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ANALYSIS OF THE TIME-DEPENDENT BEHAVIOUR OF COMPOSITE CROSS-SECTIONS BY LAPLACE-TRANSFORM

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Abstract. Composite structures consisting of precast and cast in-situ concrete elements are increasingly common. These combinations demand a mechanical model which takes into account the time-dependent behaviour and analysis of the different ages of the connected concrete components. The effect of creep and shrinkage of the different concrete components can be of relevance for the state of serviceability, as well as for the final state. The long-time behaviour of concrete can be described by the rate-of-creep method, combined with a discretisation of time. The internal forces are described for each time interval using a system of linear differential equations, which can be solved by Laplace-transform.

Keywords: concrete, creep, shrinkage, Laplace-transform, rate-of-creep method, composite cross-section, time-dependent behaviour, reinforced and prestressed concrete construction.

1. Introduction

Composite structures consisting of precast and cast in-situ concrete sections are increasingly common in reinforced and prestressed concrete structures. Depending upon span, load intensity and special requirements, the prefabricated units are equipped with reinforcement and prestressed steel. Due to the time gap between the production of the prefabricated units and the cast-in-situ sections of the elements, which are added at a later stage, the different long-time behaviour of the concrete parts in the cross-section has to be considered. The influence of creep and shrinkage can be essential for the state of serviceability, as well as for the final state, particularly when the prefabricated units are prestressed.

The theory of creep allows a sufficiently realistic description of the long-time behaviour of concrete. In general terms, creep-induced deformation depends on the concrete age at the moment of observation t and at the moment of loading or change of stresses τ. The creep is connected to a redistribution of the stresses, and the change of the stresses itself as a result of this redistribution will influence the development of time-dependent deformations.

In the general theory of a linear elastic-creeping body, the creep coefficient ϕ is a function of t and τ: $\phi = \phi(t,\tau)$ [1, 2]. Analytical solutions of the problem can be found only for special cases [3, 4]. More general problems have to be solved numerically. Creep analysis based on the principle of complementary strain energy and mathematical optimisation [5–8] is methodically similar to the approach of Čyras et al [9] used for the analysis of elastic-plastic structures.

This paper deals with the case that creep function can be approximately defined as the difference $\phi(t,\tau) = \phi(t) - \phi(\tau)$. Such an assumption is acceptable when sufficiently short time intervals are considered. This method permits applying Laplace-transform to get solutions in an explicit form and is an alternative to the widespread practice to take creep-induced redistributions of stresses in account by modification of the Young's modulus of elasticity [10–12], by a relaxation coefficient [13–17] or simply by the lump-sum method of AASHTO [18].

The internal forces in a segment of the cross-section increase during increase all time intervals (t_{j-1},t_j). This increase has to be taken into account as an additional load when the creep for t_k is considered.

2. Assumptions and conditions

The entire cross-section consists of n effective cross-section segments. Concerning compatibility of deformation and material behaviour, the following assumptions are made:
Assumption 1:

There is a rigid bond between the shear-resistant cross-section segments. It is assumed that the entire cross-section remains plane during deformation. This means that all segments of the cross-section have the same rotation angle κ.

Assumption 2:

The concrete is uncracked (state I) and participates in the transmission of tensile stresses at all times.

Assumption 3:

The reinforcement (index *s*) and pre-stressed steel (index *p*) are creep-resistant and linear-elastic. The strains $\varepsilon_s(t)$, $\varepsilon_p(t)$ and the stresses $\sigma_s(t)$, $\sigma_p(t)$ are related according to the Hook's law:

$$\varepsilon_s(t) = \frac{\sigma_s(t)}{E_s}\,; \quad d\varepsilon_s(t) = \frac{d\sigma_s(t)}{E_s}\,, \tag{1}$$

$$\varepsilon_p(t) = \frac{\sigma_p(t)}{E_p}\,; \quad d\varepsilon_p(t) = \frac{d\sigma_p(t)}{E_p}\,.$$

Assumption 4:

The "old" and the "new" concrete components of the cross-section behave like elastic-ageing bodies. The intensity of creep deformation depends only on the age of the concrete, not on the age at loading. Thus the creep coefficient is defined by:

$$\phi_c(t,\tau) = \varphi_c(t) - \varphi_c(\tau)\,. \tag{2}$$

Under these conditions, the concrete strain $\varepsilon_c(t)$ at the time t due to a variable concrete stress $\sigma_c(\tau)$ ($t_0 \le \tau \le t$) during the creep period ($t - t_0$) can be described by the formula

$$\varepsilon_c(t) = \frac{\sigma_c(t)}{E_c(t)} + \int_{t_0}^{t} \frac{\sigma_c(\tau)}{E_c(\tau)} \cdot \frac{\partial \varphi_c(\tau)}{\partial \tau} d\tau + \varepsilon_{cs}(t,\tau_s)\,. \tag{3}$$

Thus the differential increase in concrete deformation, assuming a constant modulus of elasticity E_c, is

$$d\varepsilon_c(t) = \frac{d\sigma_c(t)}{E_c} + \frac{\sigma_c(t)}{E_c} \cdot d\varphi_c(t) + d\varepsilon_{cs}(t)\,. \tag{4}$$

3. Computation model

The computation model has to take into account that stress-strain relations, the equations of equilibrium and the conditions of compatibility of deformation are time-dependent.

Equilibrium conditions

For a time interval (t_{j-1}, t_j) the equilibrium of the internal normal forces $N_{x,r}$ and bending moments $M_{y,r}$, $M_{z,r}$ ($r = 1, 2, .., n$) is defined by the differential equations

$$\begin{aligned}
\sum_{r=1}^{n}\left[N'_{x,r}(t)\right] &= N'_{x,i}(t),\\
\sum_{r=1}^{n}\left[N'_{x,r}(t)\cdot z_{i,r} + M'_{y,r}(t)\right] &= M'_{y,i}(t),\\
\sum_{r=1}^{n}\left[N'_{x,r}(t)\cdot y_{i,r} + M'_{z,r}(t)\right] &= M'_{z,i}(t),
\end{aligned} \tag{5}$$

where $(..)_r' = d(..)_r / d\varphi$. The internal forces are related to the centroid I (Fig 1).

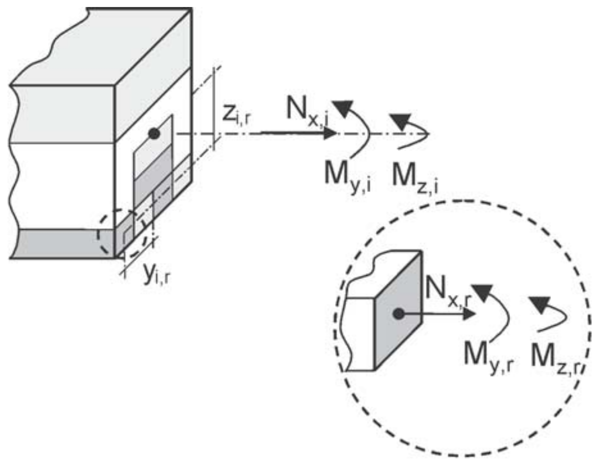


Fig 1. Internal forces of the cross-section

For sufficiently short finite time intervals, the increases in the internal forces S_i' can be approximated by the gradient

$$S_i' \approx \left(S_{i,j} - S_{i,j-1}\right) / \left(\varphi(t_j) - \varphi(t_{j-1})\right).$$

Compatibility conditions

The infinitesimal rates of the strain $d\varepsilon_{x,r}(t)$ and the curvatures $d\kappa_{y,r}(t)$ and $d\kappa_{z,r}(t)$ of the cross-section segment *r* are related to the internal forces by the differential equations

