



RISK ASSESSMENT OF CONSTRUCTION PROJECTS

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Abstract. The paper presents risk assessment of construction projects. The assessment is based on the multi-attribute decision-making methods. The risk evaluation attributes are selected taking into consideration the interests and goals of the stakeholders as well as factors that have influence on the construction process efficiency and real estate value. Ranking of objects and determination of their optimality are determined by applying TOPSIS grey and COPRAS-G methods with attributes values determined at intervals. A background and a description of the proposed model are provided and key findings of the analysis are presented.

Keywords: decision-making, construction, project, risk, assessment, multi-attribute, TOPSIS grey, COPRAS-G.

1. Introduction

The risk factor in construction business is very high. Construction objects are unique and built only once. Construction objects life cycle is full of various risks. Risks come from many sources: temporary project team that is collected from different companies, construction site, etc. Moreover, the size and complexity of construction objects are increasing which adds to the risks. This is in addition to the political, economic, social conditions where the object is to be undertaken. Object risk can be defined as an uncertain event or condition that, if it occurs, has a positive or negative effect on at least one project objective, such as time, cost, quality (Project Management Institute Standards Committee 2004). The risks cause cost and time overruns in construction projects.

2. Description of the risk assessment model

Risk management is activity process about defining sources of uncertainty (risk identification), estimating the consequences of uncertain events/conditions (risk analysis), generating response strategies in the light of expected outcomes and finally, based on the feedback received on actual outcomes and risks emerged, carrying out identification, analysis and response generation steps repetitively throughout the life cycle of an object to ensure that the project objectives are met. Risk management in construction is a tedious task as the objective functions tend to change during the object life cycle (Dikmen *et al.* 2008). Tserng *et al.* (2009) presented a study of ontology-based risk management framework of construction projects through project life cycle variance – covariance. Risk value model for currency market is presented by Aniūnas *et al.* (2009). Isaac and Navon (2009) described models of building projects as a basis for change control.

Risk management processes of construction project describe the work of all project life cycle. The risk assessment problem is analysed by many authors (Shevchenko *et al.* 2008; Suhobokov 2008; Zavadskas *et al.* 2008a; Zavadskas and Vaidogas 2008; Schieg 2008, 2009; Šarka *et al.* 2008). Proper risk allocation in construction contracts has come to assume prominence because risk identification and risk allocation have a clear bearing on risk handling decisions (Perera *et al.* 2009).

Hassanein and Afify (2007) analysed risk identification procedure for construction contracts. Albert *et al.* (2008) pointed on the investigated risk assessment. El-Sayegh (2008) presented risk assessment and allocation problem, Han *et al.* (2008) described web-based integrated system, Gao (2009) presented strategies with the risk adjustment. Graves and Ringuest (2009) analysed probabilistic dominance criteria for comparing uncertain alternatives. Lahdelma *et al.* (2009) investigated uncertainties in multi-criteria decision problems.

Cost-effective solutions that meet the performance criteria can be achieved, especially if the principle of whole-life costing is being adopted (Straub 2009). The risk management in construction object's life cycle stages can be divided into: macro, meso and micro levels (Fig. 1). The project life cycle includes five steps of process management:

- Initiating;
- Planning;
- Executing;
- Monitoring and Controlling;
- Closing.

The process of risk management can be divided into three stages:

- Identification;
- Analysis;
- Control.

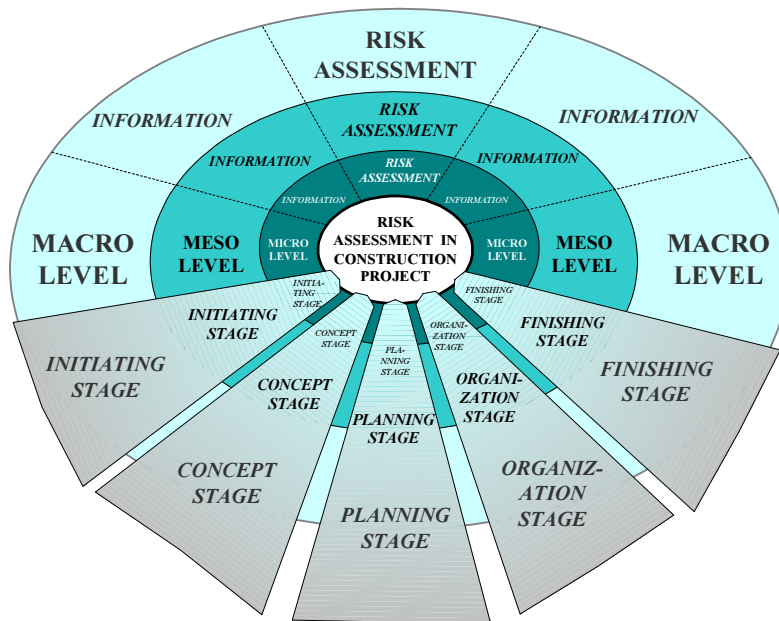


Fig. 1. Risk assessment divided according to object life cycle environment

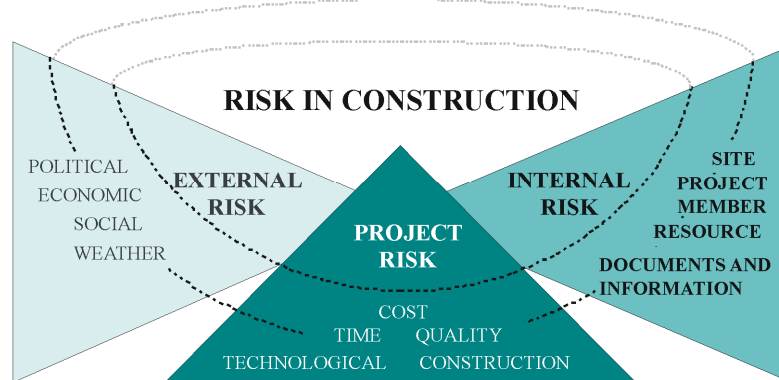


Fig. 2. Risk allocation structure by level in construction object

The risk management process in construction is extreme and important. Risk measure includes risk level determination of each objective and the risk analysis estimation by applying various approaches and technology. Risk control process evaluates performance of risk control.

2.1. Risk identification

Risk identification is the first and main step of risk management process. It is describing the competitiveness conditions and the clarification of risk and uncertainty factors (Rutkauskas 2008; Zayed *et al.* 2008), recognition of potential sources of risk and uncertainty event responsibilities. The project risks can be divided into three groups:

- External;
- Project;
- Internal.

External risks are those risks that are beyond the control of the project management team. Internal risks can be divided according to the party who might be the originator of risk events such as stakeholders, designer,

contractor, etc. There are various classification ways of risk management methods.

The risk allocation structure of construction objects is presented in Fig. 2.

External risks (environmental criteria):

- Political risk;
- Economic risk;
- Social risk;
- Weather risk.

Political risks. There are changes in government laws of legislative system, regulations and policy and improper administration system, etc. (Li and Liao 2007).

Economic risks. There is inconstancy of economy in the country, repayment situation in manufacture sphere, inflation and funding. Considering the current economic situation, this result can be reasonably expected. Tvaronavičienė and Grybaitė (2007) analysed Lithuanian economic activities in construction. Economic disasters, as referred to herein, are periodic economic disasters of such magnitude that a contractor could not properly assess either their probability or their cost impact.

Social risks are the growing importance to any effort at risk allocation. It is an area in which political and social pressures from parties having little interest in a project but having a great impact on such a project greatly influence its outcome. The impact of the financial aid on social and economic development of the region is analysed by Ginevičius and Podvezko (2009), risk communication in organizations is analysed by Conchie and Burns (2008).

Weather risk. Except for extremely abnormal conditions, it is a risk for the contractor to assume, as its impact on construction methods can be assessed by the contractor.

