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# INTERTEMPORAL PORTFOLIO ALLOCATION AND HEDGING DEMAND: AN APPLICATION TO SOUTH AFRICA[1]

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**Abstract.** This paper analyses the intertemporal hedging demand for stocks and bonds in South Africa, the United Kingdom and the United States. The analysis is done using an approximate solution method for the optimal consumption and wealth portfolio problem of an infinitely long-lived investor. Investors are assumed to have Epstein-Zin-Weil-type preferences and face asset returns described by a first-order vector autoregression in returns and state variables. The results show that the mean intertemporal hedging demands for stocks are considerably smaller in SA than in the UK or the US, whilst the mean intertemporal hedging demand for bonds are not significantly different from zero in any of the countries considered. Furthermore, it is found that stocks in the US and the UK do not present a useful hedging opportunity for an investor in SA, nor do SA stocks present a useful hedging opportunity for investors from the UK or the US.

**Keywords:** intertemporal hedging demand, multi-period portfolio choice, return predictability, South Africa, portfolio allocation, financial markents.

**JEL Classification:** C32, G11.

## Introduction

During the past two decades, a number of studies have focussed on hedging demand and the role it plays in portfolio allocation, for example Campbell and Viceira (1999, 2001), Campbell, Chan and Viceira (2003), Lynch (2001), Su and Lau (2010), Ang, Papanikolaou, Westerfield (2013). However, the majority of these studies calibrated the models using data from the United States (US) and to date there has been no empirical application that considers the effects of return predictability on hedging demand for an investor in South Africa (SA). As part of BRICS[2], the international political organisa-

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[2] Brazil, Russia, India, China and South Africa.

tion of leading emerging market economies (EMEs), SA is a prominent representative EME. Ideally the analysis could have been done for SA, Brazil, China India and Russia, however these countries were not included in the analysis due to unavailability of data. To the best of our knowledge, this is the only existing study that conducts an analysis of hedging demand in the context of an emerging economy, and, hence, provides an opportunity to investigate whether hedging demand behaviour in an emerging economy differs from that in developed economies. Therefore, in this paper the intertemporal hedging demand for an investor in SA is analysed, in addition to investors in the United Kingdom (UK) and the US. The purpose of this paper is to calculate the implied optimal demand of an infinitely-lived investor for financial assets in SA, including the myopic and intertemporal hedging components for domestic bills, bonds and stocks by using the approach of Campbell *et al.* (2003) (henceforth CCV). Following Rapach and Wohar (2009) (henceforth RW), the optimal asset demands for an investor in SA who, in addition to domestic financial assets, also has access to foreign stocks and bonds (i.e. assets from the UK and the US) is estimated. In a final exercise, the optimal asset allocation for investors in the UK and the US who have access to SA stocks and bonds is calculated. Analysing the hedging demand in such a fashion is important since it provides insight as to how the market could react in time of crises. In light of this, the financial crisis is included in the sampling period.

In all the scenarios considered it is assumed that the return dynamics are well-characterised by a first-order vector autoregression (VAR(1)) process comprising three instruments namely the bill yield, the dividend yield and the term spread. Given the estimates of the dynamic processes governing asset returns and values for the parameters relating to intertemporal preferences, CCV's approximate analytical and numerical procedure is used to solve the multi-period portfolio choice problem of the investor and subsequently estimate the implied mean total demand, mean myopic[3] demand and mean intertemporal hedging[4] demands for domestic bills, stocks and bonds in each country. Confidence intervals are computed using a parametric bootstrap procedure. The monthly historic intertemporal hedging demands for domestic stocks and bonds in each country are also presented. Note that, our study not only presents the analysis from the perspective of an emerging economy, but, given the fact that we include the financial crisis in our sample, the historic intertemporal hedging demand allows us to investigate the changes, if any, in the hedging demand for a typical South African, US and UK investor during the period of the crisis. The extended sample, thus, helps us capture the effect of a major financial market instability on the hedging behaviour of individuals, which, has not been looked into thus far, given that both the CCV and RW samples ended before the crisis period.

[3] Myopic portfolio choice focuses on a single period ahead, hence it is the portfolio that an investor would choose if the investment horizon is only one period only. It basically corresponds to asset demand generated under a static Markowitz (1952) problem.

[4] Rational investors who are risk averse may wish to hedge their exposure to wealth shocks, which leads to intertemporal hedging demands for financial assets. Hedging demand can be either negative or positive, negative hedging demand indicates a short position and positive hedging demand indicates a long position. See section 3.2 for further details.

The remainder of this paper is organised as follows. The literature review is done in Section 1, the empirical approach is discussed in Section 2, whilst Section 3 presents the empirical results. The concluding section also offers possible future avenues for research.

## 1. Literature review on portfolio allocation

According to portfolio theory, the main objective of an investor is to allocate investment between the available assets in an optimal manner. An influential theory dealing with portfolio selection, that became the foundations of modern portfolio theory, was introduced by Markowitz (1952). The mean-variance optimisation of Markowitz (1952) shows that by investing in more than one stock or diversifying a portfolio, the riskiness of a portfolio will decrease if the assets are not strongly positively correlated. Tobin (1958) also analysed portfolio demand in a mean-variance setting and added a risk-free asset[5] to Markowitz (1952)'s analysis. Although the mean-variance analysis of Markowitz (1952) provided a basic theory for portfolio analysis and usefully emphasized that diversification can reduce risk, the model is static and hence unrealistically assumes that investors are only concerned about wealth-risks one period ahead. However, since investors seek to finance lifetime consumption, they are interested beyond the current period.

Merton (1969) and Samuelson (1969) point out that the solution of a static portfolio choice model can differ significantly from the solution of a multi-period portfolio choice problem, this is also more recently shown by Liu (2006). Merton (1973) explains that when investment opportunities fluctuate over time, long-term investors are interested in shocks to investment opportunities (the productivity of wealth) in addition to shocks to wealth. Merton (1969, 1973) then introduced the concept of intertemporal hedging demand for financial assets.

Merton's (1969, 1973) intertemporal model is difficult to solve in closed form and for a number of years solutions could only be obtained when the model was reduced to a static version, hence the applicability of the Merton (1969) model was limited. This, however, changed when advances in computing power and numerical methods made it possible to solve multi-period portfolio choice models numerically by using discrete-state approximations. Interest in multi-period portfolio choice models was further encouraged by empirical evidence that stock and bond returns have important predictable components[6]. Examples of empirical research using numerical methods to solve portfolio choice problems include Brennan *et al*. (1997), Balduzzi and Lynch (1999) and Lynch (2001). In a similar framework, Kim and Omberg (1996) solve the non-myopic

[5] The rate of return on a risk-free asset is called the risk-free rate and is important to most investors since it is often used as a benchmark when measuring the return of other financial assets.

[6] For example, Kandel and Stambaugh (1996) find that the current values of predictive variables can influence the investor's portfolio decision significantly, even when the investor's former beliefs are weighed against predictability. Brennan *et al.* (1997) find that predictability of asset returns is sufficient to yield significant improvement in portfolio returns for strategies that take it into consideration.

portfolio problem analytically in a continuous-time model. A limitation of these models, however, is that for the sake of simplicity, it is unrealistically assumed that no consumption takes place before a terminal date.

Campbell and Viceira (1999) address this limitation by considering a model in which a long-lived investor chooses not only an optimal portfolio, but also consumption, to maximise utility defined over consumption. Since the portfolio choice and intertemporal consumption problem is highly intractable when expected returns are time-varying, the authors use an analytical method to solve the optimal consumption and portfolio choice problem of an infinitely-lived investor. The Euler equation and budget constraint of the exact problem is replaced by approximate equations that are less complicated. An advantage of this approach is that the model can be calibrated using real data and asset returns – specifically, in this case US stock market data was used. Campbell and Viceira (2001) apply the approximation technique of Campbell and Viceira (1999) to develop a model of optimal consumption and portfolio allocation for an infinitely-lived investor with recursive utility, facing stochastic interest rates.

The literature on multi-period portfolio choice models discussed above focus almost exclusively on domestic investments in US assets. RW extends the literature and apply the methodology of CCV to investigate return predictability and the intertemporal hedging demands for stocks and bonds for investors in Australia, Canada, France, Germany, Italy, the US and the UK using a sample period of 1952:04–2004:05[7]. In this paper, following CCV and RW, empirical literature is extended by analysing the domestic portfolio allocation of a SA investor. In addition to the optimal portfolio allocation of an investor who can invest in domestic assets, the allocations for domestic investors who can also invest in foreign assets (from the US and the UK) are also analysed. SA has recently become a member of the BRICS bloc of powerful and influential emerging-market economies and as such SA's financial markets and the hedging demand therein should be explored, researched and compared to other countries.

It is important to note that the asset demand from multi-period portfolio choice problems in this paper, similar to recent empirical literature, is partial in nature and hence the return processes are treated as exogenous. Furthermore, the estimated asset allocation can be interpreted in two ways according to extant literature. Firstly, following Campbell and Viceira (2002), the estimated asset demands can be viewed as normative descriptions of investor behaviour. Hence, for a given return process, the estimated asset demands are those that an investor with an assumed set of preferences is expected to have. Alternatively, following Lynch (2001), estimated asset demands can be interpreted as a positive description of the behaviour of a small group or a unique individual (representative agent), who exploits the return predictability that is created by a large number of investors, created by habit persistence (Campbell, Cochrane 1999) or may be of the type assumed in models of behavioural finance, for example Barberis *et al.* (2000).

[7] Due to data unavailability, the sampling period for some countries differs.

## 2. Empirical approach

Following CCV's and RW's approach, the multi-period portfolio choice problem is discrete in time with the investor assumed to have an infinite horizon and Epstein-Zin-Weil recursive preferences. This approach is different from Kim and Omberg (1996), for example, who utilise a finite-horizon model with power utility defined over terminal wealth. Furthermore, the CCV approach does not impose borrowing or short-term sales nor does it make provision to include transaction costs.

