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THE MODEL OF BRITTLE MATRIX COMPOSITES FOR DISTRIBUTION OF STEEL FIBRES

Wiesława Głodkowska[1], **Janusz Kobaka**[2]
[1]*Concrete Structures Department, Faculty of Civil and Environmental Engineering,*
[2]*Laboratory of Building Engineering, Faculty of Civil and Environmental Engineering,*
Technical University of Koszalin, ul. Sniadeckich 2, 75-453 Koszalin, Poland
E-mails: [1]*glod@wbiis.tu.koszalin.pl (corresponding author);* [2]*kobaka@wbiis.tu.koszalin.pl*

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Abstract. The paper deals with the distribution of steel fibres in the mineral composite of fine aggregate. The authors have proposed the distribution of steel fibres in a composite space model based on statistical grounds. The model provides for the randomness of fibre distribution in composite space in accordance with the adopted probability distribution. The developed model has been experimentally verified. The results concerning the distribution of steel fibres in mineral composite have been obtained from the statistical model and compared with those of the model frequently applied by other authors on the basis of geometric grounds. Good compatibility of steel fibre distribution for a description of both models has been ascertained. As the amount of fibres influences the strength of composite tensile, the relationship between the above introduced feature and the quantity of fibres in the cross-section located nearby tensile failure surface has been developed with reference to the experimental tests.

Keywords: mineral composite, fibre reinforcement, fibre distribution, model, studies.

1. Introduction

Mineral composites with an addition of steel fibre reinforcement gain more and more popularity becoming frequently used structural material in civil engineering. Composites with steel fibres were first used more than 130 years ago when in 1874. A. Bernard announced the first patent for steel fibre reinforced concrete.

Steel fibres added to the composite mix act as reinforcement and considerably improve some mechanical properties of the composite. Such properties are, for example, tensile strength, flexural strength (Yazici *et al.* 2007), abrasion resistance, mechanical impact resistance or freezing and thawing test sustainability. Advantageous properties of the composite containing steel fibres allows using this material in tunnel construction (Kasper *et al.* 2008), bridge surfacing, industrial flooring or reinforcing mine headings.

Within the period of more than 130 years of applying similar types of composites in civil engineering, many authors made attempts to describe fibre distribution in composite space (Krenchel 1975; Soroushian, Lee 1990; Li *et al.* 1991; Stroeven 1991; Kooiman 2000) which is a complex problem due to the fact that real fibre orientation to composite space remains unknown. Fibres are oriented to various angles considering different distances between them and mould walls. Steel fibre reinforcement is dispersed within the entire composite space the final distribution of which is influenced by a number of factors such as mould wall restrictions, fibre content, the mix preparation method (mixing, vibration, putting in the mould) or composite matrix (filling) composition. Furthermore, fibre distribution has key influence on the main mechanical composite properties such as tensile strength.

The authors have proposed steel fibre distribution in the composite space model based on statistical grounds. The model provides for the randomness of fibre distribution within composite space in accordance with the adopted probability distribution. Based on this model, the correlation between the amount of fibres in the crack failure cross-section and tensile strength during composite split test has been established. The proposed model has been compared with another model described in literature on the subject and based on the geometric grounds of fibre distribution. The models based on statistical and geometric grounds have been named in this paper as statistical and geometric models respectively. The geometric model has been used for the purposes of comparative analysis because it takes into account, just like a statistical model, the influence of the wall of the formed element on fibre orientation.

2. A statistical model of fibre distribution

The authors have assumed for the statistical model that steel fibres are distributed randomly in composite space. The probability of fibre occurrence in the given area was determined with uniform statistical distribution that is a statistical distribution of probability distributed uniformly within <a, b> interval. Probability outside this interval is equal to zero. The probability density function has been defined looking at the following relationship:

$$f(x) = \begin{cases} 0 & x < a, \\ \dfrac{1}{b-a} & a \le x \le b, \\ 0 & x > b. \end{cases} \qquad (1)$$

An assumption has been made that composite space is the 150×150 mm cube with stiff sides. Fibre length in the introduced composite was 50 mm. Furthermore, the following simplifications have been applied in the statistical model:

- fibre is straight;
- fibre diameter is close to zero;
- fibres are rigid and stiff.