Project risks (*construction process criteria*):

- Time risk;
- Cost risk;
- Work quality;
- Construction risk;
- Technological risk.

Time risk can be determined by appraisal of the delay at construction, technology and for all works.

Cost risk. The cost of opportunity product rises due to neglecting of management (Zavadskas *et al.* 2008a).

Work quality. Deflective work is considered a significant risk factor in this category because not only does it result in construction delays and additional cost to the contractor but it easily leads to disputes on the liability for the deflection.

Construction risk. The risks are involved in construction delay, changes in the work and construction technology.

Technological risk. Designing errors; lack of technologies; management errors; shortage of the qualified labour.

Internal risks (*intrinsic criteria*):

- Resource risk;
- Project member risk;
- Construction site risk;
- Documents and information risk.

Resource risk. Materials and equipment involve considerable risks. The availability and productivity of the resources necessary to construct the project are risks which are proper for the contractor to assume (Fisk 2003).

Project member risk. Team risk refers to issues associated with the project team members, which can increase the uncertainty of a project's outcome, such as team member turnover, staffing build up, insufficient knowledge among team members, cooperation, motivation, and team communication issues.

Stakeholders' risks rightfully belong to the stakeholder alone and should be retained by stakeholders except to the extent that they are influenced by construction methods determined by the contractor, or created by suppliers controlled by the contractor. Stakeholders' influence on the external environment is analysed by Mitkus and Šostak (2008).

Designers risk. The expansion of construction has placed great burdens upon the design professions. Main-

taining performance standards in the face of this is quite difficult, and occasionally, design or specification deflections occur that create construction problems. Design failures or constructability errors are becoming more and apparent, and the architect should bear the true cost of such failures.

Contractor risk. The prime or general contractors are in the best position to assess the capacity of their subcontractors, and therefore it is they who should bear the risk of not assessing the risk properly.

Subcontractor risk is that is properly assumed by the contractor except where it arises from one of the other listed risks attributable to stakeholder or architect (Fisk, 2003).

Suppliers risk. Default from obligations of the supplier (Fisk 2003).

Team risk. Team risk refers to issues associated with the project team members that can increase the uncertainty of a project's outcome, such as team member turnover, staffing build up, insufficient knowledge among team members, cooperation, motivation, and team communication issues. Working team must analyse the business activities of all alliance members and identify various risk factors in business activities and their characters (Gunstone 2003; Li and Liao 2007; Li *et al.* 2007).

Construction site risk. Accident exposures in workplace are inherent in the nature of the work and are best assessed by the contractors and their insurance and safety advisors (Fisk 2003).

Documents and information risk assumes: contradiction in documents; pretermission; legal and communication. Changed order negotiation and delayed dispute resolution are significant risks during project construction. Communication is very important at all construction period and after finishing construction work.

Protracted negotiation on disputes or valuation of changed work is undesirable to most contractors.

Connections of the contractors with subcontractors and suppliers are analysed by Mitkus and Trinkūnienė (2008).

2.2. Risk analysis and control

Risk analysis. Risk and uncertainty rating identifies the importance of the sources of risk and uncertainty about the goals of the project. Risk assessment is accomplished by estimating the probability of occurrence and severity of risk impact.

More detailed information available in the construction process can be effectively used for traditional risk management schemes such as *risk control*. Risk control can be described as the five-stage process (Han *et al.* 2008):

- Identification;
- Analysis;
- Evaluation;
- Response;
- Monitoring.

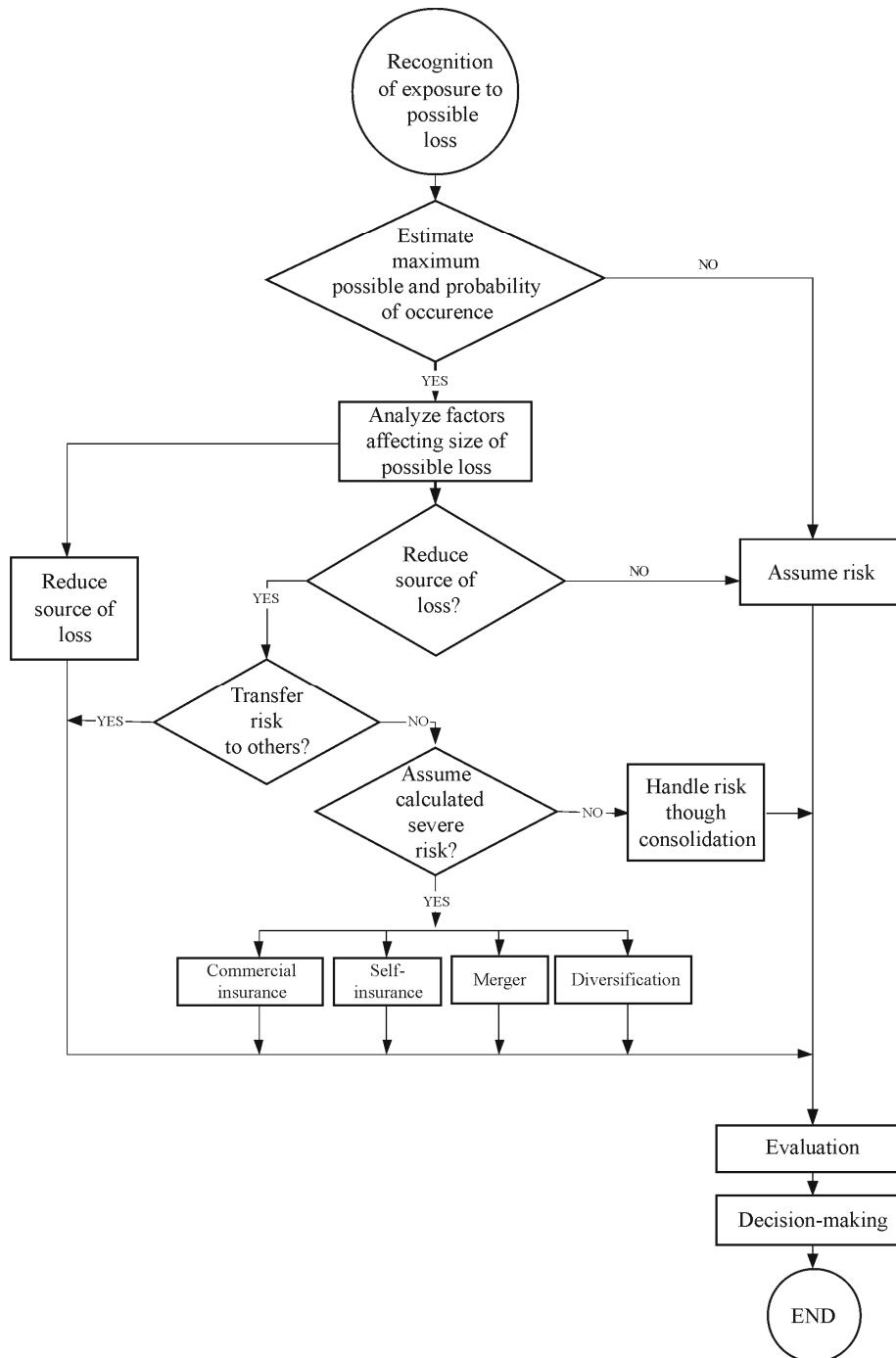


Fig. 3. Decision making model of risk assessment

Risk control establishes a plan, which reduces or eliminates sources of risk and uncertainty impact on the project's deployment.

Options available for mitigation are:

- Commercial insurance;
- Self-insurance;
- Merger and diversification.

Decision making model of risk assessment is shown in Fig. 3.

This model must be filled at every turn of risk management process.