### 2.1. Securities

Suppose that an investor can invest in $n$ risky assets, using after-consumption wealth[8]. Let $R_{1,t+1}$ be the real return on a benchmark asset. Then, the real return on the investor's portfolio is given by:

$$R_{p,t+1} = \sum_{i=2}^{n} \propto_{i,t} \left( R_{i,t+1} - R_{1,t+1} \right) + R_{1,t+1}, \tag{1}$$

where $\propto_{i,t}$ is the portfolio weight on risky asset $i$ at time $t$. The vector of log excess returns, $x_{t+1}$, can be defined as:

$$x_{t+1} = \left[ r_{2,t+1} - r_{1,t+1}, \ldots, r_{n,t+1} - r_{1,t+1} \right]', \tag{2}$$

where $r_{1,t+1} = \log(R_{1,t+1})$ for all $i$ with n=3. In the empirical application in this paper, $r_{2,t+1}$ and $r_{3,t+1}$ are the logs of the total return index for stocks and bonds respectively. The benchmark asset, $r_{1,t+1}$, is the difference in the logs of the total return index for bills for the current and previous month. Following RW and CCV, it is assumed that the benchmark asset is not riskless since the real return on bills is subject to inflation risk.

The state variables further include a vector of instruments, $s_{t+1}$. In the empirical application of this paper the instruments comprise the nominal bill yield, the yield spread and the log of dividend yield. Details on the calculation of these variables can be found in Section 4.

By stacking the log benchmark asset, the vector of log excess returns and the vector of state instruments, $r_{1,t+1}$, $x_{t+1}$ and $s_{t+1}$, into an ($m$×1) vector, the vector of state variables, $z_{t+1}$, is obtained:

$$z_{t+1} = \begin{bmatrix} r_{1,t+1} \\ x_{t+1} \\ s_{t+1} \end{bmatrix}. \tag{3}$$

CCV hypothesise that a first-order VAR process[9] captures the dynamics of the relevant state variables well. Thus it is assumed that the data generating process is given by the first-order vector autoregressive system:

[8] RW's notation is adopted.

[9] Campbell (1991), Balduzzi and Lynch (1999), Campbell and Viceira (1999), Lynch (2001) and Campbell *et al.* (2004), amongst others, have also used this type of dynamic specification. The Schwarz information criterion, however, also confirmed one as the optimal lag length. This result is available upon request from the authors.

$$z_{t+1} = \Phi_0 + z_t\Phi_1 + v_{t+1}, \tag{4}$$

where $\Phi_0$ is an $(m\times 1)$ vector of intercepts and $\Phi_1$ is an $(m\times m)$ matrix of slope coefficients. Furthermore, $v_{t+1}$ is an $m$-vector of VAR innovations (shocks to the state variables) that are independently and identically distributed, and follow a normal distribution with zero mean and variance $\sum_v$. This variance-covariance matrix can be represented as:

$$\sum_v \equiv Var_t\left(v_{t+1}\right) = \begin{bmatrix} \sigma_1^2 & \sigma'_{1x} & \sigma'_{1s} \\ \sigma_{1x} & \Sigma_{xx} & \Sigma'_{xs} \\ \sigma_{1s} & \Sigma_{xs} & \Sigma_{ss} \end{bmatrix}. \tag{5}$$

In the first column of this matrix, the variance of the innovation to the benchmark asset return is given by $\sigma_1^2$, whilst the covariances between innovations to the benchmark asset return and innovations to the excess returns is found in the $(n - 1)$ vector $\sigma_{1x}$. Similarly, $\sigma_{1s}$ is an $(m-n)$ vector of covariances between innovation to the benchmark asset return and innovations to the instruments. In the second column, $\Sigma_{xx}$ represents the $(n - 1)\times(n - 1)$ variance-covariance matrix for innovations to the excess returns, whilst $\Sigma_{xs}$ is the $(m - n)\times(n - 1)$ matrix of covariances between innovations to excess returns and innovations to the instruments. In the final column, $\Sigma_{ss}$ is the $(m - n)\times(m - n)$ variance-covariance matrix for innovations to the instruments. The innovations are thus allowed to be cross-sectionally correlated, but it is assumed that they are homoskedastic[10] and independently distributed over time, similar to Kandel and Stambaugh (1996). The homoskedastic assumption facilitates the derivation of the unconditional distribution of $z_t$ and the linearity of the VAR system implies that $z_t$ inherits the normality of the shocks $v_{t+1}$.

## 2.2. Preferences and optimality conditions

The investor is assumed to have Epstein-Zin-Weil recursive preferences, maximised over an infinite horizon. Following Epstein-Zin-Weil (1989, 1991), let $\gamma > 0$ and $\psi > 0$ represent the coefficient of relative risk aversion (CRRA) and elasticity of intertemporal substitution (EIS) respectively and let $\theta \equiv \frac{(1-\gamma)}{(1-\psi^{-1})}$. The recursive preferences characterised by this utility is then given by:

$$U\left(C_t, E_t\left(U_{t+1}\right)\right) = \left[\left(1-\delta\right)C_t^{\frac{(1-\gamma)}{\theta}} + \delta\left(E_t[U_{t+1}^{1-\gamma}]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{(1-\gamma)}}, \tag{6}$$

where $C_t$ is the consumption at time $t$, $E_t\left(\cdot\right)$ is the conditional expectation operator and $0 < \delta < 1$ is the time discount factor. An advantage of using this utility function is that the notion of risk aversion is separated from that of the elasticity of intertemporal

[10] CCV point out that this assumption is restrictive since state variables can only affect portfolio choice by predicting changes in expected returns – state variables cannot predict changes in risk. However, Harvey (1991), for example, have found that state variables have only a limited ability to predict risk, relative to the effects state variables have on expected returns.

substitution. Thus $\gamma$ and $\psi$ are conceptually distinct notions relating to intertemporal preferences.

The investor selects $C_t$ and portfolio weights $[\propto_{2,t},\ldots,\propto_{n,t}]'$ at each time $t$, using all available information in order to maximise the utility (thus make optimal consumption and portfolio choices) subject to the intertemporal budget constraint:

$$W_{t+1} = (W_t - C_t)R_{p,t+1}, \tag{7}$$

where $W_t$ is wealth at time $t$ and $p$ is a specific portfolio selection. Given this budget constraint, the Euler equation for consumption for any asset $i$ that an investor's optimal consumption and portfolio policies have to satisfy, as derived by Epstein and Zin (1989, 1991), is:

$$E_t\left\{\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{\frac{-1}{\psi}}\right]^{\theta} R_{p,t=1}^{-(1-\theta)} R_{i,t+i}\right\} = 1. \tag{8}$$

This first-order condition can be reduced to the standard first-order condition in the power utility case where $\gamma = \frac{1}{\psi}$ and $\theta = 1$. When investment opportunities are constant, optimal policies imply a myopic rule. CCV show that the solution is myopic when $\psi$ = $\gamma$ = 1, implying log utility. However, exact analytical solutions for this problem are generally not available with time-varying investment opportunities except for specific values of $\gamma$ and $\psi$. Therefore, CCV combine a relatively simple numerical procedure with an extension of Campbell and Viceira (1999, 2001) in a multivariate framework to solve the optimal rules for all values of $\gamma$ and $\psi$.

### 2.3. Solution methodology

CCV use an approximation of the log real return on the portfolio in equation (1) which, in continuous time, holds exactly and is highly accurate over short time intervals. Hence, using monthly data should help to ensure the accuracy of the approximation in this paper's empirical application. The non-linear budget constraint in equation (7) is log-linearised to obtain an approximation of the budget constraint. Furthermore, a second-order Taylor expansion is applied to the Euler equation (equation (8)) to obtain the log-linearised Euler equation. Both these approximations are exact when $\psi$ = 1, hence the solution to the approximate model is suitable when $\psi$ is close to unity.

### 2.4. Solving approximate model

In order to solve the model, CCV presume that the optimal portfolio and consumption rules are linear and quadratic respectively in $z_t$, and take the following forms:

$$\alpha_t = A_0 + A_1 z_t, \tag{9}$$

$$c_t - w_t = b_0 + B_1' z_t + z_t' B_2 z_t, \tag{10}$$

where $c_t$ and $w_t$ are the log levels of $C_t$ and $W_t$ respectively. In equations (9) and (10),

$A_0$ is of dimension $(n - 1) \times 1$ and $A_1$ has dimension $(n - 1) \times (m)$, $b_0$ has dimension $(1 \times 1)$, $B_1$ is $(m \times 1)$ and $B_2$ is $(m \times m)$. These are coefficient matrices that have to be determined and due to the infinite-horizon assumption, are constant through time. This assumption implies that the problem does not have to be solved backward recursively starting from the last date. The focus of this paper is mainly on equation (9) which shows the investor's optimal asset allocations. An iterative numerical procedure is used to compute estimates of the constant coefficient matrices of $A_0, A_1, b_0$, $B_1$ and $B_2$, assuming that the coefficients of the optimal portfolio rule for each value of $\gamma$ are independent of $\psi$ given $\rho$, where $\rho \equiv 1 - \exp\left[E\left(c_t - w_t\right)\right]$. For every $\gamma$, a value is fixed for $\rho$ and an arbitrary value for $\psi$ is chosen. Furthermore $\delta$ is set to $(0.92)^{1/12}$ on a monthly basis whilst considering different values of $\gamma$ and $\psi$. For further details, see CCV.