$$\begin{aligned}
d\varepsilon_{x,r}(t) &= \frac{dN_{x,r}(t)}{E_r \cdot A_r} + \frac{N_{x,r}(t)}{E_r \cdot A_r} \cdot d\varphi_{r,u}(t) + d\varepsilon_{cs,x,r}(t)\\
&= d\varepsilon_{x,i}(t) + d\kappa_{y,i}(t) \cdot z_{i,r} + d\kappa_{z,i}(t) \cdot y_{i,r}\\
d\kappa_{y,r}(t) &= \frac{dM_{y,r}(t)}{E_r \cdot I_{y,r}} + \frac{M_{y,r}(t)}{E_r \cdot I_{y,r}} \cdot d\varphi_{r,u}(t) + d\kappa_{cs,y,r}(t)\\
&= d\kappa_{y,i}(t)\\
d\kappa_{z,r}(t) &= \frac{dM_{z,r}(t)}{E_r \cdot I_{z,r}} + \frac{M_{z,r}(t)}{E_r \cdot I_{z,r}} \cdot d\varphi_{r,u}(t) + d\kappa_{cs,z,r}(t)\\
&= d\kappa_{z,i}(t).
\end{aligned} \tag{6}$$

The index "*u*" indicates the time t_u when the load increment is imposed.

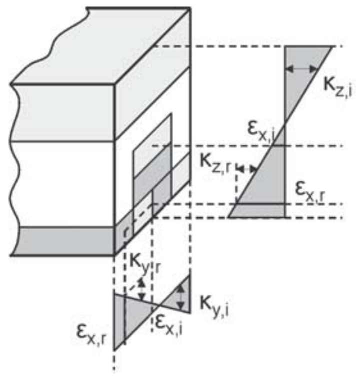


Fig 2. Deformations of the cross-section

Due to the different time-dependent material properties the creep and shrinkage of each cross-section segment can be related to a reference function $\phi_u(t)$ by the multiplier

$$\alpha_{\varphi,r,u}(t) = \frac{d\varphi_{r,u}(t)}{d\varphi_u(t)}. \quad (7)$$

Thus the differential increase of the creep function $d\varphi_{r,u}(t)$ of the cross-section segment r, starting at time t_u, can be expressed by the differential increase $d\varphi_u(t)$ of a reference creep function. The deformations due to shrinkage can be treated in a similar way:

$$\beta_{\varepsilon,x,r,u}(t) = \frac{d\varepsilon_{cs,x,r,u}(t)}{d\varphi_u(t)},$$

$$\beta_{\kappa,y,r,u}(t) = \frac{d\kappa_{cs,y,r,u}(t)}{d\varphi_u(t)}, \quad (8)$$

$$\beta_{\kappa,z,r,u}(t) = \frac{d\kappa_{cs,z,r,u}(t)}{d\varphi_u(t)}.$$

Introducing these relations, the equations (6) can be transformed as follows:

$$\begin{aligned}
\varepsilon'_{x,r}(t) &= \frac{N'_{x,r}(t)}{D_r} + \frac{N_{x,r}(t)}{D_r}\cdot\alpha_{\varphi,r,u}(t) + \beta_{\varepsilon,x,r,u}(t) \\
&= \varepsilon'_{x,i}(t) + \kappa'_{y,i}(t)\cdot z_{i,r} + \kappa'_{z,i}(t)\cdot y_{i,r} \\
\kappa'_{y,r}(t) &= \frac{M'_{y,r}(t)}{B_{y,r}} + \frac{M_{y,r}(t)}{B_{y,r}}\cdot\alpha_{\varphi,r,u}(t) + \beta_{\kappa,y,r,u}(t) \\
&= \kappa'_{y,i}(\tau) \\
\kappa'_{z,r}(t) &= \frac{M'_{z,r}(t)}{B_{z,r}} + \frac{M_{z,r}(t)}{B_{z,r}}\cdot\alpha_{\varphi,r,u}(t) + \beta_{\kappa,z,r,u}(t) \\
&= \kappa'_{z,i}(t),
\end{aligned} \quad (9)$$

where $D_r = E_r A_r$ is the tensile stiffness and $B_r = E_r I_r$ the flexural stiffness of the segments.

The equilibrium conditions (5) and the compatibility conditions (9) represent a first-order linear differential equation system with constant coefficients, that is used to determine the internal forces of the segments $N_{x,r,j}$, $M_{y,r,j}$ and $M_{z,r,j}$, as well as the entire cross-section deformations $\varepsilon_{x,i,j}$, $\kappa_{y,i,j}$ and $\kappa_{y,i,j}$ for $t = t_j$ (Fig 2).