3. Grey research methodology of risk assessment

3.1. Grey system theory

Deng (1982) developed the Grey system theory. Grey relational analysis possesses advantages (Deng 1988, 1989):

- involves simple calculations,
- requires smaller samples,
- a typical distribution of samples is not needed,
- the quantified outcomes from the Grey relational grade do not result in contradictory conclusions to qualitative analysis and

–the Grey relational grade model is a transfer functional model that is effective in dealing with discrete data.

The risk assessment always deals with future and values of criteria cannot be expressed exactly. This multi-criteria decision-making problem can be determined not with exact criteria values, but with fuzzy values or with values at some intervals (Fig. 4).

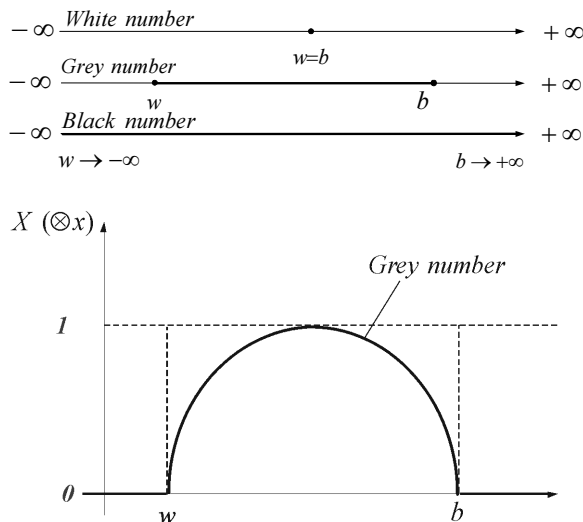


Fig. 4. White, grey and black numbers by Deng (Yamaguchi *et al.* 2007)

The Deng’s grey numbers are given as follows:

$$X(k, \otimes x) = \begin{cases} [0, 1] & k \in [w, b] \\ 0 & k < w, b < k \end{cases} \quad (1)$$

The use of grey relational analysis in solving multiple attribute decision-making problems is analysed by Kuo *et al.* (2008) and Cakir (2008).

Grey theory was applied in evaluating of national economic strength (Lin and Liu 2007), selection of an ERP system and intelligent sensors (Yang *et al.* 2007) and in the assessment of information security (Shen *et al.* 2009).

3.2. MADM methods for solving the problem

Multiple attributes decision aid provides several powerful solution tools (Hwang and Yoon 1981; Figueira *et al.* 2005) for confronting sorting the problems. There can be used very simplified techniques for the evaluation such as the Simple Additive Weighting – SAW; TOPSIS – Technique for Order Preference by Similarity to Ideal Solution (Hwang and Yoon 1981).

A more detailed survey of multi-attribute decision-making methods in the construction context is presented by many authors. Zavadskas and Antuchevičienė (2006) presented construction objects renewal modelling by applying multi-attribute evaluation of rural buildings’ regeneration alternatives; the multi-alternative design and multiple criteria analysis of the life cycle of a building is described by Banaitienė *et al.* (2008); selection of the

effective dwelling house walls by applying attribute values determined at intervals is described by Zavadskas *et al.* (2008c).

The purpose is to be achieved by using attributes of effectiveness, which have different dimensions, different significances as well as different directions of optimization (Kendall 1970; Zavadskas 1987). The discrete criteria values can be normalized by applying different normalization methods (Zavadskas and Turskis 2008; Ginevičius 2008). The purpose of analysis also can be different (Kaklauskas *et al.* 2007; Ginevičius *et al.* 2007; Bregar *et al.* 2008). Multiple criteria decision aid (Hwang and Yoon 1981) provides several powerful and effective tools (Figueira *et al.* 2005; Zavadskas *et al.* 2008b; Dzemyda *et al.* 2007; Ginevičius *et al.* 2008a, b; Ginevičius and Podvezko 2009) for confronting sorting the problems.

There is a wide range of methods (Ulubeyli and Kazaz 2009; Jakimavicius and Burinskiene 2009a, b; Plebankiewicz 2009; Liaudanskiene *et al.* 2009; Liu 2009; Dytczak and Ginda 2009; Podvezko 2009) based on multi-criteria utility theory: SAW – Simple Additive Weighting (Ginevičius *et al.* 2008b); SAW-G (Zavadskas *et al.* 2010); MOORA – Multi-Objective Optimization on the basis of Ratio Analysis (Brauers and Zavadskas 2006; Brauers *et al.* 2007, 2008a, b; Kalibatras and Turskis 2008); TOPSIS – Technique for Order Preference by Similarity to Ideal Solution (Hwang and Yoon 1981); VIKOR – compromise ranking method (Opricovic and Tseng 2004); COPRAS – COmplex PROportional ASsessment (Zavadskas and Kaklauskas 1996); Game theory methods (Zavadskas and Turskis 2008; Peldschus 2008, 2009; Ginevičius and Krivka 2008; Turskis *et al.* 2009) and other methods (Turskis 2008).

TOPSIS is a method to identify solutions from a finite set of alternatives based upon simultaneous minimization of distance from an ideal point and maximization of distance from a negative ideal point. The TOPSIS method was developed by Hwang and Yoon (1981). The only subjective input needed is relative weights of attributes. An extension of TOPSIS for group decision making is analysed by Shih *et al.* (2007) and incremental analysis of MCDM with an application to group TOPSIS is developed by Shih (2008). Lin *et al.* (2008) applied TOPSIS method with grey number operations.

The COPRAS method determines a solution with the ratio to the ideal solution and the ratio with the ideal-worst solution. Zavadskas *et al.* (2008c, d) applied COPRAS-G method with grey number operations to the problem with uncertain information.

The algorithm of problem solution applying TOPSIS grey and COPRAS-G methods is presented in Fig. 5. Either method is applicable to the solution of problems in construction: Lin *et al.* (2008) applied TOPSIS method with grey number operations to the contractor selection problem solution with uncertain information. Zavadskas *et al.* (2008c) applied COPRAS-G method with grey number operations to the selection of the effective dwelling house walls problem with uncertain information, Zavadskas *et al.* (2009).