## 2.5. Optimal portfolio choice

In order to divide the total demand into its myopic and hedging components, following Merton (1969, 1973) CCV derive equations (11) and (12):

$$A_0 = \left(\frac{1}{\gamma}\right)\sum_{xx}^{-1}[H_x\Phi_0 + \frac{1}{2}\sigma_x^2 + \left(1-\gamma\right)\sigma_{1x}\Bigg] + \left(1-\frac{1}{\gamma}\right)\sum_{xx}^{-1}\left[\frac{-\Lambda_0}{1-\psi}\right], \quad (11)$$

$$A_1 = \frac{1}{\gamma}\sum_{xx}^{-1}H_x\Phi_1 + \left(1-\frac{1}{\gamma}\right)\sum_{xx}^{-1}[\frac{-\Lambda_1}{1-\psi}], \quad (12)$$

where $H_x$ is a selection matrix that selects the vector of excess returns, $x_t$, from the full state vector, $z_t$, and $\sigma_x^2$ is the vector consisting of the diagonal elements in $\Sigma_{xx}$, the variances of excess returns. Furthermore, $\Lambda_0$ and $\Lambda_1$ are coefficient matrices that depend on $b_0$, $B_1$, $B_2$, $\gamma$, $\psi$, $\delta$, $\rho$, $\Phi_0$, $\Phi_1$, and $\Sigma_v$. The first term on the right-hand side of equations (11) and (12) represents the myopic part of asset demand, thus the part that only focuses on the single-period-ahead. Following Merton (1969, 1973), a rational investor, who is more risk-averse than a logarithmic investor, will hedge against unfavourable changes in investment opportunities. The effect of intertemporal hedging demand on optimal portfolio choice, which arises in a multi-period problem, is reflected by the second term on the right-hand side of both equations (11) and (12). Therefore, intertemporal hedging considerations influence both the mean optimal portfolio allocation to risky assets (through $A_0$ and $A_1$) and the sensitivity of the optimal allocation to changes in the state variables (through $A_1$). It can be seen that the intertemporal hedging demand is zero when $\gamma = 1$; when an investor is not sufficiently risk averse to have an intertemporal hedging demand. Furthermore, if there is no return predictability or investment opportunities are constant over time the second term on the right-hand side of equations (11) and (12) also becomes zero.

CCV show that the optimal portfolio rule is independent of $\psi$ given $\rho$. Thus $\psi$ only affects portfolio choice to the extent that it enters into the determination of $\rho$. This result forms an important part of the numerical procedure. Furthermore, RW's approach is followed and 90 per cent confidence intervals are constructed for the mean demands using a parametric bootstrap procedure in order to get a sense of uncertainty associated with the point estimates of the total, myopic and mean hedging demands for each asset in the respective countries[11]. See RW for further details.

## 3. Empirical application for investors in SA, the US and the UK

The CCV procedure is used to estimate equations (9) and (10) for an infinitely-lived investor in SA, the UK and the US who can invest in 3-month Treasury Bills, a domestic stock index and domestic 10-year government bonds. Investment opportunities in each country respectively are described by a VAR(1) system that includes the real short-term interest rate, excess stock returns and excess bond returns. Other variables that have been identified as return predictors identified in empirical research, namely short-term nominal interest rates, the dividend yield and the difference between the yields of long-term bonds and Treasury bills[12] are also used.

Optimal portfolio rules are calculated for three different values of $\gamma$ (4, 7 and 10) following RW[13], assuming that $\psi = 1$, and that the time discount factor ($\delta$) is equal to 0.92 at an annual frequency. The optimal portfolios are then also calculated for different values of $\psi$. The parameters for equation (4) is estimated using maximum-likelihood, given $\widehat{\Phi_0}$ , $\widehat{\Phi_1}$ and $\widehat{\sum_v}$. Equations (11) and (12) are used to estimate mean myopic and hedging demand for each asset for each value of $\gamma$. Since the main focus of this paper is intertemporal hedging demand, figures containing the monthly hedging demand for domestic stocks, bonds and cash (bills) for the sample period are also presented.

### 3.1. Data

Monthly data for SA, the UK and the US were obtained from the Global Financial Database. The sample starts in 1960:02 for each country and ends in 2010:09. The analysis is repeated using a second sampling period (1960:02–2004:05) in order to facilitate the comparison of results to RW's results[14]. This is also assumed to be a good representa-

[11] Note that in CCV, the VAR is estimated imposing the restriction that the unconditional means of the variables implied by the VAR coefficient estimates equal their full-sample arithmetic counterparts. Standard, unconstrained least-squares fits exactly the mean of the variables in the VAR excluding the first observation. CCV use constrained least-squares to ensure that they fit the full-sample means. In our case, as in RW, we use constrained maximum-likelihood estimates to ensure that we fit the full-sample means. Unlike CCV, to get a sense of the sampling uncertainty associated with the point estimates of the mean total, myopic, and hedging demands for each asset in each country, we construct 90% confidence intervals for the mean demands using a parametric bootstrap procedure, and since, maximum-likelihood estimates are more commonly used when conducting bootstraps, we follow RW, in using constrained maximum-likelihood estimates.

[12] Empirical studies using the dividend yield to predict asset returns include Balduzzi and Lynch (1999), Barberis (2000), Brandt (1999), Brennan *et al.* (1997) and Campbell and Viceira (1999). Empirical studies using the term spread include Brandt (1999), amongst others, whilst Brennan *et al.* (1997) utilise the Treasury bill yield.

[13] CCV use $\gamma = 1, 2, 5$ and 20.

[14] The results for the second sampling period are not included in this paper in order to conserve space, but are available upon request.

tion of portfolio allocation of investors in SA, the UK and the US prior to the financial crisis. Following RW, six state variables are calculated for each respective country[15]. The log real return on the 3-month Treasury bill ($rtbr_t$) is treated as the benchmark asset and is measured as the difference in the logs of the total return index for bills for the current and previous month, minus the difference in the logs of the consumer price index for the given and previous month. The two excess real returns are therefore the excess returns on the stock market index[16] and the excess returns on a 10-year government bond. The log excess stock return ($xsr_t$) is measured as the difference in the logs of the total return index for stocks for the current and previous month, minus the difference in the logs of the total return index for bills for the current and previous month. The excess bond return ($xbr_t$) is measured similarly, using the total return index on 10-year government bonds instead of the total return index for stocks. Regarding the instruments (return predictors), the difference between the yield on a 3-month Treasury bill and the 12-month backward looking moving average (following Campbell (1991) and Hodrick (1992)) is used as the nominal bill yield ($bill_t$), whilst the log of the dividend yield is the second instrument ($div_t$). The third instrument, the term spread ($spread_t$), is measured as the yield on a 10-year government bond minus the yield on a 3-month Treasury bill.

The summary statistics of the state variables (three risky assets as well as the three instruments) for the three respective countries are reported in Table 1, which contains the mean, standard deviation and first-order autocorrelation coefficient. The mean and standard deviation for the three risky assets are expressed in annualised percentage units and the Sharpe ratio, measured as the ratio of the annualised mean to the annualised standard deviation for excess stock and bond returns, is also included. A higher Sharpe ratio indicates either a higher mean (return) or a lower standard deviation, hence it is expected that assets with a higher Sharpe ratio will be in higher demand, all else equal. This higher demand is be related to myopic demand.

It can be seen in Table 1 that the mean excess stock return is the highest in SA at over 6 per cent, followed by 3.9 per cent in the US and 3.5 per cent in the UK[17]. However the standard deviation is also the highest in SA and despite this, the Sharpe ratio for the excess stock return is the highest in SA. The standard deviations for mean excess stock returns are the lowest in the US.

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[15] See Appendix for a graphical representation of state variables.

[16] The stock return indices are the Johannesburg Stock Exchange return index, the UK FTSE All-share return index and the S&P 500 total return index for SA, the UK and the US respectively.

[17] When comparing the analysis for the full sample period with that of the sub-sample (that excludes the financial crisis) it is found that the Sharpe ratio for excess stock returns in SA and the UK are lower when sample does not include the financial crisis, whilst the ratio is higher in the US. Judging purely by the Sharpe ratios, it is thus expected that the myopic demand for stocks in the US is relatively higher prior to the financial crisis and the myopic demand for stocks in SA is higher in the sample period that includes the financial crisis. In the sub-sample the mean excess bond returns are lower in all the countries considered, whilst the standard deviations remain largely unchanged, resulting in lower Sharpe ratios.

**Table 1.** Summary statistics for the full sampling period

| 1960:02–2010:09 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | SA | | | | UK | | | | US | | | |
| | Mean | Std Dev | Sharpe | $\rho_1$ | Mean | Std Dev | Sharpe | $\rho_1$ | Mean | Std Dev | Sharpe | $\rho_1$ |
| $rtbr_t$ | 1.03 | 1.96 | | 0.47 | 1.68 | 2.00 | | 0.33 | 1.27 | 1.11 | | 0.46 |
| $xsr_t$ | 6.24 | 21.88 | 0.29 | 0.09 | 3.51 | 18.75 | 0.19 | 0.11 | 3.86 | 15.13 | 0.25 | 0.06 |
| $xbr_t$ | 0.93 | 7.37 | 0.13 | 0.29 | 1.25 | 5.34 | 0.23 | 0.27 | 1.89 | 7.89 | 0.24 | 0.10 |
| $bill_t$ | 0.03 | 1.48 | | 0.94 | (0.03) | 1.22 | | 0.90 | (0.03) | 0.94 | | 0.88 |
| $div_t$ | 1.25 | 0.32 | | 0.97 | 1.43 | 0.28 | | 0.98 | 1.06 | 0.40 | | 0.99 |
| $spread_t$ | 1.69 | 2.35 | | 0.98 | 0.82 | 1.60 | | 0.95 | 1.49 | 1.24 | | 0.94 |

**Notes*:*** $rtbr_t$ = log real 3-month Treasury bill return; $xsr_t$ = log excess stock return; $xbr_t$ = log excess bond return, $bill_t$ = 3-month Treasury bill yield; $div_t$ = log dividend yield; $spread_t$ = difference between a 10-year government bond yield and a 3-month Treasury bill yield. The first-order autocorrelation coefficient is given by $\rho_1$.