To simulate fibre distribution in composite space, *Fibredist* computer program was developed. The calculation algorithm, which can be used for defining steel fibre orientation in composite space, is shown in Fig. 1.

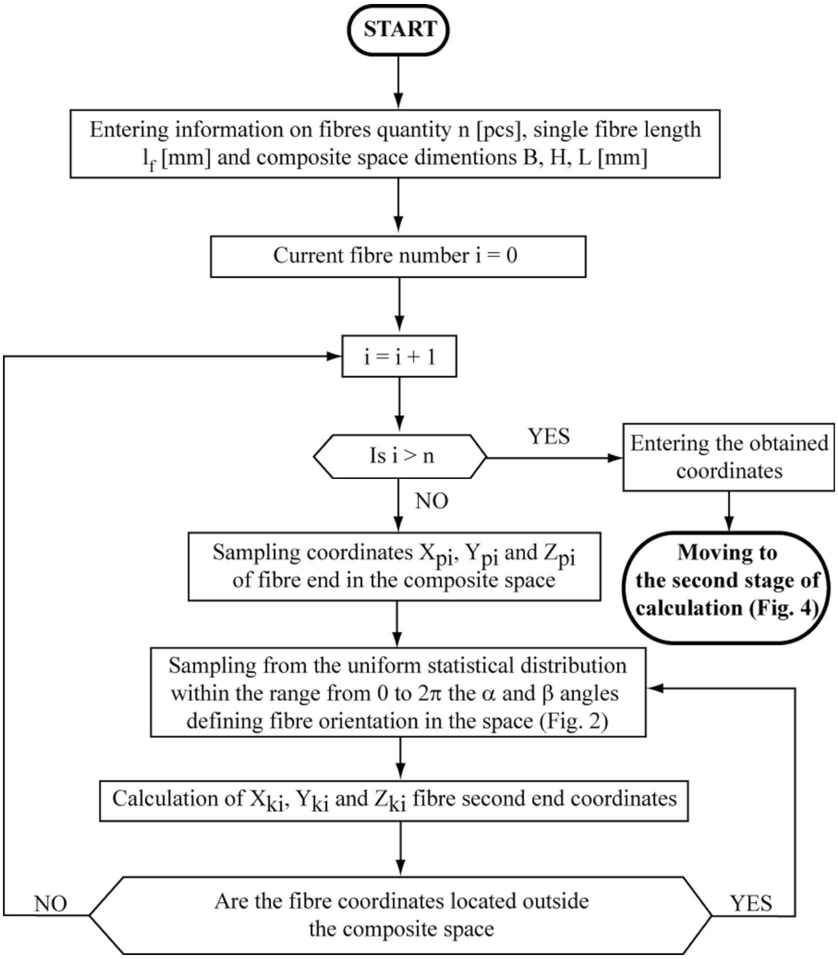


Fig. 1. The algorithm determining fibre orientation in composite space – the first stage of calculation

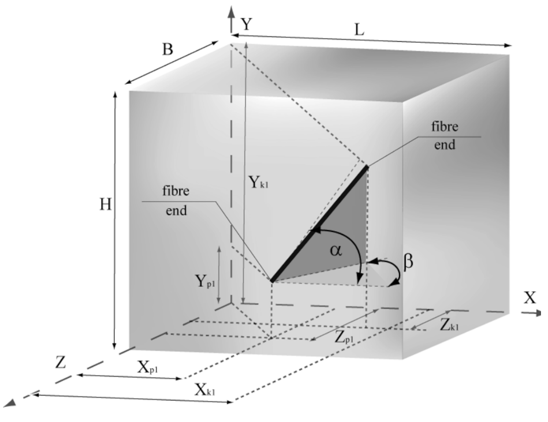


Fig. 2. The orientation of a single fibre with relation to the axis of the Cartesian system

The quantity of all steel fibres "*n*" contained in composite space (Fig. 1) is defined on the basis of the assumed volumetric fibre content in the given composite, fibre dimensions and steel density. The final effect of the calculation process is data on the coordinates of fibre ends within composite space (Fig. 2).