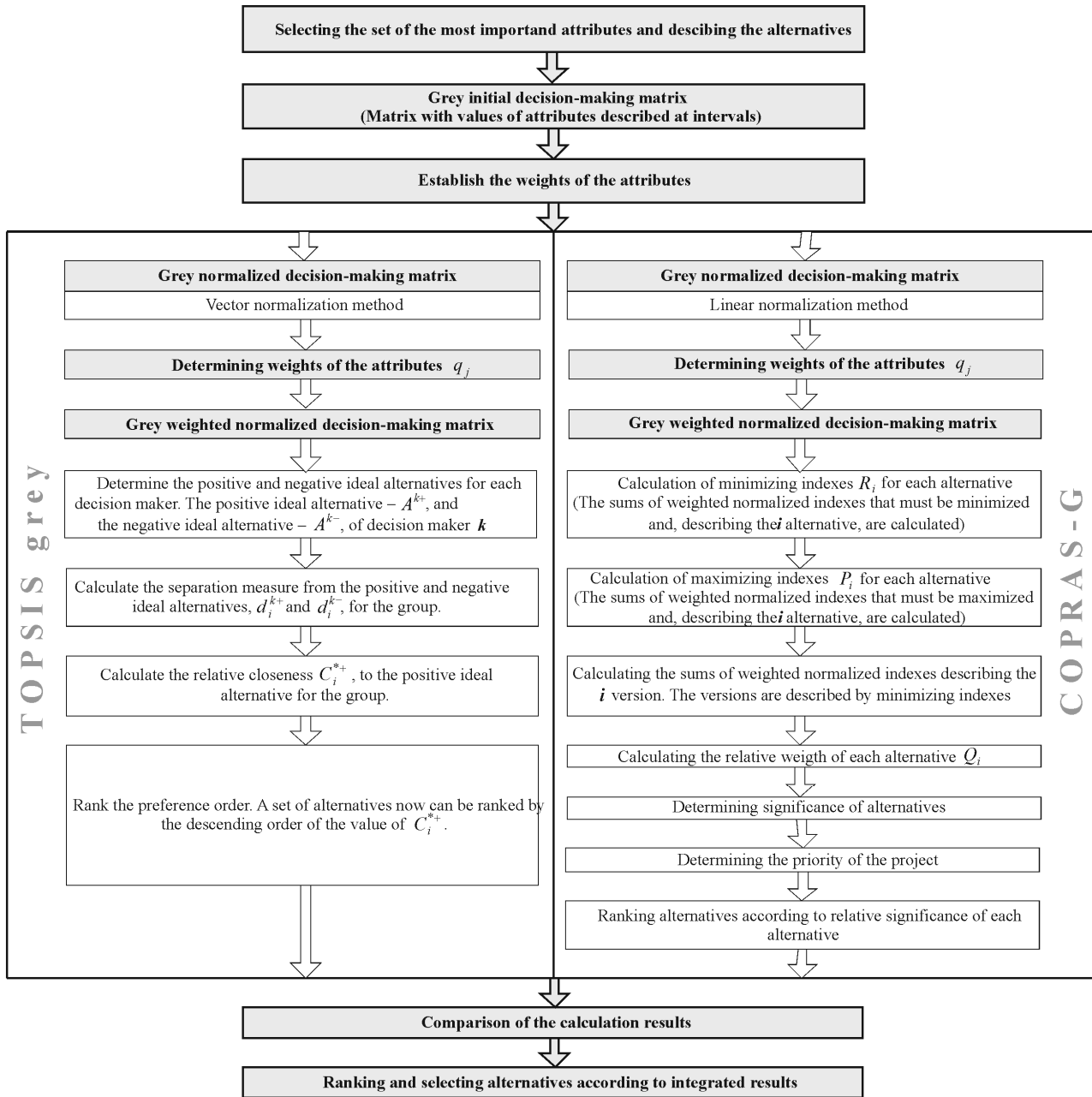


Fig. 5. The main algorithm of problem solution applying TOPSIS grey and COPRAS-G methods

3.2.1. TOPSIS method with attributes values determined at intervals

The TOPSIS method is one of the best described mathematically and not simple for practical using. Lin *et al.* (2008) proposed the model of TOPSIS method with attributes values determined at intervals which includes the following steps:

- Step 1: Selecting the set of the most important attributes, describing the alternatives;
- Step 2: Constructing the decision-making matrix $\otimes X$. Grey number matrix $\otimes X$ can be defined as:

$$\otimes X^k = \begin{bmatrix} \otimes x_{11}^k & \otimes x_{12}^k & \dots & \otimes x_{1m}^k \\ \otimes x_{21}^k & \otimes x_{22}^k & \dots & \otimes x_{2m}^k \\ \vdots & \vdots & \ddots & \vdots \\ \otimes x_{n1}^k & \otimes x_{n2}^k & \dots & \otimes x_{nm}^k \end{bmatrix}; i = \overline{1, n}; j = \overline{1, m}, (2)$$

where $\otimes x_{ij}^k$ denotes the grey evaluations of the i -th alternative with respect to the j -th attribute by decision maker $k(k = 1, \dots, K)$; $[\otimes x_{i1}^k, \otimes x_{i2}^k, \dots, \otimes x_{im}^k]$ is the grey number evaluation series of the i -th alternative given by decision maker k . It is noted that there should be K grey decision matrices for the K members of the group.

Step 3: Establish the weights of the attributes q_j .

Step 4: Construct the normalized grey decision matrices:

$$\otimes \bar{x}_{ij,b}^k = \frac{\otimes x_{ij}^k}{\max_i(b_{ij}^k)} = \left(\frac{w_{ij}^k}{\max_i(b_{ij}^k)}, \frac{b_{ij}^k}{\max_i(b_{ij}^k)} \right). \quad (3)$$

On the other hand, the normalization of the smaller-the-better type attribute can be calculated as:

$$\otimes \bar{x}_{ij,w}^k = -\frac{\otimes x_{ij}^k}{\min_i(w_{ij}^k)} + 2 = \left(\frac{-b_{ij}^k}{\min_i(w_{ij}^k)} + 2; \frac{-w_{ij}^k}{\min_i(w_{ij}^k)} + 2 \right). \quad (4)$$

Step 5: Determining weights of the attributes q_j .

Step 6: Construct the grey weighted normalized decision making matrix.

Step 7: Determine the positive and negative ideal alternatives for each decision maker. The positive ideal alternative – A^{k+} , and the negative ideal alternative – A^{k-} , of decision maker k can be defined as:

$$A^{k+} = \left\{ \left(\max_i b_{ij}^k | j \in J \right), \left(\min_i w_{ij}^k | j \in J' \right) \mid i \in n \right\} = [x_1^{k+}, x_2^{k+}, \dots, x_m^{k+}] \quad \text{and} \quad (5)$$

$$A^{k-} = \left\{ \left(\min_i w_{ij}^k | j \in J \right), \left(\max_i b_{ij}^k | j \in J' \right) \mid i \in n \right\} = [x_1^{k-}, x_2^{k-}, \dots, x_m^{k-}]. \quad (6)$$

Step 8: Calculate the separation measure of the positive and negative ideal alternatives, d_i^{k+} and d_i^{k-} , for the group. There are two sub-steps to be considered: the first one concerns the separation measure for individuals; the second one aggregates their measures for the group.

Step 8.1: Calculate the measures of the positive and negative ideal alternatives individually. For decision maker k , the separation measures of the positive ideal alternative – d_i^{k+} and negative ideal alternative – d_i^{k-} are computed through weighted grey number as:

$$d_i^{k+} = \left\{ \frac{1}{2} \sum_{j=1}^m q_j \left[\left| x_j^{k+} - \bar{w}_{ij}^k \right|^p + \left| x_j^{k+} - \bar{b}_{ij}^k \right|^p \right] \right\}^{1/p}; \quad (7)$$

$$d_i^{k-} = \left\{ \frac{1}{2} \sum_{j=1}^m q_j \left[\left| x_j^{k-} - \bar{w}_{ij}^k \right|^p + \left| x_j^{k-} - \bar{b}_{ij}^k \right|^p \right] \right\}^{1/p}. \quad (8)$$

In equations (6) and (7), for $p \geq 1$ and integer, q_j is the weight for the attribute j which can be determined by attributes' weight determination methods. If $p = 2$, then the metric is a weighted grey number Euclidean distance function. Equations (6) and (7) will be as follows:

$$d_i^{k+} = \sqrt{\frac{1}{2} \sum_{j=1}^m q_j \left[\left| x_j^{k+} - \bar{w}_{ij}^k \right|^2 + \left| x_j^{k+} - \bar{b}_{ij}^k \right|^2 \right]}, \quad (9)$$

$$d_i^{k-} = \sqrt{\frac{1}{2} \sum_{j=1}^m q_j \left[\left| x_j^{k-} - \bar{w}_{ij}^k \right|^2 + \left| x_j^{k-} - \bar{b}_{ij}^k \right|^2 \right]}. \quad (10)$$

Step 8.2: Aggregate the measures for the group. The group separation measure of each alternative will be aggregated through an operation, \otimes for all decision makers. Thus, the two group measures of the positive and negative ideal alternatives: d_i^{*+} and d_i^{*-} , respectively, are the following two equations:

$$d_i^{*+} = \otimes d_i^{1+}, \dots, \otimes d_i^{K+} \quad \text{for alternative } i, \quad \text{and} \quad (11)$$

$$d_i^{*-} = \otimes d_i^{1-}, \dots, \otimes d_i^{K-} \quad \text{for alternative } i. \quad (12)$$

Geometric mean is adopted, and the group measures of each alternative will be:

$$d_i^{*+} = \left(\prod_{k=1}^K d_i^{k+} \right)^{1/K}, \quad \text{for alternative } i, \quad (13)$$

$$d_i^{*-} = \left(\prod_{k=1}^K d_i^{k-} \right)^{1/K}, \quad \text{for alternative } i. \quad (14)$$

Step 9: Calculate the relative closeness C_i^{*+} , to the positive ideal alternative for the group. The aggregation of relative closeness for the i -th alternative with respect to the positive ideal alternative of the group can be expressed as:

$$C_i^{*+} = \frac{d_i^{*-}}{d_i^{*+} + d_i^{*-}}, \quad (15)$$

where $0 \leq C_i^{*+} \leq 1$. The larger the index value is, the better evaluation of alternative will be.