The mean excess bond returns are much lower than the mean excess stock returns in the three countries, ranging from 0.93 per cent in SA to 1.89 per cent in the US. The standard deviations are also much lower for excess bond return than for excess stock returns, in all three countries. The relatively lower standard deviations for mean excess bond returns in comparison to mean excess stock returns were also found by RW and CCV. The Sharpe ratio for excess bond returns is the highest in US, followed by the UK and then SA. The Sharpe ratio for excess stock returns is higher than the Sharpe ratio for excess bond returns in SA and in the US, however the opposite is observed in the UK.

The first-order correlation coefficients for instruments are much higher than for the asset returns, indicating relatively higher persistence. The first-order correlation coefficients of the mean excess stock returns are lower than for the mean excess bond returns in all three countries and range from 0.06 (in the US) to 0.11 (in the UK), indicating fairly limited persistence. The first-order correlation coefficients of excess bond returns are higher than for excess stock returns in all countries, and range from 0.10 in the US to 0.29 in SA. The real bill return shows the highest persistence of the three risky assets, with first-order correlation coefficients ranging from 0.33 in the UK to 0.47 in SA.

A VAR for two sampling periods is estimated for SA, the UK and the US respectively in an attempt to capture periods that include and exclude respectively, the recent financial crisis[18]. Note, we choose to end in 2004:05 for the pre-crisis sample to coincide with the sample of RW. However, to save space, we report the results from the pre-crisis sample in footnotes at relevant parts of the paper, and draw parallels to the results from the larger sample.

[18] The VAR results and correlation coefficients for SA can be found in the Tables 7–9. The results for the UK and the US are available upon request.

### 3.2. Strategic domestic asset allocations of investors in SA, the UK and the US

The optimal portfolio allocation to stocks, bonds and bills changes over time since the optimal portfolio rule is linear in the vector of state variables. Therefore the mean allocation to each asset and the mean hedging portfolio demand is used to analyse the level effects.

Tables 2 reports the mean total demand, mean myopic demand and mean intertemporal hedging demand in percentages for domestic stocks, bonds and bills. Three different values of $\gamma$ (4, 7 and 10) are considered while $\psi$ is kept constant and equal to one. The total mean demand across the three asset classes adds up to 100 (per cent) and the mean hedging demand add up to zero. Also, within a certain asset class, the myopic demand and the hedging demand add up to the total demand for that specific asset. The 90% confidence intervals for the mean asset demands that were generated following RW's bootstrap procedure are also included in Table 2. Note that the 90% confidence intervals are relatively wide for total demand, myopic demand and hedging demand for all three asset classes. This suggests that merely reporting point estimates hides sampling uncertainty in empirical multi-period portfolio choice problems. By comparing numbers for a specific country within any column, the effect of increasing risk aversion (higher $\gamma$) on total asset allocation and intertemporal hedging demand can be observed. Recall that in the case where $\gamma = 1$, i.e., the logarithmic investor, the optimal portfolio rule is purely myopic.

In all three countries the mean total demand for stocks declines as risk aversion ($\gamma$) increases. This is in line with expectations since stocks would be considered to be the riskiest investment choice in this portfolio. It can be seen that the mean hedging demand for stocks also decreases as $\gamma$ increases in each country respectively. Campbell and Viceira (1999) explain that the hedging demand is not monotonic in risk aversion, because an extremely risk-averse investor will limit their exposure to the risky asset. Therefore the magnitude of hedging demand first increases and then falls as $\gamma$ increases. Accordingly, CCV found that intertemporal hedging demand for stocks eventually becomes negative when the investor becomes extremely risk averse, and this would contribute to the lower mean total demand for stocks. Similarly, the mean total demand for bonds decreases as $\gamma$ increases. In all the countries, the mean hedging demands for bonds are negative, irrespective of $\gamma$ and this contributes to the lower mean total demands for bonds in comparison to the mean total demand for stocks. In contrast, the mean total demand for bills increases as $\gamma$ increases. This is also in line with expectations since bills would be the asset with the lowest risk in this portfolio.

Hence, in general, investors who are more risk averse (higher values of $\gamma$), have lower total demand for stocks and bonds and higher total demand for bills. When risk aversion is low, the total demand for stocks and bonds is high and in the US and the UK it can be seen that these values added together exceeds 100. Thus the investor goes short in bills (negative value for total demand for bills). In contrast, an investor in SA does not go short in bills at any level of risk aversion. However, as risk aversion increases, the total demand for stocks and bonds decrease, and the investor in the UK or the US does not go short in bills to the same extent (thus smaller negative values for total demand for bills).

**Table 2.** Mean demands and 90% confidence intervals for domestic assets for an investor in SA, the UK and US respectively (1960:02–2010:09)

| | | Stocks | | | Bonds | | | Bills | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand |
| SA | $\gamma = 4$ | **51.69** | **44.34** | 7.35 | 19.42 | 31.54 | –12.12 | 28.89 | 24.11 | 4.77 |
| | | [20, 85] | [19, 68] | [–4, 21] | [–80, 136] | [–113, 140] | [–32, 24] | [–91, 136] | [–101, 148] | [–40, 31] |
| | $\gamma = 7$ | **27.81** | **25.51** | 2.3 | 5.02 | 16.87 | –11.85 | 67.17 | 57.62 | 9.56 |
| | | [9, 49] | [11, 39] | [–5, 13] | [–51, 72] | [–50, 95] | [–26, 12] | [–12, 121] | [–14, 128] | [–20, 28] |
| | $\gamma = 10$ | **17.76** | **17.98** | –0.22 | –0.7 | 11.01 | –11.71 | **82.95** | **71.02** | 11.93 |
| | | [3, 33] | [8, 27] | [–6, 10] | [–43, 45] | [–37, 65] | [–24, 6] | [31, 124] | [21, 121] | [–10, 28] |
| UK | $\gamma = 4$ | **80.2** | **32.65** | **47.55** | 102.15 | 108.56 | –6.41 | –82.35 | –41.21 | –41.14 |
| | | [32, 124] | [14, 54] | [16, 82] | [–41, 235] | [–47, 266] | [–35, 27] | [–235, 58] | [–188, 111] | [–95, 4] |
| | $\gamma = 7$ | **52.53** | **19.16** | **33.37** | 58.41 | 62.19 | –3.78 | –10.94 | 18.65 | –29.59 |
| | | [17, 86] | [9, 31] | [7, 63] | [–19, 136] | [–28, 152] | [–24, 20] | [–104, 71] | [–64, 107] | [–70, 11] |
| | $\gamma = 10$ | 37.46 | 13.76 | 23.7 | 40.4 | 43.64 | –3.24 | 22.14 | 42.59 | –20.46 |
| | | [9, 65] | [7, 22] | [–1, 46] | [–20, 88] | [–20, 106] | [–18, 16] | [–48, 80] | [–15, 105] | [–60, 7] |
| US | $\gamma = 4$ | **98.31** | **50.2** | **48.11** | 66.55 | 81.79 | –15.24 | –64.86 | –31.99 | –32.87 |
| | | [42, 161] | [19, 62] | [19, 109] | [–48, 214] | [–53, 198] | [–28, 16] | [–220, 66] | [–142, 101] | [–115, –5] |
| | $\gamma = 7$ | **67.3** | **28.51** | **38.79** | 34.77 | 46.4 | –11.63 | –2.07 | 25.09 | –27.16 |
| | | [32, 127] | [10, 35] | [12, 93] | [–32, 121] | [–31, 113] | [–20, 12] | [–101, 78] | [–38, 101] | [–97, –8] |
| | $\gamma = 10$ | **52.26** | **19.83** | **32.43** | 22.57 | 32.24 | –9.67 | 25.16 | 47.92 | –22.76 |
| | | [21, 101] | [7, 24] | [12, 84] | [–25, 82] | [–22, 79] | [–15, 9] | [–57, 81] | [4, 101] | [–84, –6] |

**Notes:** The mean total, myopic and hedging demands are reported in percentages in this table for domestic stocks, 10-year government bonds, and 3-month Treasury bills for an investor with an elasticity of substitution ($\psi$) equal to one, the discount factor ($\delta$) equal to $0.92^{1/12}$ on a monthly basis. Three different values for the coefficient of risk aversion is applied ($\gamma = 4, 7, 10$) and the bootstrapped confidence intervals (following RW) for the 90% confidence intervals of the mean asset demands are given in brackets. Significance according to these confidence intervals is indicated by a bold entry.

If risk aversion increases further, the investor eventually becomes long in stocks, bonds and bills (see Table 2 in the case of the US and the UK when $\gamma = 10$). It can also be noted that at high levels of risk aversion ($\gamma = 10$), an investor's portfolio in the US and the UK still has positive total demand for stocks, bonds and bills.

In SA, even though the mean total demand for domestic stocks is sizable, the mean hedging demands for stocks are relatively small. The null hypothesis that the mean and total hedging demand for stocks and bonds are zero cannot be rejected for any level of risk aversion at a 10% level of significance. The mean total demand for bonds is significantly smaller than the mean demand for stocks in SA, and is close to zero. The mean hedging demands for bonds in SA are negative and the 90% confidence intervals for the mean hedging demand for bonds in SA do not result in the rejection of the null hypothesis of zero mean hedging demand for bonds. There is a preference for stocks in the optimal portfolio allocation, which can be attributed to the relatively strong estimated positive correlation between unexpected excess returns on stocks and bonds, which shifts optimal myopic allocation towards the asset with the higher Sharpe ratio, namely stocks in the case of SA.

