6] can be applied.

Determination of failure modes Z_i can be realized by minimizing the reliability index:

$$min\ \beta = \frac{\overline{\mathbf{M}}_o^T \dot{\Theta} - \overline{\mathbf{F}}^T \dot{u}}{\sqrt{A+B}}, \tag{6}$$

$$A = \dot{\Theta}^T \sigma_{M_o}^T \left[K_{M_o}\right] \sigma_{M_o} \dot{\Theta},$$

$$B = \dot{u}^T \sigma_F^T [K_F] \sigma_F \dot{u},$$

subject to geometrical equations:

$$[A]^T \dot{u} - \dot{\Theta} = 0, \tag{7}$$

$$\dot{u} \geq 0, \quad \dot{\Theta} \geq 0, \tag{8}$$

where:

$\left[K_{M_o}\right]$ - correlation matrix of limit bending moments,

$[K_F]$ - correlation matrix of external forces,

$[A]$ - matrix of equilibrium equations.

The problem (6)-(8) can be solved applying, for instance, the concave minimization technique, presented in [7]:

$$min\left(-\frac{1}{\beta^2}\right) = -A - B, \tag{9}$$

$$A = \dot{\Theta}^T \sigma_{M_o}^T \left[K_{M_o}\right] \sigma_{M_o} \dot{\Theta},$$

$$B = \dot{u}^T \sigma_F^T [K_F] \sigma_F \dot{u}$$

subject to:

$$\overline{\mathbf{M}}_o^T \dot{\Theta} - \overline{\mathbf{F}}^T \dot{u} = 1, \tag{10}$$

and linear constraints (7) and (8).

The presented above problem is nonconvex, because of the possibility of nonglobal local minimum. Every local minimum represents the certain kinematic mechanism $\dot{u}_i$, $\dot{\Theta}_i$ and reliability index β_i due to this mechanism.

Having determined and enumerated the most important mechanisms (due to prescribed reliability level P_f), we can solve the optimization problem (1)-(2). The deviations σ_{M_o}, σ_F, correlation matrices $\left[K_{M_o}\right]$, $[K_F]$, vector of mean values, $\overline{\mathbf{F}}$ and prespecified overall failure probability, P_{fd} are prescribed as known values. The vector of limit bending moment mean values $\overline{\mathbf{M}}_o$ is to be determined.

While the usual aim of many engineering structures is minimization of theoretical weight (mass), the objective function actually can be realized by [8]:

$$min\ \overline{\mathbf{M}}_o^T \mathbf{l}, \tag{11}$$

where $\mathbf{l}$ - the vector of weighting values, for instance, the lengths of corresponding bars. Optimization problem is stated as:

$$min\ \overline{\mathbf{M}}_o^T \mathbf{l}, \tag{12}$$

subject to:

$$\overline{\mathbf{M}}_o^T \dot{\Theta} - \overline{\mathbf{F}}^T \dot{u} \geq \beta_i \sqrt{A+B} \tag{13}$$

$$i = 1,...,m,$$

$$A = \dot{\Theta}^T \sigma_{M_o}^T \left[K_{M_o}\right] \sigma_{M_o} \dot{\Theta},$$

$$B = \dot{u}^T \sigma_F^T [K_F] \sigma_F \dot{u},$$

where m is the number of failure modes, taken into account to satisfy the condition:

$$P_{fd} \leq \sum_{i=1}^{m} \left(P_{fi} = \Phi(\beta_i)\right), \tag{14}$$

which for great values of β_i can be replaced as [9]:

$$\ln \sum_{i=1}^{k} c_1 \cdot e^{-2,3 \cdot c_2 \cdot \beta_k^{c_3}} \leq \ln P_{fd} \qquad (15)$$

where - c_1, c_2, c_3 - certain empiric coefficients.

At the first step, fixing the design variables, $\overline{M}_o$ the problem (9)-(10) - (7)-(8) is solved to identify the most relevant modes. In the second step, the optimization problem (12)-(14) is solved. This iterative process is repeated until required convergence is achieved.

4. Static approach

Applying static approach, performance function is replaced by equations of statical admissibility [8]. One of the first works based on static approach was proposed by A.Čyras [10]. It must be noted, that in the paper the problem was formulated and solution algorithm presented for general, variable plasticity case. Actually, loads may or may not induce plastic failure individually, although their change in time can produce cycles of plastic deformations which lead to an unrestricted growth of plastic deformations or to an alternation of their sign that result in plastic failure. The first case is usually referred to as progressive failure, and the second one, which is of a low cycle fatigue character, is called variable plasticity. Referring to the both cases of limit state of structure as cyclic plastic failure, the optimization problem in [10] was stated as:

$$min\left(\overline{M}_o^T 1 = V\right), \qquad (16)$$

subject to:

$$Pr\left(\overline{M}_o^T 1 \geq V\right) = q_v, \qquad (17)$$

$$Pr\left([G]\overline{M}_o - M_r \geq M_e^+\right) \geq q, \qquad (18)$$

$$Pr\left([G]\overline{M}_o + M_r \geq M_e^-\right) \geq q, \qquad (19)$$

$$[A]M_r = 0, \qquad (20)$$

$$\overline{M}_o \geq 0, \qquad (21)$$

where:

V - theoretical weight,
$[G]$ - structure configuration matrix,
M_r - vector of residual stresses,

$q = (q_1, ..., q_n)$ - vector of prescribed failure reliabilities $(0 \leq q_i \leq 1)$,

q_v - fixed reliability for objective function,

n - total number of stresses of discrete model.

Applying the chance constrained technique, the problem is replaced with the deterministic one and solved by usual mathematical programming methods. The problem formulated and solution method presented in the paper is related to random material and determined loads, but they can be expanded taking into account the random nature of variable loads and material simultaneously.

The same approach for constant random loads and random material was applied in [11].

5. Conclusions

The methods, based on static approach, seems to be more applicable for practical engineering design. It ensures good enough reliability accuracy for practical design purposes, which exceeds with the increase of correlation of limit bending moments and external loads. The approach presented in [10] is general, while it involves also variable plasticity failure modes, practically not possible to evaluate using the kinematic approach.

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APIE TIKIMYBINĘ OPTIMIZACIJĄ STANDŽIŲJŲ-PLASTINIŲ RĖMŲ SKAIČIAVIME

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S a n t r a u k a

Straipsnyje nagrinėjamas duoto patikimumo diskretinės rėminės konstrukcijos projektavimo uždavinys. Konstrukcijos geometriniai parametrai, apkrovų pridėjimo, pavojingų pjūvių vietos yra apibrėžiamos fiksuotai (determinuotai). Išorinės apkrovos ir konstrukcijos medžiaga - stochastinės. Nagrinėjamas vienintelis galimas konstrukcijos suirimo tipas - plastinio suirimo mechanizmo susidarymas. Išorinės apkrovos apibūdinamos kaip kvazistatinės, nesukeliančios dinaminių efektų. Konstrukcijos įtempimų būvis charakterizuojamas lenkimo momentų vektoriumi $\mathbf{M}$, i.e. neįvertinant šlyties ir sukimo efektų. Taigi konstrukcijos laikomoji galia apibūdinama ribinių lenkimo momentų vektoriumi $\mathbf{M}_o$, kurio komponentai priklauso nuo skerspjūvio charakteristikų ir medžiagos takumo ribos. Konstrukcijos plastinio suirimo mechanizmas apibūdinamas deformacijų (deviacijų) $\dot{\Theta}$ ir poslinkių ů greičių vektoriais.

Optimizacijos uždavinys (1)-(2) gali būti formuluojamas naudojantis kinematiniu ir statiniu principais.

Naudojant kinematinį principą ribinį būvį aprašančios lygtys (4) išreiškiamos per deformacijų ir greičių vektorius. Atskirų plastinių suirimo mechanizmų tikimybės (5) yra nustatomos naudojant patikimumo indeksus β_i. Šių mechanizmų identifikavimui sprendžiamas uždavinys (6)-(8). Šį uždavinį galima spręsti formuluojant jį kaip neiškiliojo matematinio programavimo uždavinį (9)-(10) - (7)-(8). Sprendinio lokalinius minimumus atitinka galimi suirimo mechanizmai, atitinkantys patikimumo indeksus β_i. Nustačius visus suirimo mechanizmus, atitinkančius reikiamą patikimumo lygį (3), sprendžiamas optimizacijos uždavinys (11)-(14). Sprendimo eiga: fiksuojant vektorių $\overline{\mathbf{M}}_o$ sprendžiamas uždavinys (9)-(10) - (7)-(8), randami pagrindiniai plastiniai suirimo mechanizmai; po to sprendžiamas uždavinys (12)-(14). Šis iteracinis procesas kartojamas tol, kol pasiekiamas reikiamas konvergavimo tikslumas.

Naudojant statinį principą, ribinį būvį apibūdina statinio leistinumo sąlygos. Pagal A.Čyro pasiūlytą sprendimo būdą optimizacijos uždavinys formuluojamas kaip matematinio programavimo uždavinys (16)-(21). Taip formuluojant uždavinį galima įvertinti ir kintamo plastiškumo sąlygotus suirimo būdus. Sprendžiant šį stochastinį uždavinį jis pakeičiamas determinuotu matematinio programavimo uždaviniu. Toks sprendimo būdas yra labiau taikytinas praktiniams projektavimo uždaviniams spręsti.

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