Step 10: Rank the preference order. A set of alternatives now can be ranked by the descending order of the value of C_i^{*+} .

3.2.2. COPRAS-G method with attributes values determined at intervals

The procedure of using the COPRAS-G method includes the following steps:

Step 1: Selecting the set of the most important attributes, describing the alternatives;

Step 2: Constructing the grey decision-making matrix $\otimes X$:

$$\otimes X = \begin{bmatrix} [\otimes x_{11}] & [\otimes x_{12}] & \dots & [\otimes x_{1m}] \\ [\otimes x_{21}] & [\otimes x_{22}] & \dots & [\otimes x_{2m}] \\ \vdots & \vdots & \ddots & \vdots \\ [\otimes x_{n1}] & [\otimes x_{n2}] & \dots & [\otimes x_{nm}] \end{bmatrix} = \begin{bmatrix} [w_{11}; b_{11}] & [w_{12}; b_{12}] & \dots & [w_{1m}; b_{1m}] \\ [w_{21}; b_{21}] & [w_{22}; b_{22}] & \dots & [w_{2m}; b_{2m}] \\ \vdots & \vdots & \ddots & \vdots \\ [w_{n1}; b_{n1}] & [w_{n2}; b_{n2}] & \dots & [w_{nm}; b_{nm}] \end{bmatrix}; \quad (16)$$

$i = \overline{1, n}; \quad j = \overline{1, m},$

where $\otimes x_{ij}$ is determined by w_{ij} and b_{ij} .

Step 3: Establishing the weights of the attributes q_j .

Step 4: Normalizing the decision-making matrix $\otimes X$:

$$\begin{aligned}\bar{w}_{ij} &= \frac{w_{ij}}{\frac{1}{2} \left(\sum_{i=1}^n w_{ij} + \sum_{i=1}^n b_{ij} \right)} = \frac{2w_{ij}}{\sum_{i=1}^n w_{ij} + \sum_{i=1}^n b_{ij}}; \\ \bar{b}_{ij} &= \frac{b_{ij}}{\frac{1}{2} \left(\sum_{i=1}^n w_{ij} + \sum_{i=1}^n b_{ij} \right)} = \frac{2b_{ij}}{\sum_{i=1}^n (w_{ij} + b_{ij})}; \\ i &= \overline{1, n} \text{ and } j = \overline{1, m}.\end{aligned}\quad (17)$$

In formula (17), w_{ij} is the lower value of the j attribute in the alternative i of the solution; b_{ij} is the upper value of the attribute j in the alternative i of the solution; m is the number of attributes; n is the number of the alternatives compared.

Then, the decision-making matrix is normalized:

$$\otimes \bar{X} = \begin{bmatrix} [\otimes \bar{x}_{11}] & [\otimes \bar{x}_{12}] & \dots & [\otimes \bar{x}_{1m}] \\ [\otimes \bar{x}_{21}] & [\otimes \bar{x}_{22}] & \dots & [\otimes \bar{x}_{2m}] \\ \vdots & \vdots & \ddots & \vdots \\ [\otimes \bar{x}_{n1}] & [\otimes \bar{x}_{n2}] & \dots & [\otimes \bar{x}_{nm}] \end{bmatrix} = \begin{bmatrix} [\bar{w}_{11}; \bar{b}_{11}] & [\bar{w}_{12}; \bar{b}_{12}] & \dots & [\bar{w}_{1m}; \bar{b}_{1m}] \\ [\bar{w}_{21}; \bar{b}_{21}] & [\bar{w}_{22}; \bar{b}_{22}] & \dots & [\bar{w}_{2m}; \bar{b}_{2m}] \\ \vdots & \vdots & \ddots & \vdots \\ [\bar{w}_{n1}; \bar{b}_{n1}] & [\bar{w}_{n2}; \bar{b}_{n2}] & \dots & [\bar{w}_{nm}; \bar{b}_{nm}] \end{bmatrix}. \quad (18)$$

Step 5: Determining weights of the attributes q_j .

Step 6: Calculating the weighted normalized decision matrix $\otimes \hat{X}$. The weighted normalized values $\otimes \hat{x}_{ij}$ are calculated as follows:

$$\otimes \hat{x}_{ij} = \otimes \bar{x}_{ij} \cdot q_j; \quad \hat{w}_{ij} = \bar{w}_{ij} \cdot q_j; \quad \hat{b}_{ij} = \bar{b}_{ij} \cdot q_j. \quad (19)$$

In formula (19), q_j is the weight of the j -th attribute.

$$\otimes \hat{X} = \begin{bmatrix} [\otimes \hat{x}_{11}] & [\otimes \hat{x}_{12}] & \dots & [\otimes \hat{x}_{1m}] \\ [\otimes \hat{x}_{21}] & [\otimes \hat{x}_{22}] & \dots & [\otimes \hat{x}_{2m}] \\ \vdots & \vdots & \ddots & \vdots \\ [\otimes \hat{x}_{n1}] & [\otimes \hat{x}_{n2}] & \dots & [\otimes \hat{x}_{nm}] \end{bmatrix} = \begin{bmatrix} [\hat{w}_{11}; \hat{b}_{11}] & [\hat{w}_{12}; \hat{b}_{12}] & \dots & [\hat{w}_{1m}; \hat{b}_{1m}] \\ [\hat{w}_{21}; \hat{b}_{21}] & [\hat{w}_{22}; \hat{b}_{22}] & \dots & [\hat{w}_{2m}; \hat{b}_{2m}] \\ \vdots & \vdots & \ddots & \vdots \\ [\hat{w}_{n1}; \hat{b}_{n1}] & [\hat{w}_{n2}; \hat{b}_{n2}] & \dots & [\hat{w}_{nm}; \hat{b}_{nm}] \end{bmatrix}. \quad (20)$$

Step 7: Calculating the sums P_i of the attribute values, whose larger values are more preferable, for each alternative:

$$P_i = \frac{1}{2} \sum_{j=1}^k (\hat{w}_{ij} + \hat{b}_{ij}). \quad (21)$$

Step 8: Calculating the sums R_i of attribute values, whose smaller values are more preferable, for each alternative:

$$R_i = \frac{1}{2} \sum_{j=k+1}^m (\hat{w}_{ij} + \hat{b}_{ij}). \quad (22)$$

In formula (22), $(m-k)$ is the number of attributes which must be minimized.

The sum of all R_i and P_i equals 1.

$$\sum_{i=1}^n P_i + \sum_{i=1}^n R_i = 1. \quad (23)$$

Step 9: Calculating the relative weight of each alternative Q_i :

$$Q_i = P_i + \frac{\sum_{i=1}^n R_i}{R_i \sum_{i=1}^n \frac{1}{R_i}}. \quad (24)$$

Step 9*: If all attributes should be minimized then $P_i = 0$ and $\sum_{j=1}^n R_i = 1$. The formula (24) can be written as follows:

$$Q_i = \frac{1}{R_i \cdot \sum_{i=1}^n \frac{1}{R_i}}. \quad (25)$$

Step 10: Determining the optimality criterion L :

$$L = \max_i Q_i; \quad i = \overline{1, n}. \quad (26)$$

Step 11: Determining the priority of the project.