In the UK, the mean total and hedging demands for domestic stocks are larger than in the case of SA. Despite wide confidence intervals, the intervals are tight enough to conclude that for lower levels of risk aversion the mean hedging demand for stocks in the UK is sizable and significantly different from zero. The mean hedging demand for bonds are, similar to SA, negative and small in magnitude and the null hypothesis of zero mean hedging demand for bonds cannot be rejected according to the 90% confidence intervals.

With regard to the US, for each reported $\gamma$ value large positive mean total and mean hedging demand for stocks can be observed in Table 2. Similar to the results of RW, the null hypothesis of a zero mean for hedging demand for stocks can be rejected according to the 90% confidence intervals. Regarding the hedging demand for bonds, however, in line with results from SA and the UK, the null hypothesis cannot be rejected and hence the hedging demand for bonds is not significantly different from zero. In the US, similar to the UK, there is a preference for bonds in the optimal myopic allocation because of the estimated large positive correlation between unexpected excess returns on stocks and bonds in both countries since the Sharpe ratio for bonds is larger than for stocks in the US and the UK.

A prominent result in Table 2 is that the mean total and hedging demand for stocks is significantly lower in SA than in the US and the UK. Following CCV and RW, two factors could contribute to this. First, the positive coefficient for the lagged dividend yield in the expected excess stock return equations of the VARs. Second, a strong negative correlation between innovations to excess stock returns and dividends. In order to understand how these factors contribute to hedging demand for stocks, consider the following: relatively high Sharpe values for stocks for all three countries, as seen in Table 1, imply that investors are usually long in the stock market. Hence a negative shock in expected stock returns corresponds to a deterioration in the investor's opportunity set. The positive coefficient for the lagged dividend yield in the expected excess stock return equations in the VAR models for the 3 countries considered implies that expected stock returns in the next period increase when the dividend yield increases in the current period. Furthermore, according to the strong negative correlation between innovations to excess stock returns and dividends, a negative innovation to stock returns in the next period would be accompanied by a positive innovation to the dividend yield in the next period. Then, following from the positive coefficient on the lagged dividend yield in the excess stock return equation of the VAR, the higher dividend yield in the next period would lead to higher expected stock returns in the period after that (two periods from now). Therefore, poor stock returns are correlated with an improvement in future investment opportunities, and thus stocks hedge exposure to future unfavourable return shocks. Hence, stocks can be described as a good hedge against themselves, and this increases the demand of conservative investors.

In accordance with this argument, hedging demand for stocks is the highest in the US, since the negative correlation between innovations to excess stock returns and dividends was the largest in absolute value (–0.97) even though the coefficient for lagged dividend is not the largest (0.006). The hedging demand for stocks in the UK is also large, but since the negative correlation between innovations to excess stock returns and dividends is slightly weaker than in the US (–0.77), the hedging demand for stocks is lower in the UK than in the US. Therefore, in the UK and particularly in the US, domestic stocks provide attractive intertemporal hedging instruments for domestic investors[19]. There is a weaker negative correlation between innovations to excess stock returns and dividends in SA (–0.43), hence the relatively lower hedging demand for stocks in SA is attributable to a relatively smaller correlation coefficient in absolute value between innovations to excess stock returns and dividend yield. The low hedging demand for stocks in SA contributes to the lower mean total demand for stocks. The decreases in hedging demand for stocks, as risk aversion increases, are more prominent in the UK and the US. Following RW, this can be explained by the idea that stocks are the riskiest asset with the potential to be a good hedge against themselves: when risk aversion increases, demand for risky assets (stocks) will decrease and the investor will require fewer stocks as a hedge against adverse stock returns. However, in accordance with a high Sharpe ratio, mean myopic demand for stocks is sizable in SA.

Another observation mentioned above is the negative intertemporal hedging demand for bonds in all countries. The intertemporal hedging demand for bonds becomes larger (less negative) as risk aversion increases. This is in line with CCV, who found that the intertemporal hedging demand for bonds is negative at intermediate levels of risk aversion, but turns positive for extremely risk averse investors, thus when $1/\gamma$ (risk tolerance) approaches zero. This negative mean hedging effect is large enough that for investors with intermediate risk aversion, the total mean demand for bonds becomes negative. This can be seen in Table 2 where in SA the total mean demand for bonds in SA becomes negative when $\gamma = 10$. A possible reason for this result is related to the

[19] These results are in line with the results that RW found for the US and the UK, and also compare favourably with Campbell and Viceira (2000).

positive correlation between excess stock and bond returns that were observed for all countries.

Although there are similarities between the results presented in Table 2 and the results of RW, there are also notable differences. One of the most pertinent differences is related to the magnitude of the mean total and intertemporal hedging demands for stocks in the US and the UK – which RW found to be larger. Historical hedging demand is shown in Figure 1 and used to investigate the reason for these differences in the mean hedging demand and also to determine what effects the financial crisis had on the hedging demand for stocks and bonds. Knowing that the financial crisis had devastating effects on global equity markets[20], it is to be expected that the hedging demand for assets would also be affected.

In Figure 1 it can be seen that the hedging demand for bonds in SA was positive in the years prior to the financial crisis and after a period of volatility fell to below zero. From 2009 onwards it can also be observed that the hedging demand for stocks and bonds are closer to zero than before the financial crisis, however it is clear that the financial crisis had limited effects on the mean hedging demand for domestic stocks and bonds of an SA investor[21]. Furthermore, between 1960 and 2010, the hedging demand in SA for stocks is less volatile than for bonds, and fluctuates around zero.

By examining the hedging demand for stocks over time in the UK, it can be seen in Figure 1 that the hedging demand for stocks is typically above hedging demand for bonds, however between the late 1990s and 2002 the hedging demand for bonds moves above the hedging demand for stocks. Between the early 2000s and 2010, the difference between hedging demand for stocks and hedging demand for bonds is smaller in magnitude. It can be observed, that during the financial crisis hedging demand for bonds fell to one of its lowest levels and whilst at the same time the hedging demand for stocks peaked.

The hedging demand for bonds in the US is relatively less volatile than in the UK or in SA. Furthermore, the hedging demand for stock is generally higher than the hedging demand for bonds. Similar to the UK, however, the hedging demand for bonds move above the hedging demand for stock roughly between 1998 and 2002 and falls below the hedging demand for stocks again roughly until 2005. The hedging demand for stocks and bonds are then both positive and roughly at same levels until 2007, after which the hedging demand for stocks once again moves above the hedging demand for bonds. The effect of the financial crisis is more prominent in the US than in the UK or SA, where towards the end of 2008, hedging demand for stocks increases significantly and then decreases, settling at a higher level than previously. Hedging demand for bonds follows roughly the opposite path. If the mean hedging demand for stocks in the US in the two sampling periods is compared it is seen that similar to the UK, the mean hedging demand for stocks decrease when the financial crisis is included in the sample.

[20] After reaching US$62.57 trillion in October 2007, world market capitalisation fell to US$25.50 trillion in March 2009 (Moody, Lynn & Lieberson, Inc., 2009).

[21] When comparing the results from Table 2 to the results from the sub-sample, it is found that the mean hedging demand for SA did not change significantly between the two sampling periods.

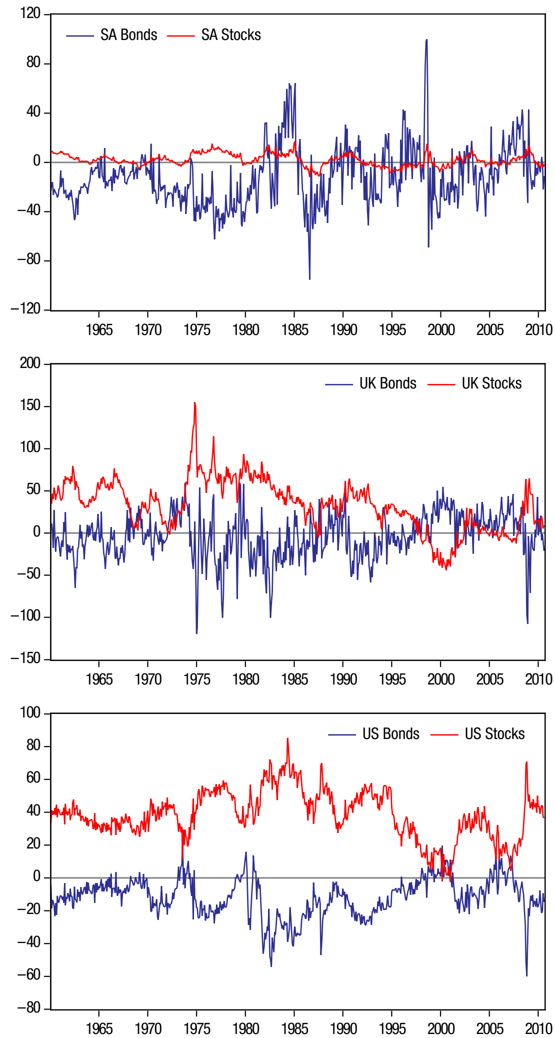


**Fig. 1.** Historical intertemporal hedging demands for domestic stocks and bonds in SA, the UK and the US when $\gamma = 7$ and $\psi = 1$

The movements in the hedging demands for stocks and bonds around the time that the financial crisis started can be explained by the return predictability and its effect on intertemporal hedging demand. Since domestic UK and US stocks were found to be attractive intertemporal hedging instruments for domestic investors, stocks can be described as a good hedge against themselves in that they hedge exposure to future adverse return shocks. Therefore, when the financial crisis started and stock markets collapsed, hedging demand for stocks increased. Regarding the decrease in hedging demand for domestic UK and US bonds, as discussed above a possible reason for this result, following CCV, can be related to the positive correlation between excess stock and bond returns that were observed for all countries. It can be argued that when the financial crisis started and hedging demand for stocks increased, the short-term risk related to this positive intertemporal hedging demand for stocks was offset by taking short(er) positions in long-term bonds.