Information about fibre orientation in composite space is used in further calculations of fixing the number of fibres and their distribution in the composite crack failure cross-section. An assumption has been made that a plane perpendicular to x-axis would run through composite space (150 mm cube). This surface cuts through composite space at L/2 (Fig. 3).

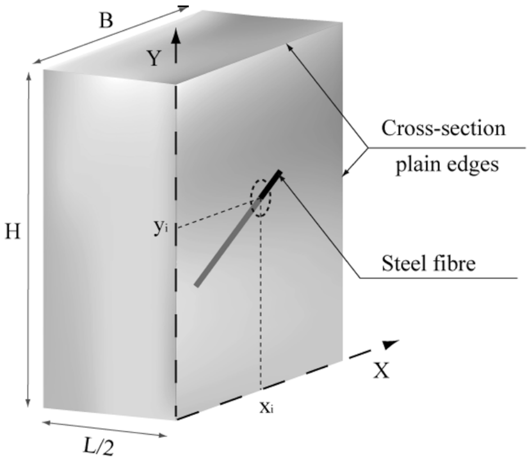


Fig. 3. Steel fibre cut-through point on the cross-section surface of composite space

The method of calculating the coordinates of fibre cut-through points on the cross-section surface is illustrated in Fig. 4.

3. A geometrical model of fibre distribution

The number of fibres cutting-through the given cross-section surface of composite space in the so-called geometric model has been defined applying the fibre orientation factor (α) (Dupont, Vandewalle 2005) defining the average proportion of fibre length projection (onto the horizontal axis) to their length.

To calculate α factor, the analysed cross-section has been divided into sections subjected to various limitations (Fig. 5). In the first section, α is calculated with the assumption that the fibre can rotate freely in all directions. In the second section, fibre rotation is limited to one edge which is the wall of the element moulded. In the third section, fibre rotation is limited to two edges – the element walls perpendicular to each other.