To get a matrix formulation of the equation system the following vectors and matrixes are introduced:

vector functions s_j and $s`_j$ for the internal forces of the segments and their first derivatives:

$$s_j = \left[N_{x,1,j} \quad M_{y,1,j} \quad M_{z,1,j} \quad \cdots \quad N_{x,n,j} \quad M_{y,n,j} \quad M_{z,n,j}\right]^T,$$

$$s'_j = \left[N'_{x,1,j} \quad M'_{y,1,j} \quad M'_{z,1,j} \quad \cdots \quad N'_{x,n,j} \quad M'_{y,n,j} \quad M'_{z,n,j}\right]^T,$$

stiffness matrix $Q_{s,j}$, matrix of creep parameters $A_{\alpha,j,u}$, vector $\beta_{j,u}$ of shrinkage parameters:

$$Q_{s,j} = \begin{bmatrix} D_1 & 0 & \cdots & & & & 0 \\ 0 & B_{y,1} & & & & & \\ \vdots & & B_{z,1} & & & & \\ & & & \ddots & & & \\ & & & & D_n & & \vdots \\ & & & & & B_{y,n} & 0 \\ 0 & & & & \cdots & 0 & B_{z,n} \end{bmatrix}.$$

$$A_{\varphi,j,u} = \begin{bmatrix} \alpha_{\varphi,1,j,u} & 0 & \cdots & & & & 0 \\ 0 & \alpha_{\varphi,1,j,u} & & & & & \\ \vdots & & \alpha_{\varphi,1,j,u} & & & & \\ & & & \ddots & & & \\ & & & & \alpha_{\varphi,n,j,u} & & \vdots \\ & & & & & \alpha_{\varphi,n,j,u} & 0 \\ 0 & & & & \cdots & 0 & \alpha_{\varphi,n,j,u} \end{bmatrix},$$

$$\beta_{j,u} = \left[\beta_{\varepsilon,x,1,j,u} \quad \beta_{\kappa,y,1,j,u} \quad \beta_{\kappa,z,1,j,u} \quad \cdots \quad \beta_{\varepsilon,x,n,j,u} \quad \beta_{\kappa,y,n,j,u} \quad \beta_{\kappa,z,n,j,u}\right]^T,$$

equilibrium matrix A_G:

$$A_G = \begin{bmatrix} 1 & 0 & 0 & \cdots & 1 & 0 & 0 \\ z_{i,1} & 1 & 0 & \cdots & z_{i,n} & 1 & 0 \\ y_{i1} & 0 & 1 & \cdots & y_{i,n} & 0 & 1 \end{bmatrix},$$

vector b_j of external load increments

$$b_j = \left[\Delta N_{x,i}/\Delta\varphi_{j-1} \quad \Delta M_{y,i}/\Delta\varphi_{j-1} \quad \Delta M_{z,i}/\Delta\varphi_{j-1}\right]^T,$$

vector q'_j of the derivatives of the cross-section deformations

$$q'_j = \left[\varepsilon'_{x,i,j} \quad \kappa'_{y,i,j} \quad \kappa'_{z,i,j}\right]^T.$$

Thus the differential equations (5) and (9) can be represented as follows:

$$\begin{aligned} A_G s'_j &= b_j \\ Q_{s,j}^{-1} s'_j + Q_{s,j}^{-1} A_{\varphi,j,u} s_j - A_G^T q'_j &= -\beta_{j,u}. \end{aligned} \quad (10)$$