Step 12: Calculating the utility degree of each alternative:

$$N_i = \frac{Q_i}{L}. \quad (27)$$

3.3. Establishing the general solution

There are two different multi-attribute decision-making methods presented: TOPSIS grey and COPRAS-G. Solution results for the problem under investigation are obtained compared to solution results and generated aggregated results of problem solution.

4. Case study: risk assessment of construction projects

Application of different solution methods sometimes yields different results. It is recommended to use several multi-attribute decision making methods for real problem solution and compare the results.

Due to a lack of information the attributes were determined at intervals. The TOPSIS method with attributes values determined at intervals and COPRAS-G

method were applied to construction objects risk assessment of small-scale objects in construction. Risk assessment of four small-scale objects was made by 3 experts. The small-scale objects are of different design, architecture, construction technology, area, different number of floors and they are in different sites of the Vilnius region.

The initial decision making data are presented in Table 1. In Table 1 q_j is the attribute weight and alternative objects are v_1, \dots, v_4 .

To determine the weights of the attributes, the experts' judgment method is applied (Kendall 1970), which has been successfully used in research by the authors since 1987 (Zavadskas 1987). In order to establish the weights, a survey has been carried out and 43 experts have been questioned. These experts, basing their answers on their knowledge, experience and intuition, had to rate attributes of effectiveness starting with the most important ones. The rating was done on a scale from 1 to 13, where 13 meant "very important" and 1 "not important at all". The weights of attributes were established according to the rating methods (Zavadskas 1987) of these experts and also demonstrated the priorities of the user (stakeholder). The weights of the attributes obtained by this method are presented in Table 1. All risks in construction should be as minimal as possible – optimization direction is minimum. In Table 1 data on the following attributes are presented:

a) External risk assessment:

- ⊗ x_1 – political,
- ⊗ x_2 – economic,

⊗ x_3 – social,

⊗ x_4 – weather;

b) Project risk assessment:

⊗ x_5 – time,

⊗ x_6 – cost,

⊗ x_7 – quality,

⊗ x_8 – technological,

⊗ x_9 – construction;

c) Internal risk assessment:

⊗ x_{10} – resource,

⊗ x_{11} – project member,

⊗ x_{12} – site,

⊗ x_{13} – documents and information.

Each attribute is given zero to ten score. Every expert is allowed to give grey number evaluations.

In Table 2 the normalised decision-making matrix is presented with value of each attribute expressed at intervals, for the calculation of both: TOPSIS grey and COPRAS-G methods. Fig. 6 is a graphic view showing the calculation results according to TOPSIS grey method. The calculation results according to COPRAS-G method are presented in Fig. 7. Fig. 8 is a graphic view showing the aggregated results.

The calculation results for each project are presented in Table 3.

Overall least risk according to calculation results by applying TOPSIS grey method (Table 3) ranks as follows:

Project 1 > Project 3 > Project 4 > Project 2.

Table 1. Initial decision-making matrix with values at some intervals

Attribute	Weight q_j	Expert 1				Expert 2				Expert 3				
		Project				Project				Project				
		v_1	v_2	v_3	v_4	v_1	v_2	v_3	v_4	v_1	v_2	v_3	v_4	
⊗ x_1	$[w_1; b_1]$	0.05	[6.0;7.0]	[6.5;7.5]	[5.0;5.5]	[6.0;6.5]	[7.0;8.0]	[7.5;8.0]	[6.5;8.5]	[7.0;8.0]	[7.5;8.5]	[6.0;8.0]	[6.5;8.0]	[7.5;9.0]
⊗ x_2	$[w_2; b_2]$	0.09	[6.0;6.5]	[7.0;7.5]	[5.0;6.0]	[6.0;7.0]	[7.0;8.5]	[7.5;8.5]	[6.0;7.0]	[6.5;7.0]	[6.5;8.0]	[4.0;5.5]	[5.5;6.0]	[7.0;7.5]
⊗ x_3	$[w_3; b_3]$	0.06	[6.0;6.5]	[5.0;5.5]	[4.0;5.0]	[5.5;6.0]	[8.0;8.5]	[6.5;7.5]	[5.5;6.5]	[8.0;8.5]	[7.0;8.5]	[7.0;8.0]	[5.5;6.5]	[5.5;6.5]
⊗ x_4	$[w_4; b_4]$	0.04	[4.5;5.5]	[5.0;6.5]	[5.5;7.5]	[6.0;6.5]	[4.0;5.0]	[4.5;5.0]	[6.5;7.0]	[6.5;7.5]	[4.5;5.0]	[5.5;6.0]	[5.5;7.0]	[8.0;8.5]
⊗ x_5	$[w_5; b_5]$	0.09	[8.0;8.5]	[8.5;9.0]	[6.0;6.5]	[7.0;8.5]	[4.0;5.0]	[6.0;6.5]	[7.0;7.5]	[5.0;7.0]	[6.5;7.0]	[6.0;7.0]	[5.5;6.5]	[6.0;7.0]
⊗ x_6	$[w_6; b_6]$	0.11	[7.0;7.5]	[8.0;8.5]	[4.5;5.0]	[8.0;8.5]	[6.0;6.5]	[7.0;7.5]	[5.0;5.5]	[7.5;8.0]	[8.0;9.0]	[7.0;8.0]	[5.0;6.0]	[7.5;8.5]
⊗ x_7	$[w_7; b_7]$	0.12	[5.0;5.5]	[6.0;6.5]	[5.5;7.0]	[4.0;6.0]	[4.5;5.5]	[5.5;7.5]	[7.5;8.0]	[5.0;6.5]	[7.0;7.5]	[4.0;5.0]	[6.5;7.5]	[6.0;7.0]
⊗ x_8	$[w_8; b_8]$	0.07	[2.0;4.0]	[5.0;6.5]	[4.5;5.5]	[4.0;6.5]	[4.0;5.5]	[4.0;6.0]	[4.0;5.5]	[3.5;5.0]	[4.0;4.5]	[5.5;6.5]	[3.5;6.0]	[6.0;5.0]
⊗ x_9	$[w_9; b_9]$	0.09	[8.0;9.0]	[7.5;8.0]	[7.0;8.5]	[5.0;7.5]	[7.0;8.5]	[6.5;7.0]	[7.5;9.0]	[6.0;7.0]	[6.0;8.0]	[7.0;7.5]	[6.5;7.0]	[7.0;7.5]
⊗ x_{10}	$[w_{10}; b_{10}]$	0.06	[7.0;7.5]	[6.0;7.5]	[5.0;6.5]	[5.0;6.5]	[4.5;6.5]	[7.5;8.0]	[6.5;7.5]	[7.0;8.0]	[5.0;6.0]	[7.5;8.0]	[6.5;8.0]	[7.0;7.5]
⊗ x_{11}	$[w_{11}; b_{11}]$	0.11	[5.0;6.5]	[7.0;8.0]	[5.5;6.0]	[6.0;7.5]	[4.0;5.0]	[7.0;7.5]	[5.0;5.5]	[6.5;7.0]	[5.0;6.0]	[6.5;7.0]	[6.0;6.5]	[6.5;7.0]
⊗ x_{12}	$[w_{12}; b_{12}]$	0.04	[7.0;7.5]	[4.0;5.5]	[6.0;6.5]	[5.0;6.0]	[5.0;5.5]	[7.0;8.0]	[7.0;7.5]	[8.0;8.5]	[7.0;8.0]	[5.0;6.0]	[6.5;7.0]	[6.0;6.5]
⊗ x_{13}	$[w_{13}; b_{13}]$	0.07	[5.0;6.0]	[3.0;4.5]	[6.0;7.0]	[6.0;6.5]	[4.0;5.0]	[4.5;5.0]	[6.5;7.5]	[4.5;5.0]	[4.0;5.0]	[4.5;5.5]	[5.0;5.5]	[7.0;7.5]