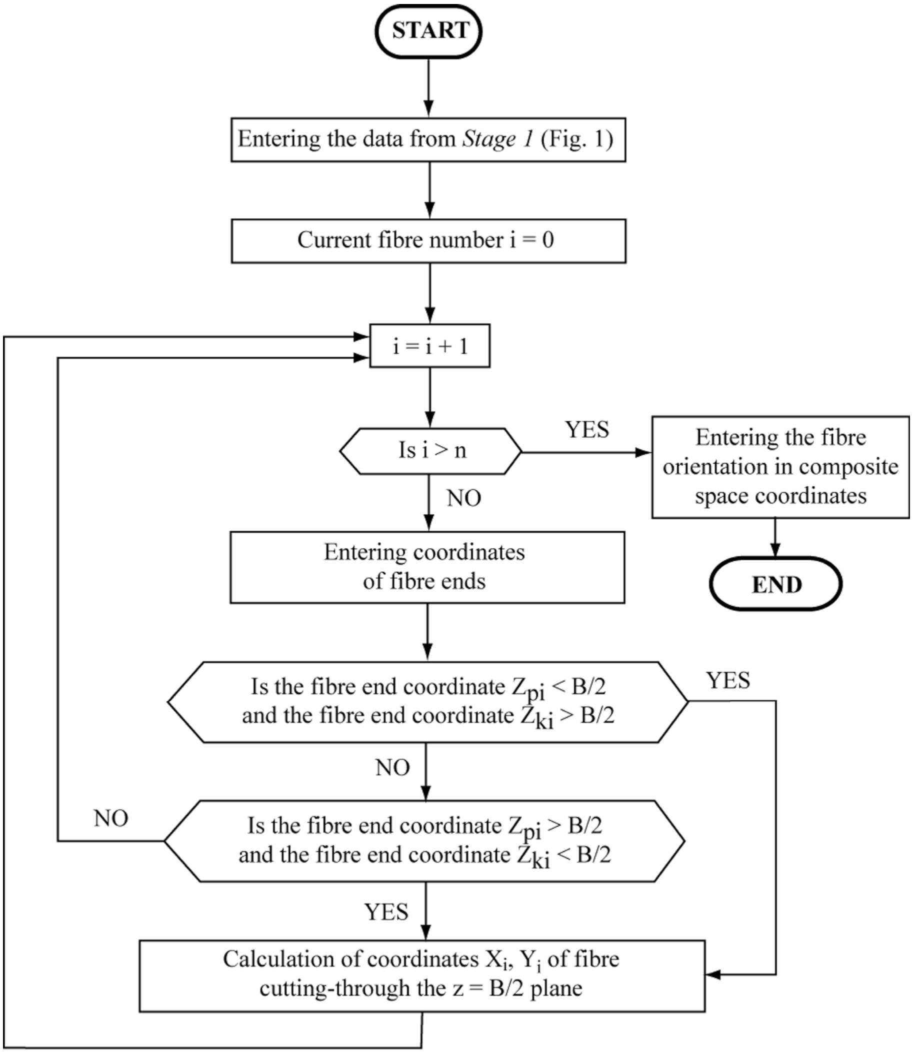


Fig. 4. Calculation algorithm for the coordinates of fibre cut-through points on the cross-section surface of composite space – the second stage of calculation

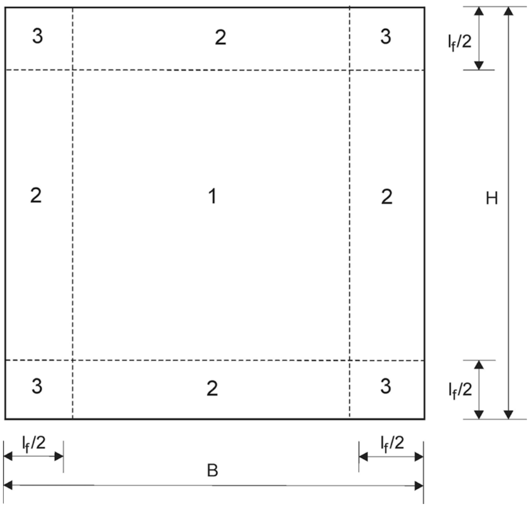


Fig. 5. Composite space cross-section divided into sections (2): l_f – fibre length; H –element height; B – element breadth

A detailed description of the method for determining fibre orientation factors in particular sections was provided in the papers by (Dupont 2003; Dupont, Vandewalle 2005).

After fixing fibre orientation factors for particular sections, a general form of coefficient α for the analysed cross-section can be defined (Dupont 2003; Dupont, Vandewalle 2005):

$$\alpha = \frac{\left[\alpha_1 \cdot \left(B - l_f\right) \cdot \left(H - l_f\right) + \alpha_2 \cdot l_f \cdot \left[\left(B - l_f\right) + \left(H - l_f\right)\right] + \alpha_3 \cdot l_f^2\right]}{BH}, \tag{2}$$

where: B – cross-section breadth [mm]; H – cross-section height [mm]; l_f – fibre length [mm]; α_1 – fibre orientation factor in the first section; α_2 – fibre orientation factor in the second section; α_3 – fibre orientation factor in the third section.

The number of fibres cutting-through the analysed cross-section has been defined using the following formula (Dupont, Vandewalle 2005):

$$N = \alpha \frac{V_f}{A_f} A_c, \tag{3}$$

where: N – the number of fibres in the analysed cross-section [pcs]; α – fibre orientation factor in the general cross-section; V_f –volumetric fibre content within composite space [%]; A_f – single fibre cross-section area [mm^2]; A_c – sample cross-section area [mm^2].