By transforming the differential equations (10), the derivatives of the functions $s(t_j)=s_j$ and $q(t_j)=q_j$ can be separated:

$$\begin{aligned} q'_j - R_{j,u} s_j &= P_j \beta_{j,u} + Q_{i,j}^{-1} b_j \\ s'_j + G_{j,u} s_j &= F_j \beta_{j,u} + K_j b_j \end{aligned} \qquad (11)$$

with $R_{j,u} = Q_{i,j}^{-1} A_G A_{\phi,j,u}$

$$P_j = Q_{i,j}^{-1} A_G Q_{s,j}$$

$$G_{j,u} = A_{\phi,j,u} - Q_{s,j} A_G^T Q_{i,j}^{-1} A_G A_{\phi,j,u}$$

$$F_j = Q_{s,j} A_G^T Q_{i,j}^{-1} A_G Q_{s,j} - Q_{s,j}$$

$$K_j = Q_{s,j} A_G^T Q_{i,j}^{-1}$$

$$Q_{i,j} = A_G Q_{s,j} A_G^T = \begin{bmatrix} D_{x,i} & S_{y,i} & S_{z,i} \\ S_{y,i} & B_{y,i} & B_{yz,i} \\ S_{z,i} & B_{yz,i} & B_{z,i} \end{bmatrix}.$$

Further transformations are necessary in order to get an explicit solution for the vector function s_j of the linear differential equation system (11). First, the matrices of the eigenvalues $L_{j,u}$ and eigenvectors $T_{j,u}$ of the matrix $G_{j,u}$ are determined, then a transformation with the matrix $G_{j,u} = T_{j,u} L_{j,u} T_{j,u}^{-1}$ is carried out:

$$L_{j,u} = \begin{bmatrix} \lambda_{1,j,u} & & 0 \\ & \ddots & \\ 0 & & \lambda_{3n,j,u} \end{bmatrix}, T_{j,u} = \begin{bmatrix} \nu_{1,j,u} & \cdots & \nu_{3n,j,u} \end{bmatrix}.$$

Thus the equation (11) can be represented as follows:

$$s_j'^* + L_{j,u} s_j^* = T_{j,u}^{-1} F_j \beta_{j,u} + T_j^{-1} K_j b_j \qquad (12)$$

with $s_j^* = T_{j,u}^{-1} s_j$ and $s_j^{*`} = T_{j,u}^{-1} s_j`$.

By using Laplace-transform, this separated differential equation system can be transferred from the original space to the image space

$$\begin{aligned} F(s_j^*) = \; & \left(pE + L_{j,u}\right)^{-1} s_{j-1}^* \\ & + \left(p^2 E + pL_{j,u}\right)^{-1} T_{j,u}^{-1} F_j \beta_{j,u} \\ & + \left(p^2 E + pL_{j,u}\right)^{-1} T_{j,u}^{-1} K_j b_j , \end{aligned} \qquad (13)$$

where $F(s_j^*)$ is the Laplace-transform of the vector function s_j.

For the time interval (t_{j-1},t_j), the retransformation into the original space, as well as the transformation of the internal forces vector $s_j = T_{j,u} s_j^*$, leads to the solution of the differential equation system:

$$\begin{aligned} s_j = T_{j,j-1} s_j^* = \; & T_{j,j-1} Y_{j,j-1} T_{j,j-1}^{-1} s_{j-1} \\ & + T_{j,j-1}\left(L_{j,j-1}^{-1} - L_{j,j-1}^{-1} Y_{j,j-1}\right) T_{j,j-1}^{-1} F_j \beta_{j,j-1} \\ & + T_{j,j-1}\left(L_{j,j-1}^{-1} - L_{j,j-1}^{-1} Y_{j,j-1}\right) T_{j,j-1}^{-1} K_j b_j \end{aligned} \qquad (14)$$

with $Y_{j,j-1} = \begin{bmatrix} e^{-\lambda_1 \Delta\varphi_{j,j-1}} & & 0 \\ & \ddots & \\ 0 & & e^{-\lambda_{3n} \Delta\varphi_{j,j-1}} \end{bmatrix}$,

$u=j-1$ beginning of loading and shrinkage by $t_u = t_{j-1}$,
$\Delta\varphi_{j,j-1}$ increases in the creep function of the load, acting since the point of time t_{j-1}.