Table 2. Normalized decision-making matrix

Attribute	Expert 1				Expert 2				Expert 3				
	Project												
	v ₁	v ₂	v ₃	v ₄	v ₁	v ₂	v ₃	v ₄	v ₁	v ₂	v ₃	v ₄	
TOPSIS grey method													
⊗ \bar{x}_1	$[\bar{w}_1; \bar{b}_1]$	[0.80;0.60]	[0.92;0.50]	[1.00;0.90]	[0.80;0.70]	[0.92;0.77]	[0.85;0.92]	[1.00;0.69]	[0.92;0.77]	[0.75;0.58]	[1.00;0.67]	[0.92;0.67]	[0.75;0.50]
⊗ \bar{x}_2	$[\bar{w}_2; \bar{b}_2]$	[0.80;0.70]	[0.60;0.50]	[1.00;0.80]	[0.80;0.60]	[0.83;0.67]	[0.75;0.58]	[1.00;0.83]	[0.92;0.83]	[0.38;0.13]	[1.00;0.63]	[0.63;0.50]	[0.25;0.13]
⊗ \bar{x}_3	$[\bar{w}_3; \bar{b}_3]$	[0.50;0.38]	[0.75;0.63]	[1.00;0.75]	[0.63;0.50]	[0.55;0.45]	[0.82;0.64]	[1.00;0.82]	[0.55;0.45]	[0.60;0.30]	[0.60;0.40]	[1.00;0.70]	[0.90;0.70]
⊗ \bar{x}_4	$[\bar{w}_4; \bar{b}_4]$	[1.00;0.78]	[0.89;0.56]	[0.78;0.73]	[0.67;0.78]	[1.00;0.75]	[0.88;0.75]	[0.38;0.25]	[0.38;0.13]	[1.00;0.89]	[0.78;0.67]	[0.78;0.44]	[0.22;0.11]
⊗ \bar{x}_5	$[\bar{w}_5; \bar{b}_5]$	[0.67;0.58]	[0.58;0.50]	[1.00;0.92]	[0.83;0.58]	[1.00;0.75]	[0.50;0.38]	[0.25;0.13]	[0.75;0.25]	[0.82;0.73]	[0.91;0.64]	[1.00;0.82]	[0.91;0.73]
⊗ \bar{x}_6	$[\bar{w}_6; \bar{b}_6]$	[0.44;0.33]	[0.33;0.11]	[1.00;0.89]	[0.44;0.11]	[0.80;0.70]	[0.60;0.50]	[0.55;0.90]	[0.50;0.40]	[0.40;0.20]	[0.60;0.40]	[1.00;0.80]	[0.50;0.30]
⊗ \bar{x}_7	$[\bar{w}_7; \bar{b}_7]$	[0.75;0.63]	[0.50;0.38]	[0.63;0.25]	[1.00;0.50]	[1.00;0.78]	[0.78;0.33]	[0.33;0.22]	[0.89;0.56]	[0.25;0.13]	[1.00;0.75]	[0.38;0.13]	[0.50;0.25]
⊗ \bar{x}_8	$[\bar{w}_8; \bar{b}_8]$	[1.00;0.86]	[0.57;0.14]	[0.71;0.43]	[0.86;0.14]	[0.86;0.43]	[0.86;0.29]	[0.86;0.43]	[1.00;0.57]	[0.86;0.71]	[0.43;0.14]	[1.00;0.29]	[0.29;0.57]
⊗ \bar{x}_9	$[\bar{w}_9; \bar{b}_9]$	[0.40;0.20]	[0.50;0.40]	[0.60;0.30]	[1.00;0.50]	[0.83;0.58]	[0.92;0.83]	[0.75;0.50]	[0.80;0.82]	[1.00;0.67]	[0.83;0.75]	[0.92;0.83]	[0.83;0.75]
⊗ \bar{x}_{10}	$[\bar{w}_{10}; \bar{b}_{10}]$	[0.60;0.40]	[0.80;0.50]	[1.00;0.70]	[0.80;0.70]	[1.00;0.56]	[0.33;0.22]	[0.56;0.33]	[0.44;0.22]	[1.00;0.80]	[0.50;0.30]	[0.70;0.40]	[0.60;0.50]
⊗ \bar{x}_{11}	$[\bar{w}_{11}; \bar{b}_{11}]$	[1.00;0.70]	[0.60;0.40]	[0.90;0.80]	[0.80;0.50]	[1.00;0.75]	[0.25;0.13]	[0.75;0.63]	[0.38;0.25]	[1.00;0.80]	[0.70;0.60]	[0.80;0.70]	[0.70;0.60]
⊗ \bar{x}_{12}	$[\bar{w}_{12}; \bar{b}_{12}]$	[0.25;0.13]	[1.00;0.63]	[0.50;0.38]	[0.86;0.50]	[1.00;0.90]	[0.60;0.40]	[0.60;0.50]	[0.40;0.30]	[0.60;0.40]	[1.00;0.80]	[0.70;0.60]	[0.80;0.70]
⊗ \bar{x}_{13}	$[\bar{w}_{13}; \bar{b}_{13}]$	[0.75;0.50]	[1.00;0.88]	[0.38;0.25]	[0.50;0.38]	[1.00;0.75]	[0.88;0.75]	[0.38;0.13]	[0.88;0.75]	[1.00;0.75]	[0.88;0.63]	[0.75;0.63]	[0.25;0.13]
COPRAS-G method													
⊗ \bar{x}_1	$[\bar{w}_1; \bar{b}_1]$	[0.24;0.28]	[0.26;0.30]	[0.20;0.22]	[0.24;0.26]	[0.23;0.26]	[0.25;0.26]	[0.21;0.28]	[0.23;0.26]	[0.25;0.28]	[0.20;0.26]	[0.21;0.26]	[0.25;0.30]
⊗ \bar{x}_2	$[\bar{w}_2; \bar{b}_2]$	[0.24;0.25]	[0.27;0.29]	[0.20;0.24]	[0.24;0.27]	[0.24;0.28]	[0.26;0.30]	[0.21;0.24]	[0.23;0.24]	[0.26;0.30]	[0.16;0.22]	[0.22;0.24]	[0.28;0.30]
⊗ \bar{x}_3	$[\bar{w}_3; \bar{b}_3]$	[0.28;0.30]	[0.23;0.25]	[0.18;0.23]	[0.25;0.28]	[0.27;0.29]	[0.22;0.25]	[0.19;0.22]	[0.27;0.29]	[0.26;0.31]	[0.26;0.30]	[0.19;0.24]	[0.20;0.24]
⊗ \bar{x}_4	$[\bar{w}_4; \bar{b}_4]$	[0.19;0.23]	[0.21;0.28]	[0.23;0.32]	[0.26;0.28]	[0.17;0.22]	[0.20;0.22]	[0.28;0.30]	[0.28;0.33]	[0.18;0.20]	[0.22;0.24]	[0.22;0.28]	[0.32;0.34]
⊗ \bar{x}_5	$[\bar{w}_5; \bar{b}_5]$	[0.26;0.27]	[0.27;0.29]	[0.19;0.21]	[0.23;0.27]	[0.17;0.21]	[0.25;0.27]	[0.29;0.31]	[0.21;0.29]	[0.25;0.27]	[0.23;0.29]	[0.21;0.25]	[0.23;0.27]
⊗ \bar{x}_6	$[\bar{w}_6; \bar{b}_6]$	[0.25;0.27]	[0.27;0.31]	[0.16;0.18]	[0.25;0.31]	[0.23;0.25]	[0.26;0.28]	[0.19;0.21]	[0.28;0.30]	[0.27;0.31]	[0.24;0.27]	[0.17;0.20]	[0.25;0.29]
⊗ \bar{x}_7	$[\bar{w}_7; \bar{b}_7]$	[0.22;0.24]	[0.26;0.29]	[0.24;0.31]	[0.18;0.26]	[0.18;0.22]	[0.22;0.30]	[0.30;0.32]	[0.20;0.26]	[0.28;0.30]	[0.16;0.20]	[0.26;0.30]	[0.24;0.28]
⊗ \bar{x}_8	$[\bar{w}_8; \bar{b}_8]$	[0.18;0.20]	[0.25;0.33]	[0.23;0.28]	[0.20;0.33]	[0.21;0.29]	[0.21;0.32]	[0.21;0.29]	[0.19;0.27]	[0.20;0.22]	[0.27;0.32]	[0.17;0.29]	[0.24;0.29]
⊗ \bar{x}_9	$[\bar{w}_9; \bar{b}_9]$	[0.26;0.30]	[0.25;0.26]	[0.23;0.28]	[0.17;0.25]	[0.24;0.29]	[0.22;0.24]	[0.26;0.31]	[0.21;0.24]	[0.21;0.28]	[0.25;0.27]	[0.23;0.25]	[0.25;0.27]
⊗ \bar{x}_{10}	$[\bar{w}_{10}; \bar{b}_{10}]$	[0.27;0.30]	[0.23;0.29]	[0.19;0.25]	[0.23;0.25]	[0.16;0.23]	[0.27;0.29]	[0.23;0.27]	[0.25;0.29]	[0.18;0.21]	[0.27;0.30]	[0.23;0.29]	[0.25;0.27]
⊗ \bar{x}_{11}	$[\bar{w}_{11}; \bar{b}_{11}]$	[0.19;0.25]	[0.27;0.31]	[0.21;0.23]	[0.23;0.29]	[0.17;0.21]	[0.29;0.32]	[0.21;0.23]	[0.27;0.29]	[0.20;0.24]	[0.26;0.28]	[0.24;0.26]	[0.26;0.28]
⊗ \bar{x}_{12}	$[\bar{w}_{12}; \bar{b}_{12}]$	[0.29;0.32]	[0.17;0.23]	[0.25;0.27]	[0.21;0.25]	[0.18;0.19]	[0.25;0.28]	[0.25;0.27]	[0.28;0.30]	[0.27;0.31]	[0.19;0.23]	[0.25;0.27]	[0.23;0.25]
⊗ \bar{x}_{13}	$[\bar{w}_{13}; \bar{b}_{13}]$	[0.22;0.26]	[0.18;0.20]	[0.29;0.31]	[0.26;0.29]	[0.19;0.24]	[0.21;0.24]	[0.31;0.36]	[0.21;0.24]	[0.18;0.23]	[0.20;0.25]	[0.23;0.25]	[0.32;0.34]