4. Used materials and methods for testing

To verify the statistical model of steel fibre distribution, the mineral composite of fine aggregate containing variable steel fibre content was prepared. Eight cubic samples of 150 mm were made for each fibre content: 0%, 0.5%, 1%, 1.5%, 2% and 2.5%. To form elements for testing, post-glacial sand, CEM II/A-V 42.5R, Portland cement and silica dust super plasticiser containing active pozzolana were used. Fibre reinforcement was made of hook-shaped steel fibres featuring slenderness ratio $\lambda = l_f/d = 62.5$ ($l_f = 50$ mm, $d = 0.8$ mm).

The mineral composite matrix of fine aggregate was designed applying the analytical/experimental method. The composition of the matrix was modified by adding silica dust and plasticising/fluidising admixture which allowed for obtaining a proportion of $w/c = 0.38$. As variables of composite composition, the quantities of steel fibres such as 0.5, 1.0, 1.5, 2.0 and 2.5% with relation to the volume of the composite matrix, were adopted. Fibres in the composite mix were randomly located.

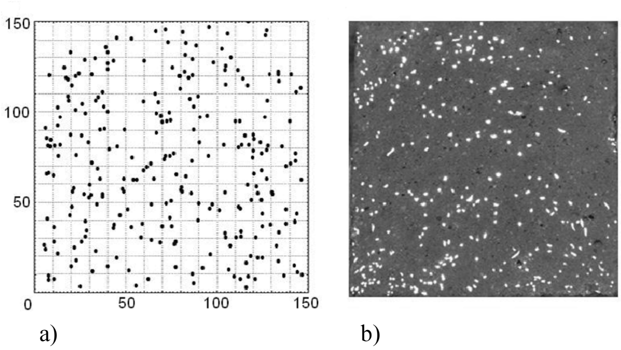
Tests on the split tensile strength (f_{ct}) of the composite were conducted in order to define the dependence of that feature on the quantity of steel fibre and distribution in the plane (European Standard EN 12390-6 2000). The test elements were demoulded within the next 2 days and kept until testing time for 28 days at a temperature of 20±2 ºC and relative air humidity of 100%. The number of samples needed for defining the average statistical value of the given feature was determined on the basis of the statistical analysis of preliminary test results at a tolerance of $\upsilon = 10\%$ and significance level of $\alpha = 0.05$. This paper presents a detailed description of the physical and mechanical features of the researched mineral composite (Głodkowska, Kobaka 2009).

Subsequently to the definition of strength f_{ct}, to define the quantity and distribution of steel fibres in the cross-section, each sample was cut vertically to the top finished surface nearby the plane of crack failure. The surfaces of the cut samples were reproduced using Fuji-Film S5000 digital camera with 6MPix matrix and the lens focal length of 5.7÷57 mm; then, the created images were subjected to the analysis of the digital image (Computer program GSA Image Analyser).

5. Research results and discussion

To check the correctness of the assumptions made for the statistical model, the results of calculating the distribution of steel fibre in composite space were compared with those of the test. The results of our research were also related to the geometric model. The number of steel fibres cutting through the cross-section at a half of the element breadth of 150 mm was used as the basis for a comparison.

In accordance with the assumptions of the statistical model made for mineral composite having a variable content of steel fibre, coordinates for fibre orientation in composite space were calculated. For each fibre content, 8 numerical simulations were performed; in effect, coordinates for fibre orientation in composite space were obtained which allowed for calculating coordinates at points where fibres penetrated the cross-section plane of the considered element (see Figs 3 and 6). Also, a test for the compatibility of fibre distribution (χ^2 test) with the assumed uniform distribution was accomplished. It appears from statistical analysis that the uniform statistical distribution of probability was correctly assumed in the analysis at a value of $\chi^2 = 13.69 < \chi^2_{\mathrm{kr}} = 21.03$ for the applied significance level $\alpha = 0.05$.



a) b)

Fig. 6. The distribution of steel fibres in the composite crack failure cross-section: a) theoretical image; b) real image

To identify steel fibre distribution in composite space, the photographs of sample cross-section obtained during the testing procedure (Fig. 6) were subjected to computer image analysis (applying *GSA Image Analyser* computer program) that allowed for indicating the real number of steel fibres in the analysed cross-section. The analysis of steel fibre distribution in composite space images observed during the test (Fig. 6b) as well as of those originating from the statistical model (Fig. 6a) has shown their high resemblance.