In summary, the hedging demand for stocks is generally less volatile than the hedging demand for bonds. In the case of the US and the UK, the hedging demand for stocks is mostly larger than the hedging demand for bonds, however, in SA the hedging demand for stocks remains close to zero whilst the hedging demand for bonds is very volatile. When the financial crisis started, changes in hedging demand was the most pronounced in the US. In general, it can be observed that the hedging demand for stocks increased in all three countries at the start of the financial crisis and then decreased, whilst the hedging demand for bonds fell and later increased. These effects are the least pronounced in SA.

Bhamra and Uppal (2006) find that both risk aversion and the elasticity of intertemporal substitution influence the consumption and portfolio decisions in general. Therefore, the mean demand for assets is also computed for different values of the elasticity of intertemporal substitution ($\psi$), whilst holding the coefficient of relative risk aversion constant and equal to 7, in line with other empirical research[22]. The results are given in Table 3[23].

**Table 3.** Mean demands for domestic assets for investors in different countries assuming different $\psi$ values with $\gamma$ = 7, 1960:02–2010:09

| | Stocks | | | Bonds | | | Bills | | |
|---|---|---|---|---|---|---|---|---|---|
| SA | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand |
| $\psi$ = 0.3 | 27.66 | 25.51 | 2.15 | 4.70 | 16.87 | –12.18 | 67.64 | 57.62 | 10.02 |
| $\psi$ = 1.0 | 27.81 | 25.51 | 2.30 | 5.02 | 16.87 | –11.85 | 67.17 | 57.62 | 9.56 |
| $\psi$ = 1.5 | 27.92 | 25.51 | 2.41 | 5.27 | 16.87 | –11.61 | 66.81 | 57.62 | 9.20 |
| UK | | | | | | | | | |
| $\psi$ = 0.3 | 46.74 | 19.16 | 27.58 | 55.88 | 62.19 | –6.31 | –2.62 | 18.65 | –21.27 |
| $\psi$ = 1.0 | 52.53 | 19.16 | 33.37 | 58.41 | 62.19 | –3.78 | –10.94 | 18.65 | –29.59 |
| $\psi$ = 1.5 | 58.07 | 19.16 | 38.92 | 60.79 | 62.19 | –1.40 | –18.86 | 18.65 | –37.51 |
| US | | | | | | | | | |
| $\psi$ = 0.3 | 58.31 | 28.51 | 29.80 | 33.45 | 46.40 | –12.95 | 8.24 | 25.09 | –16.86 |
| $\psi$ = 1.0 | 67.30 | 28.51 | 38.79 | 34.77 | 46.40 | –11.63 | –2.07 | 25.09 | –27.16 |
| $\psi$ = 1.5 | 80.87 | 28.51 | 52.36 | 36.88 | 46.40 | –9.52 | –17.75 | 25.09 | –42.84 |

**Notes:** The mean total, myopic and hedging demands are reported in percentages in this table for domestic stocks, 10-year government bonds, and 3-month Treasury bills for an investor with $\psi$ equal to 0.3, 1 or 1.5. The discount factor ($\delta$) equal to $0.92^{1/12}$ on a monthly basis and $\gamma$ is equal to 7.

[22] Lynch (2001) set $\psi$ equal to four in his assessment of return predictability on portfolio choice. Campbell and Viceira (1999) and Campbell *et al.* (2004) used a range of values for $\gamma$ between 0.75 and 40, and $\psi$ between 1/0.75 and 1/40.

[23] The mean asset demands from Table 2, for $\psi$ = 1, are repeated in Table 3 to facilitate the comparison.

For different values of $\psi$, the mean total and hedging demand for stocks in SA changes little, but for both the US and the UK the mean and total intertemporal hedging demands change more significantly as $\psi$ increases. For the US and the UK, the results are in line with RW, who argue that intuitively, as $\psi$ increases, agents become more willing to trade future for current consumption and they hold more stocks with relatively high expected return. As investors hold more stocks, the hedging demand for stocks would increase. This effect is more pronounced in the UK and the US since domestic stocks can be described as a good hedge against themselves in these countries. For an investor in SA, the hedging demand and the total demand for stocks remain largely unchanged for different values of $\psi$. Following the argument for the US and the UK, this can be explained by the idea that SA stocks were not found to be a good hedge against themselves. The mean myopic demand for all assets remains constant, irrespective of changes in the elasticity of intertemporal substitution[24].

The results in Table 3 are also in line with Bhamra and Uppal (2006), who find that the sign of the intertemporal hedging demand for the risky asset is not affected by EIS, only the magnitude is affected. This is in contrast to risk aversion, which affects both the sign and the magnitude of the intertemporal hedging demand for the risky asset.

### 3.3. Asset demands for an investor in SA who can also invest in assets from the UK and the US

Following RW the multi-period portfolio choice problem is extended to an investor in SA who also has access to foreign assets, in addition to domestic stocks and bonds. The foreign assets comprise stocks and bonds from the UK and the US respectively. In an attempt to keep the VAR parameter space manageable, the foreign countries are added to the VAR in turn. The benchmark asset remains the log real return on the 3-month Treasury bill for SA – the same asset that was used as the benchmark for an investor's portfolio problem in SA in the analysis above. The state vector is expanded to also include excess returns on foreign bonds, foreign stocks, and foreign instruments thus becoming:

$$z_{t+1} = [\mathrm{rtbr}_{t+1}, \mathrm{xsr}_{t+1}, \mathrm{xbr}_{t+1}, \mathrm{xsr}^*_{t+1}, \mathrm{xbr}^*_{t+1}, \mathrm{bill}_{t+1}, \mathrm{div}_{t+1}, \mathrm{spread}_{t+1}, \mathrm{bill}^*_{t+1}, \mathrm{div}^*_{t+1}, \mathrm{spread}^*_{t+1}]', \tag{13}$$

where $\mathrm{bill}^*_{t+1}$, $\mathrm{div}^*_{t+1}$ and $\mathrm{spread}^*_{t+1}$ are the instruments of the foreign country. Furthermore, $\mathrm{xsr}^*_{t+1}$ ($\mathrm{xbr}^*_{t+1}$) is the log excess stock (bond) return and log excess bond return in rand relative to the 3-month Treasury bill rate in SA (the benchmark asset). To measure the log excess return on foreign bonds or stocks, the observations are converted to rand following Harvey (1991), using monthly exchange rates obtained from the Global Financial Database. Subsequently, the excess return in rand is calculated (relative to the return on a SA 3-month Treasury bill).

Similar to the analysis in the previous section, it is assumed that the state vector (equation 13) is generated by a VAR(1) process[25]. Different values of risk aversion ($\gamma = 4,7$

[24] This is because myopic demand in equations (11) and (12) is independent of $\psi$.

[25] The VAR results and correlation coefficients for SA are included in the Tables 7–9. Results for the UK and the US are available upon request.

**Table 4.** Mean demands for assets when an investor in SA can invest in foreign assets

| | Domestic stocks | | | Domestic bonds | | | Foreign stocks (UK) | | | Foreign bonds (UK) | | | Domestic Bills | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| γ | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand |
| | Foreign country: UK, 1960:02–2010:09 | | | | | | | | | | | | | | |
| 4 | **54.11** | **42.47** | 11.64 | 33.97 | 52.41 | –18.43 | 22.83 | 18.35 | 4.48 | 42.76 | 47.01 | –4.25 | –53.67 | –60.24 | 6.57 |
| | [20,89] | [12,68] | [–1, 25] | [–86, 153] | [–87, 192] | [–52, 21] | [–24, 76] | [–1, 43] | [–26, 47] | [–12, 108] | [–2, 100] | [–35, 22] | [–219, 74] | [–215, 106] | [–41, 24] |
| 7 | **32.03** | **24.50** | 7.53 | 12.39 | 28.80 | –16.41 | 3.86 | 10.21 | –6.35 | 31.27 | 26.96 | 4.30 | 20.46 | 9.53 | 10.93 |
| | [12, 53] | [7, 39] | [–1, 18] | [–54, 82] | [–51, 108] | [–37, 11] | [–24, 47] | [–1, 24] | [–27, 31] | [–8, 68] | [–1, 58] | [–23, 22] | [–66, 103] | [–79, 105] | [–31, 8] |
| 10 | **22.74** | **17.32** | 5.43 | 3.57 | 19.36 | –15.79 | –6.04 | 6.95 | –12.99 | 28.40 | 18.95 | 9.46 | 51.33 | 37.43 | 13.90 |
| | [9, 39] | [5, 27] | [0, 15] | [–42, 54] | [–36, 75] | [–35, 3] | [–27, 32] | [–4, 14] | [–29, 21] | [–5, 54] | [–1, 40] | [–17, 22] | [–13, 107] | [–24, 105] | [–18, 4] |
| | Foreign country: US, 1960:02–2010:09 | | | | | | | | | | | | | | |
| 4 | **52.72** | **44.46** | 8.26 | 34.89 | 52.82 | –17.93 | 34.12 | 19.96 | 14.16 | 25.33 | 44.08 | –18.75 | –47.06 | –61.31 | 14.26 |
| | [5, 87] | [8, 78] | [–4, 21] | [–92, 234] | [–112, 267] | [–62, 21] | [–59, 94] | [–29, 37] | [–49, 62] | [–27, 114] | [–3, 101] | [–49, 29] | [–225, 83] | [–266, 99] | [–55, 72] |
| 7 | **31.45** | **25.54** | 5.92 | 13.57 | 28.99 | –15.42 | 7.37 | 11.35 | –3.98 | 22.57 | 24.85 | –2.28 | 25.02 | 9.26 | 15.76 |
| | [5, 54] | [5, 45] | [–2, 16] | [–59, 127] | [–66, 151] | [–43, 10] | [–60, 54] | [–17, 20] | [–46, 39] | [–19, 77] | [–2, 57] | [–35, 29] | [–82, 94] | [–108, 101] | [–35, 49] |
| 10 | **22.75** | **17.97** | 4.78 | 5.34 | 19.46 | –14.12 | –6.56 | 7.91 | –14.47 | 24.11 | 17.16 | 6.95 | 54.36 | 37.49 | 16.87 |
| | [5, 41] | [4, 32] | [–2, 12] | [–46, 85] | [–46, 106] | [–33, 7] | [–54, 36] | [–12, 14] | [–48, 25] | [–16, 58] | [–2, 40] | [–25, 29] | [–24, 104] | [–45, 101] | [–19, 46] |