Table 3. Solution results by applying TOPSIS grey and COPRAS-G methods

		Expert 1		Expert 2		Expert 3		Aggregated		C_i^{*+}	Rank
		d^{1+}	d^{1-}	d^{2+}	d^{2-}	d^{3+}	d^{3-}	d^{*+}	d^{*-}		
Calculation results by applying TOPSIS grey method	v_1	0.449	0.510	0.489	0.637	0.513	0.385	0.484	0.511	0.514	1
	v_2	0.532	0.307	0.725	0.318	0.387	0.448	0.548	0.358	0.395	4
	v_3	0.379	0.447	0.503	0.402	0.393	0.421	0.425	0.423	0.499	2
	v_4	0.496	0.438	0.493	0.555	0.520	0.224	0.503	0.406	0.446	3
Calculation results by applying COPRAS-G method		N^1		N^2		N^3		N^4			Rank
	v_1	0.931		1.000		0.946		0.959			1
	v_2	0.877		0.857		0.984		0.906			3
	v_3	1.000		0.865		1.000		0.955			2
v_4	0.931		0.877		0.883		0.897			4	

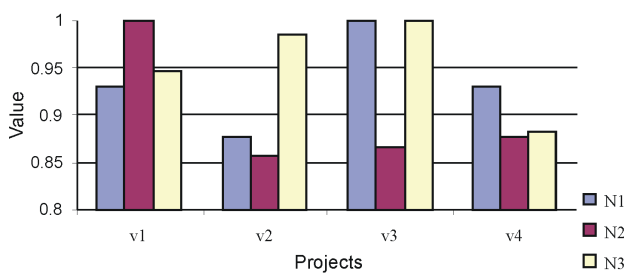


Fig. 6. Calculation results according to COPRAS-G method

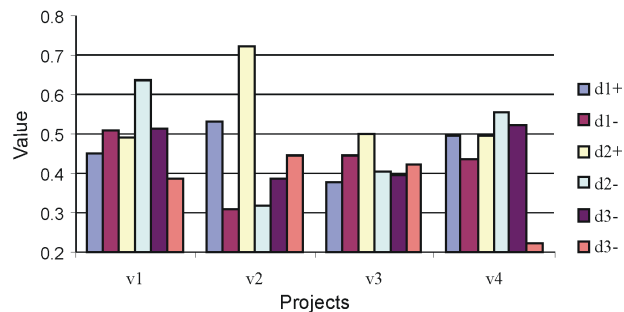


Fig. 7. Calculation results according to TOPSIS grey method

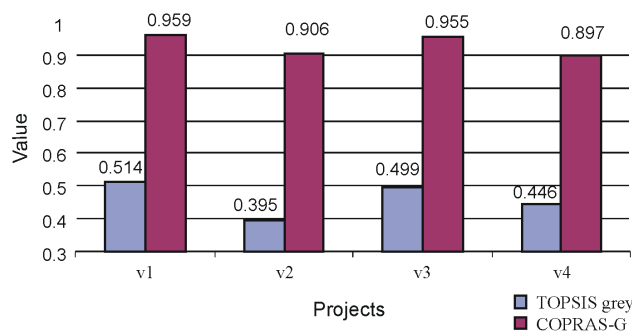


Fig. 8. Aggregated calculation results (COPRAS-G and TOPSIS grey methods)

The projects risk according to COPRAS-G method ranks as follows:

Project 1 > Project 3 > Project 2 > Project 4.

The calculation results showed that the first project has the least risk and the second or the fourth project are most risky. The first alternative was selected and implemented.

5. Conclusions

Decision-making is very important in the construction management, such as risk assessment results in construction projects, contractor and supplier selection, etc.

In real life multi-attribute modelling of multi-alternative assessment problems have some attribute values, which deal with the future and must be expressed at intervals.

Sometimes calculation according to different methods yields different results. For decision-making it is reasonable to apply several methods and select the best alternative according to aggregated results.

This model and solution results are of both practical and scientific interest. It allows all members of the construction business to make a decision by evaluating multiple attributes when values of initial data are given at intervals.

The research results show the different risk levels of construction objects.

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STATYBOS PROJEKTŲ RIZIKOS VERTINIMAS

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S a n t r a u k a

Straipsnyje vertinama statybos projektų rizika. Vertinimas pagrįstas įvairiais daugiakriterio vertinimo metodais. Rizikos vertinimo rodikliai atrenkami, atsižvelgiant į suinteresuotų šalių interesus, tikslus ir veiksnius, kurie turi įtakos statybos proceso efektyvumui ir nekilnojamojo turto vertės didinimui. Projektai surikiuoti pagal naudingumą, nustatyti santykiniai jų optimalumo dydžiai. Uždavinio modeliui aprašyti ir jam išspręsti taikomi TOPSIS grey ir COPRAS-G metodai. Projektų savybės aprašomos efektyvumo rodiklių reikšmėmis, apibrėžiamomis intervaluose. Straipsnyje aprašomas taikomas modelis, atlikta uždavinio analizė ir pateikiamos trumpos išvados.

Reikšminiai žodžiai: sprendimų priėmimas, statyba, rizika, įvertinimas, rodikliai, TOPSIS grey, COPRAS-G, rangavimas.

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