The coefficient of steel fibre distribution (α) was calculated for the geometric model. For the fibres featuring the length of $l_f = 50$ mm and the area of the considered cross-section having space of 150×150, the value of coefficient α was 0.58. Then, the numbers of fibres cutting-through the cross-section were calculated for various fibre contents.

Fig. 7 illustrates the dependence of the quantity of steel fibres per 1 cm^2 of 150 mm element cross-section on the percentage of fibre content in the composite volume. The values calculated for statistical and geometric models and experimental values were then compared.

The analysis of the results presented in Fig. 7 indicates high compatibility of statistical and geometric models with test results. Differences in the calculated values of fibre numbers in the considered cross-sections are statistically insignificant. Furthermore, the determined values of the standard deviation for the quantity of fibres in the cross-section have indicated that along with an increase in the content of steel fibre in composite space, standard deviation from experimental results has also increased. This trend can be associated with an increase in disturbances caused by the influence of some fibres onto other ones during composite mix preparation at the stage of components mixing. This has been indicated by a standard deviation of 0.42 for the highest tested volumetric fibre content in the composite, i.e. 2.5%. Standard deviation for such quantity of fibres, as per statistical model, is approximately 3-fold lower than that originating from experimental studies which indicates the necessity of accounting for an increase in the irregularity of the real distribution of steel fibre in composite space for composites having high fibre content.

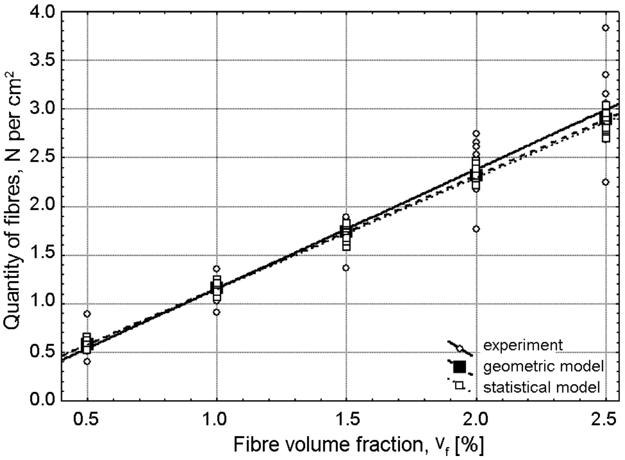


Fig. 7. Relationship between the quantity of steel fibres N per 1 cm^2 of the element cross-section and their percentage in composite volume v_f

The results of the split tensile test (f_{ct}) have indicated a clear dependence of strength on the quantity of fibres (N) in the composite crack failure cross-section. The quantity of steel fibres increases along with split tensile strength in the composite. The trend line illustrating this relationship (Fig. 8) is not, however, straight as it bends gradually. It appears from Fig. 8 that the highest increase in f_{ct} strength was observed within the tested scope for the average quantity of fibres making 1.2 (the quantity of fibres per 1 cm^2 of the cross-section) which is equivalent to 1% of fibre content within the composite volume. In the case of high fibre content (2%), an increase in split tensile strength does not compensate the cost of making such composite that increases in direct proportion to the volume of the fibre reinforcement used.