For the computation of the subsequent time step, the increase of internal forces $s_j - s_{j-1}$ has to be introduced as a new load and be superimposed on the creep function $\phi(t_{j+1},t_j)$. Abrupt changes in the load are accounted for in the equation (15) using an additional term $K_j d_j$. The vector d_j is defined as follows for a load increase Δs_j at the time t_j, acting at the centroid i:

$$d_j = \begin{bmatrix} \Delta N_{x,i,j} & \Delta M_{y,i,j} & \Delta M_{z,i,j} \end{bmatrix}^T .$$

Thus the computation of the internal forces s_k at the time t_k, takes into account the redistribution over the entire observation period $(t_0 - t_k)$, as the sum of all changes in the internal forces $\Delta s_j = s_j - s_{j-1}$ $(j=0..k)$. It should be noted that, at different times, initial segments of the internal forces vector s_j have to be superimposed on the appropriate creep functions. Thus the equation (14) can be represented as follows:

$$\begin{aligned} s_k = & \sum_{u=1}^{k-1} \sum_{j=u+1}^{k} \left[\left(T_{j,u} Y_{j,u} T_{j,u}^{-1} - E\right)\left(s_j - s_{j-1}\right)\right]^{1)} \\ & + \sum_{l=1}^{n} \sum_{j=u_l+1}^{k} \left[T_{j,u_l}\left(L_{j,u_l}^{-1} - L_{j,u_l}^{-1} Y_{j,u_l}\right) T_{j,u_l}^{-1} F_j \beta_{j,u_l}\right]^{2)} \\ & + \sum_{j=1}^{k} \left[T_{j,j-1}\left(L_{j,j-1}^{-1} - L_{j,j-1}^{-1} Y_{j,j-1}\right) T_{j,j-1}^{-1} K_j b_j\right]^{3)} \\ & + \sum_{j=1}^{k} \left[K_j d_j\right]^{4)} \end{aligned} \qquad (15)$$

1) redistribution as a result of creep with $s_0 = \begin{bmatrix} 0 & \cdots & 0 \end{bmatrix}^T$,
2) redistribution as a result of shrinkage for n cross-section segments with initial shrinkage by $t_{u,n}$,
3) internal forces of segments as a result of creep of an equal loading change,
4) internal forces of the segments as a result of abrupt loading change.

Using this procedure, it is possible to determine the internal forces s_k at the time t_k directly after the determination of the eigenvectors and eigenvalues of the matrix $G_{j,u}$ of the different times t_j. The size of the matrices and vectors can change due to newly added or eliminated cross-section segments.

4. Example

The computation of a floor system with a span of 15,30 m will illustrate the application of the aforementioned computation method. Prefabricated pre-stressed elements with additional cast-in-situ concrete are used. The prestressed elements are reinforced with prestressed and unprestressed steel. The pre-stress of the tendons is σ_{p00}=103,2 kN/cm^2. After the release from the abutments (2nd day), the finished units are stored in such a way that the dead weight is not activated. On the 28th day, the precast is inserted and the cast-in-situ concrete supplemented. The composite structure consisting of old and new concrete segments is supported by yokes until the 58th day, so that the dead weight of the entire cross-section only becomes activated from this day onwards. Table 1 shows the cross-sectional parameters as well as the material characteristics. Fig 3 shows the loading regime.

The results of computation are shown in Table 2 and in Fig 4. In the old concrete cross-section tensile stresses develop up to the time t_1 as a result of shrinking. The release from the abutments at the time t_2=t_1 leads to the abrupt appearance of compressive stresses. A stress reduction can be observed up to the time t_4 in both the old concrete and the prestressed steel, which is compensated for by the stress increase in the reinforcing steel A_{sa}. The new concrete is applied at the time t_3. This is free of stress at this moment. Tension and compression stresses develop in the new concrete segment due to creep deformations in the interval (t_3, t_4). When the dead weight is activated, a stress jumps at the time t_5 in all cross-section segments involved. In the subsequent process it can be seen that the stress peaks in the concrete segments, as well as the stress differences in the contact area between old and new concrete, diminish as expected. At the time $t_8 = \infty$ the entire concrete cross-section is under compression (state I).