**Notes:** The mean total, myopic and hedging demands are reported in percentages in this table for domestic stocks, domestic 10-year government bonds, domestic 3-month Treasury bills, foreign stock and foreign 10-year government bonds for an investor with an elasticity of substitution ($\psi$) equal to one, the discount factor ($\delta$) equal to $0.92^{1/12}$ on a monthly basis. Three different values for the coefficient of risk aversion is applied ($\gamma = 4, 7, 10$) and the bootstrapped confidence intervals (following RW) for the 90% confidence intervals of the mean asset demands are given in brackets. Significance according to these confidence intervals is indicated by a bold entry.

and 10) are considered, whilst keeping $\psi$ constant and equal to one, and assuming that $\delta = 0.92$ on an annual basis. Table 4 presents the mean asset demands for an investor in SA who can invest in the UK or in the US, and the 90% confidence interval calculated using the parametric bootstrap procedure of RW. The results are reported for the sampling period 1960:02–2010:09.

The mean total demand for domestic stocks for a SA investor is roughly similar, irrespective if the foreign country that the investor can invest in is the US or the UK, although these mean total demands for domestic stocks are higher than when the investor could not invest in foreign assets. The strong demand for domestic assets can be explained by the relatively higher Sharpe ratio for SA stocks in this period which leads to higher myopic demand for domestic stocks relative to foreign stock[26]. Table 4 shows that in no case can the null hypothesis of zero mean hedging demand be rejected for domestic and foreign stocks, domestic and foreign bonds and bills, at a 10% level of significance. Hence it can be concluded that foreign stocks in the US and the UK do not present a useful hedging opportunity for an investor in SA. The hedging demand for domestic stocks is, however, larger in the case when the SA investor could only invest in domestic assets. Similarly the total demand for domestic stocks is marginally higher and in contrast to when the investor could only invest in SA assets, the investor shorts bills in this case.

In Figure 2, the historical intertemporal hedging demands related to Table 4 are shown. The small magnitude of hedging demand for SA stocks and the volatility of hedging demand for SA bonds are clearly visible, irrespective whether the UK or the US is used as the foreign country. The hedging demand for foreign stocks is positive in the late 1970s, but becomes negative in the early 1980s. Similar to Figure 1, the hedging demands for foreign stock increases (from negative levels) towards the end of 2007 and starts to decrease again in the beginning of 2009, irrespective of whether the foreign country is the US or the UK. The hedging demands for bonds follow roughly the opposite pattern, although the reaction during the financial crisis is less pronounced in the UK. For SA assets, the hedging demand for bonds is volatile during this period, whilst the hedging demands for stocks remain relatively unaffected. In summary, although UK and US stocks were found to be a good hedge against itself for domestic investors in the UK and the US respectively, access to these stocks do not provide attractive hedging opportunities to an investor in SA.

Table 5 presents the mean asset demands for $\psi$ values of 0.3 and 1.5. The mean asset demands when $\psi$ is equal to 1, from Table 4, are also included. Similar to Table 3, as EIS increases the hedging demand for domestic and foreign stocks increase, however for UK and US stocks the hedging demand is significantly smaller in comparison to Table 3. Furthermore the sign of the hedging demand does not change for different values of $\psi$.

[26] When comparing the results of Table 4 to the sub-sample, it is found that an investor from SA who can invest in US stocks holds a larger share of US stocks than domestic stocks on average in the sub-sample. This is because the excess stock returns in SA has a relatively smaller Sharpe ratio than the excess stock returns in the US in the sub-sample, hence the myopic demand for SA stocks is lower than the mean myopic demand for US stocks.

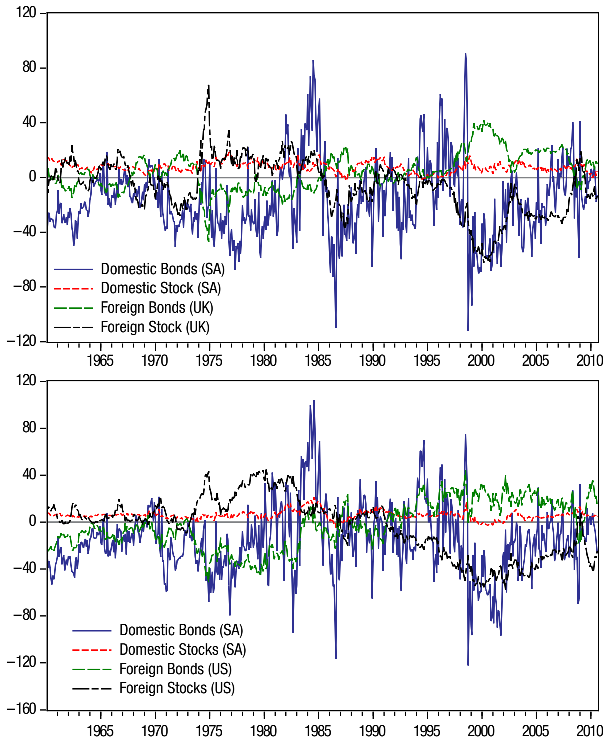


**Fig. 2.** The historical intertemporal hedging demands for domestic stocks, domestic bonds, foreign stocks and foreign bonds for investors in SA who can also invest in assets in the UK and in the US

### 3.4. Asset demands for an investor in the UK and the US, who can also invest in SA stocks

In a final application, the asset demand for an investor in the UK and the US who has access to stocks and bonds from SA in addition to domestic assets including stocks, bonds and bills is analysed. The log excess return on SA stocks and bonds are measured by first converting the stock and bonds returns from rand to the appropriate foreign currency using the relevant bilateral exchange rates. Then the excess return is calculated in the appropriate local currency, in excess of the local currency return on the benchmark asset – the log real return on a domestic (UK or US respectively) 3-month Treasury bill. Thus, there are six instruments for the investor to consider namely the domestic and SA nominal bill yields, dividend yields and term spreads.

The intertemporal hedging demands for domestic stocks continue to be fairly large in magnitude in both the UK and the US (see Table 6). According to the 90% confidence intervals the hedging demand for stocks in the US and the UK is sizable and positive. The mean hedging demand for SA stocks and bonds, however, remains small and according to the 90% confidence intervals are not significantly different from zero[27].

[27] The mean hedging demand for SA stocks and bonds for investors in the US and the UK were also not significantly different from zero in the sub-sample.

**Table 5.** Mean asset demand for an investor in SA who can invest in the UK and the US, with different values of ψ, keeping γ constant and equal to 7 (1960:02–2010:09)

| | Dom stocks | | | Dom bonds | | | Foreign stocks | | | Foreign bonds | | | Domestic bills | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ψ | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand |
| UK | | | | | | | | | | | | | | | |
| 0.3 | 31.31 | 24.50 | 6.81 | 10.75 | 28.80 | –18.05 | 3.14 | 10.21 | –7.07 | 31.85 | 26.96 | 4.89 | 22.95 | 9.53 | 13.43 |
| 1 | 32.03 | 24.50 | 7.53 | 12.39 | 28.80 | –16.41 | 3.86 | 10.21 | –6.35 | 31.27 | 26.96 | 4.30 | 20.46 | 9.53 | 10.93 |
| 1.5 | 32.72 | 24.50 | 8.22 | 14.24 | 28.80 | –14.56 | 5.33 | 10.21 | –4.87 | 30.10 | 26.96 | 3.14 | 17.60 | 9.53 | 8.08 |
| US | | | | | | | | | | | | | | | |
| 0.3 | 30.92 | 25.54 | 5.38 | 11.49 | 28.99 | –17.50 | 4.57 | 11.35 | –6.78 | 24.77 | 24.85 | – 0.08 | 28.25 | 9.26 | 18.99 |
| 1 | 31.45 | 25.54 | 5.92 | 13.57 | 28.99 | –15.42 | 7.37 | 11.35 | –3.98 | 22.57 | 24.85 | –2.28 | 25.02 | 9.26 | 15.76 |
| 1.5 | 32.10 | 25.54 | 6.56 | 17.98 | 28.99 | –11.02 | 17.27 | 11.35 | 5.92 | 15.53 | 24.85 | –9.32 | 17.12 | 9.26 | 7.86 |

**Notes:** The mean total, myopic and hedging demands are reported in percentages in this table for domestic stocks, domestic 10-year government bonds, foreign stocks, foreign 10-year government bonds and 3-month Treasury bills for an investor with ψ equal to 0.3, 1 or 1,5. The discount factor (δ) equal to $0.92^{1/12}$ on a monthly basis and γ equal to 7.