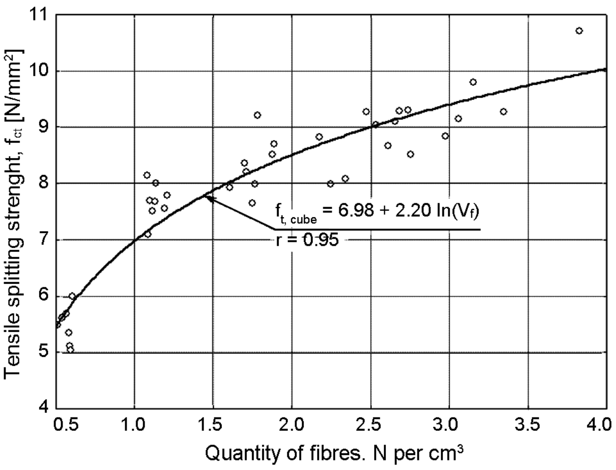


Fig. 8. The dependence of the split tensile strength of the mineral composite of fine aggregate on the quantity of steel fibres per 1 cm^2 of the composite crack failure cross-section

A high value of correlation factor r = 0.95 describing the dependence of the split tensile strength of the mineral composite of fine aggregate on the quantity of steel fibres per 1 cm^2 of the composite crack failure cross-section indicates that the applied regression equation accurately describes the results of the studies.

6. Conclusions

In the light of the test and analytical calculation results obtained, a high compatibility of both geometric and statistical models describing fibre distribution in composite space has been ascertained. The compatibility of calculation results with experimental values indicates the correctness of the developed statistical model and made assumptions.

The proposed model of steel fibre distribution based on statistical distribution allows for defining parameters of orientation towards each fibre, including orientation angle, the coordinates of both ends of the fibre, the coordinates of the points of penetration on a cross-section plane, mutual distances between fibres and the distance of fibre centres of gravity from element walls. Compared with the geometric model, the statistical model is simple in notation and use. It allows for obtaining more information on fibre distribution in composite space. The geometric model allows only for defining the quantity of fibres cutting-through the considered composite cross-section.

The developed statistical model can be applied for the composition-designing step of the composite to obtain a specific tensile strength value. The next stage for statistical model improvement takes into account an increase in the irregularity of real steel fibre distribution in composite space with increased content thereof. Also, the phenomenon of fibre settling, depending on mix consistence and vibration parameters, will be considered.

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PLIENINIŲ FIBRŲ PASISKIRSTYMO KOMPOZITUOSE SU TRAPIOMIS MATRICOMIS MODELIS

W. Głodkowska, J. Kobaka

Santrauka

Straipsnyje analizuojamas plieninių fibrų pasiskirstymas kompozite su mineraliniais užpildais. Autoriai pasiūlė plieninių fibrų pasiskirstymo kompozite modelį, grįstą statistine analize. Fibrų pasiskirstymas matricoje nagrinėjamas kaip atsitiktinis dydis, pasiskirstęs pagal tikimybinį skirstinį. Modelis yra eksperimentiškai patikrintas: plieninių fibrų pasiskirstymas kompozito matricoje pagal siūlomą statistinį modelį buvo palygintas su kitų autorių tyrimų rezultatais, taikant modelius, grįstus geometriniais pagrindais. Gauti rezultatai sutampa gerai. Kadangi plieninių fibrų kiekis turi įtakos kompozito tempiamajam stipriui, pateikta eksperimentiniais tyrimais pagrįsta priklausomybė tarp fibrų kiekio ir kompozito tempiamojo stiprio.

Reikšminiai žodžiai: mineralinis kompozitas, fibros, fibrų pasiskirstymas, modelis, analizė.

Wiesława GŁODKOWSKA. Professor at the Technical University of Koszalin, Poland. She runs the Concrete Structures and Technology Department. She's a member of the Engineering Section of Building Materials and Building Physics at the Polish Academy of Sciences. Her scientific specialty comprises concrete building. Her scientific interests pertain to repair, reinforcement and protection of concrete structures against corrosion.

Janusz KOBAKA. Headmaster of Laboratory of Building Engineering at the Technical University of Koszalin, Poland. His research interests include properties of fibre reinforced mineral composites and nondestructive testing methods used in civil engineering.