The development of the stresses, determined using the computation model presented, corresponds almost completely with the computations according to the theory of the elastic-creeping body (Fig 4 prestressed steel). The stress in the upper pre-stressed steel segment determined according to the rate of creep method is less, and that of the lower pre-stressed steel segment greater, than the stresses computed according to the theory of elastic-creeping body. This means that the calculated rotation of the entire cross-section is greater. These differences due to different assumptions concerning the time dependence of stress-strain relations are small.

5. Conclusions

Sufficiently exact solution can be found by the method presented when sufficiently small intervals of time are considered. For each time interval a system of first-order linear differential equations with constant coefficients has to be solved. This system of equations can be

Table 1. Values of cross-sections and materials

Cross-section segments	Material	Area [cm^2]	Elastic modulus [kN/cm^2]
Old concrete	C40/50	270x16 = 4320	3,500
Reinforced steel	BSt 500 S	2x14 ø10 = 21,98	20,000
Prestressed steel	St 1570/1770	2x38 ø½`` = 70,94	19,500
New concrete	C35/45	270x16 = 4320	3,350
Reinforced steel	BSt 500 S	15 ø28 = 30,15	20,000

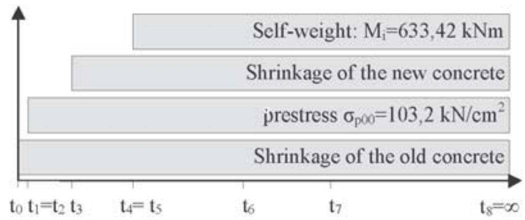


Fig 3. Loading process

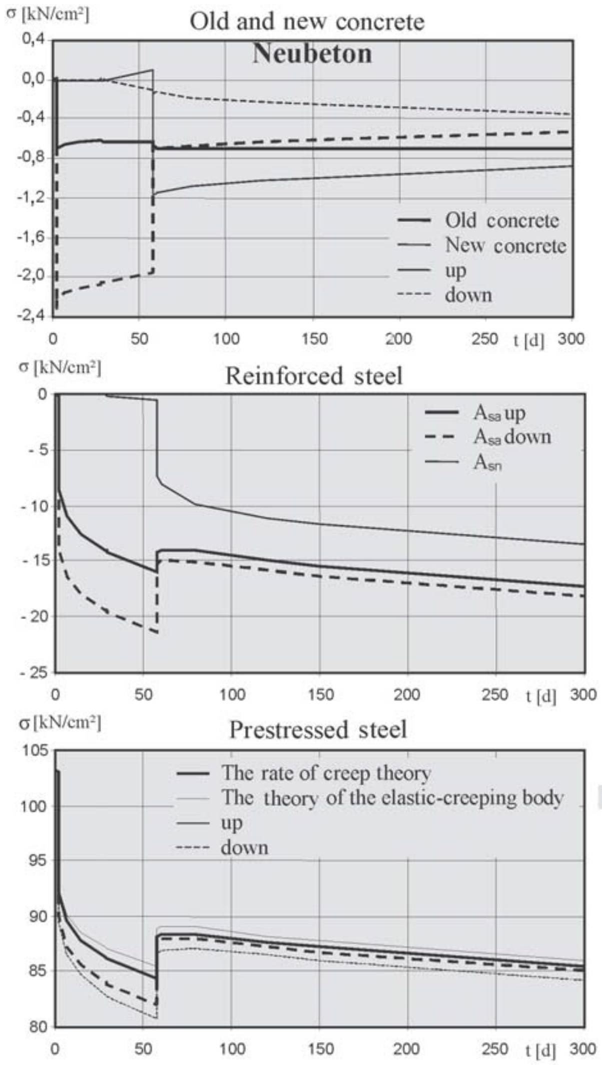
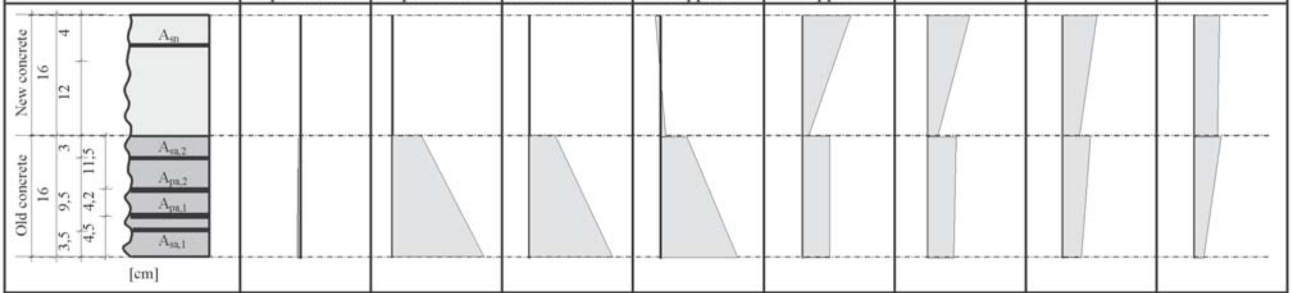


Fig 4. Process of concrete and reinforced steel stress

Table 2. Stress transfer as a result of creep and shrinkage of both concretes

Point of time [days]	t_1=2	t_2=2	t_3=28	t_4=58	t_5=58	t_6=120	t_7=600	t_8=?
	before prestress	after prestress	to apply new concrete	with support	without support			



Stress	[kN/cm²]	σ_o	σ_u	σ_o	σ_u	σ_o	σ_u	σ_o	σ_u	σ_o	σ_u	σ_o	σ_u	σ_o	σ_u	σ_o	σ_u
Old concrete	A_{ca}	0,00	0,01	–0,73	–2,32	–0,62	–2,07	–0,63	–1,95	–0,67	–0,68	–0,70	–0,64	–0,67	–0,46	–0,62	–0,18
Reinforced steel	A_{sa}	–0,18	–0,18	–6,11	–11,50	–14,03	–19,41	–15,96	–21,34	–14,76	–15,72	–14,90	–15,86	–19,15	–20,10	–26,37	–27,33
Prestressed steel	A_{pa}	103,03	103,03	94,87	92,55	86,31	83,98	84,30	81,98	87,42	87,01	87,68	87,27	83,51	83,10	76,40	75,99
New concrete	A_{cn}	0,00	0,00	0,00	0,00	0,00	0,00	0,10	–0,11	–1,17	–0,14	–1,02	–0,24	–0,83	–0,40	–0,61	–0,55
Reinforced steel	A_{sn}	0,00	0,00	0,00	0,00	0,00	0,00	–0,45	–0,45	–6,27	–6,27	–11,19	–11,19	–15,08	–15,08	–21,53	–21,53

solved with the help of Laplace-transform. The example shows that this approach is applicable to the analysis of the time-dependent load-bearing behaviour of composite sections. The method is compatible with the creep and shrinkage theories of EC 2 and DIN 1045-1 [17].

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ILGALAIKE APKROVA VEIKIAMO KOMPOZITINIO SKERSPJŪVIO ANALIZĖ, TAIKANT LAPLASO TRANSFORMACIJĄ

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S a n t r a u k a

Praktikoje vis dažniau pasitaiko kompozitinių konstrukcijų, kurias sudaro surenkamojo bei monolitinio betono elementai. Šių konstrukcijų analizei būtini skaičiavimo modeliai, kuriais galima įvertinti ilgalaikius efektus bei atskirų betono komponentų skirtingo amžiaus įtaką. Betono komponentų valkšnumo ir susitraukimo efektai yra svarbūs nagrinėjant tiek tinkamumo, tiek ir saugos ribinius būvius. Betono elgsena ilgalaikio apkrovimo atveju gali būti modeliuojama taikant valkšnumo deformacijų didėjimo metodą (angl. *rate-of-creep method*) bei diskretizaciją laikui bėgant. Vidinės jėgos kiekviename laiko intervale aprašomos taikant tiesinių diferencialinių lygčių sistemą. Ši lygčių sistema gali būti išspręsta taikant Laplaso transformaciją.

Raktažodžiai: betonas, valkšnumas, susitraukimas, Laplaso transformacija, valkšnumo deformacijų didėjimo metodas, kompozitinis skerspjūvis, elgsena veikiant ilgalaikei apkrovai, gelžbetonio ir įtemptojo gelžbetonio konstrukcija.

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