**Table 6.** Mean demands for assets when investors in the UK and the US who can invest in SA assets in addition to respective domestic assets

| | Domestic stocks | | | Domestic bonds | | | Foreign stocks (SA) | | | Foreign bonds (SA) | | | Domestic bills | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| γ | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand | Total demand | Myopic demand | Hedging demand |
| | Domestic country: UK, 1960:02–2010:09 | | | | | | | | | | | | | | |
| 4 | **67.16** | **19.89** | **47.26** | **142.07** | 137.56 | 4.51 | **48.76** | **39.59** | 9.17 | –37.48 | –38.04 | 0.57 | –120.51 | –59.00 | –61.51 |
| | [18, 114] | [–1, 41] | [12, 83] | [4, 310] | [–4, 328] | [–28, 42] | [22, 82] | [15, 66] | [–1, 21] | [–79, 18] | [–80, 18] | [–10, 11] | [–279, 37] | [–226, 76] | [–134, –7] |
| 7 | **46.65** | **11.93** | **34.72** | **83.21** | 78.79 | 4.41 | **26.90** | **22.49** | 4.41 | –21.77 | –21.64 | –0.13 | –34.99 | 8.43 | –43.42 |
| | [11, 83] | [0, 24] | [6, 66] | [10, 186] | [–2, 188] | [–21, 28] | [12, 49] | [8, 37] | [–2, 15] | [–47, 9] | [–45, 10] | [–7, 7] | [–138, 57] | [–88, 85] | [–94, 7] |
| 10 | **34.21** | **8.75** | 25.46 | **58.85** | 55.28 | 3.57 | **17.40** | **15.65** | 1.75 | –15.53 | –15.08 | –0.44 | 5.07 | 35.40 | –30.33 |
| | [7, 67] | [1, 17] | [0, 52] | [7, 131] | [–2, 131] | [–14, 24] | [4, 31] | [5, 26] | [–3, 10] | [–34, 6] | [–32, 7] | [–6, 5] | [–78, 69] | [–33, 89] | [–81, 4] |
| | Domestic country: US, 1960:02–2010:09 | | | | | | | | | | | | | | |
| 4 | **92.55** | **28.77** | **63.78** | 92.33 | 101.09 | –8.76 | **48.89** | **40.05** | 8.83 | –29.46 | –30.80 | 1.34 | –104.30 | –39.11 | –65.18 |
| | [3, 161] | [–13, 45] | [5, 130] | [–38, 245] | [4, 228] | [–49, 51] | [5, 86] | [1, 73] | [–1, 21] | [–91, 74] | [–94, 80] | [–10, 11] | [–280, 61] | [–186, 91] | [–159, 24] |
| 7 | **69.93** | **16.15** | **53.78** | 51.06 | **57.55** | –6.49 | **28.71** | 23.03 | 5.68 | –16.83 | –17.60 | 0.77 | –32.88 | 20.87 | –53.75 |
| | [10, 132] | [–6, 27] | [9, 113] | [–28, 140] | [2, 130] | [–35, 33] | [5, 53] | [0, 42] | [–1, 15] | [–53, 41] | [–54, 46] | [–6, 9] | [–158, 52] | [–63, 95] | [–129, 8] |
| 10 | **56.36** | **11.10** | **45.26** | 35.26 | 40.13 | –4.87 | **20.22** | 16.23 | 3.99 | –11.71 | –12.32 | 0.61 | –0.12 | 44.87 | –44.99 |
| | [7, 109] | [–5, 19] | [6, 94] | [–22, 97] | [–2, 88] | [–30, 20] | [4, 37] | [0, 29] | [–1, 11] | [–37, 29] | [–38, 32] | [–4, 6] | [–89, 63] | [–15, 96] | [–107, 4] |

**Notes:** The mean total, myopic and hedging demands are reported in percentages in this table for domestic stocks, domestic 10-year government bonds, foreign stock, foreign 10-year government bonds and domestic 3-month Treasury bills for an investor with an elasticity of substitution ($\psi$) equal to one and the discount factor ($\delta$) equal to $0.92^{1/12}$ on a monthly basis. Three different values for the coefficient of risk aversion is applied ($\gamma$ = 4, 7, 10) and the bootstrapped confidence intervals (following RW) for the 90% confidence intervals of the mean asset demands are given in brackets. Significance according to these confidence intervals is indicated by a bold entry.

**Table 7.** An investor from SA who can invest in domestic assets, 1960:02–2010:09
VAR estimation results[28]

| SA | rtbr | xsr | xbr | bill | div | spread | R-square |
|---|---|---|---|---|---|---|---|
| T-bill+1 | 0.287 | –0.005 | 0.030 | 0.000 | –0.002 | –0.001 | 0.318 |
| | 5.467 | –1.460 | 2.661 | –1.260 | –3.734 | –6.848 | |
| | 0.000 | 0.140 | 0.002 | 0.147 | 0.000 | 0.000 | |
| Stock+1 | 0.254 | 0.077 | 0.050 | –0.004 | 0.021 | 0.001 | 0.029 |
| | 0.454 | 1.569 | 0.351 | –2.022 | 2.294 | 0.981 | |
| | 0.629 | 0.065 | 0.687 | 0.027 | 0.014 | 0.259 | |
| Bond+1 | 0.232 | 0.030 | 0.269 | 0.000 | 0.003 | 0.001 | 0.103 |
| | 1.186 | 1.924 | 4.290 | 0.276 | 1.036 | 2.258 | |
| | 0.173 | 0.025 | 0.000 | 0.744 | 0.298 | 0.014 | |
| Short-term+1 | –0.540 | –0.628 | –4.577 | 0.965 | –0.183 | 0.055 | 0.907 |
| | –0.126 | –1.427 | –3.196 | 40.623 | –3.677 | 3.811 | |
| | 0.887 | 0.037 | 0.000 | 0.000 | 0.003 | 0.000 | |
| Dividend+1 | 0.442 | –0.352 | –0.187 | 0.009 | 0.966 | 0.003 | 0.959 |
| | 0.595 | –7.052 | –1.313 | 5.488 | 120.221 | 1.319 | |
| | 0.416 | 0.000 | 0.146 | 0.000 | 0.000 | 0.064 | |
| Spread+1 | –4.656 | 0.255 | –0.677 | –0.114 | 0.077 | 0.947 | 0.954 |
| | –0.970 | 0.476 | –0.490 | –4.127 | 1.325 | 60.780 | |
| | 0.000 | 0.447 | 0.501 | 0.000 | 0.255 | 0.000 | |

Cross correlations of VAR residuals

| | rtbr | xsr | xbr | bill | div | spread |
|---|---|---|---|---|---|---|
| rtbr | 1.000 | | | | | |
| xsr | –0.031 | 1.000 | | | | |
| xbr | 0.105 | 0.195 | 1.000 | | | |
| bill | 0.086 | –0.101 | –0.350 | 1.000 | | |
| div | –0.079 | –0.428 | –0.181 | 0.135 | 1.000 | |
| spread | –0.171 | –0.047 | –0.302 | –0.727 | 0.031 | 1.000 |

**Notes:** $rtbr_t$ = log real 3-month Treasury bill return; $xsr_t$ = log excess stock return; $xbr_t$ = log excess bond return, $bill_t$ = 3-month Treasury bill yield; $div_t$ = log dividend yield; $spread_t$ = difference between a 10-year government bond yield and a 3-month Treasury bill yield. T-statics are given below the parameter estimate.

[28] Equation (4)'s parameters are estimated using maximum likelihood. Note that maximum likelihood estimation of $\Phi_0$ and $\Phi_1$ in equation (4) is equivalent to OLS estimation.

**Table 8.** An investor from SA who can invest in domestic assets and assets from the UK, 1960:02–2010:09

| | rtbr | xsr | xbr | f_xsr | f_xbr | bill | div | spread | f_bill | f_div | f_spread | R-square |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T-bill+1 | 0.257 | –0.005 | 0.027 | 0.002 | –0.008 | –0.000 | –0.001 | –0.001 | –0.000 | –0.003 | 0.000 | 0.335 |
| | 4.842 | –1.651 | 2.489 | 0.577 | –1.301 | –0.697 | –1.531 | –6.635 | –0.849 | –3.429 | 0.571 | |
| Stock+1 | 0.177 | 0.057 | 0.061 | 0.077 | –0.045 | –0.004 | 0.028 | 0.002 | 0.002 | –0.011 | –0.001 | 0.037 |
| | 0.314 | 1.122 | 0.413 | 1.390 | –0.574 | –1.863 | 2.751 | 1.187 | 0.716 | –0.867 | –0.816 | |
| Bond+1 | 0.200 | 0.036 | 0.263 | –0.025 | 0.015 | 0.000 | 0.006 | 0.001 | –0.000 | –0.006 | –0.000 | 0.112 |
| | 1.015 | 2.147 | 4.173 | –1.546 | 0.496 | 0.496 | 1.649 | 2.585 | –0.614 | –1.510 | –0.582 | |
| F_Stock+1 | 1.166 | –0.088 | 0.256 | 0.121 | –0.044 | –0.003 | –0.002 | 0.001 | 0.002 | 0.035 | –0.000 | 0.045 |
| | 2.315 | –1.753 | 1.698 | 1.763 | –0.443 | –1.103 | –0.231 | 0.382 | 0.683 | 2.300 | –0.276 | |
| F_bond+1 | 0.404 | –0.084 | 0.093 | 0.076 | 0.021 | 0.000 | –0.003 | 0.000 | 0.001 | 0.008 | –0.001 | 0.033 |
| | 1.094 | –2.436 | 0.820 | 2.082 | 0.275 | 0.207 | –0.446 | 0.263 | 0.665 | 0.952 | –0.691 | |
| Short-term+1 | –0.965 | –0.696 | –4.521 | 0.255 | –0.087 | 0.964 | –0.166 | 0.054 | 0.014 | –0.038 | 0.007 | 0.907 |
| | –0.226 | –1.553 | –3.118 | 0.812 | –0.132 | 37.933 | –2.520 | 3.513 | 0.806 | –0.433 | 0.515 | |
| Dividend+1 | 0.707 | –0.297 | –0.195 | –0.218 | 0.217 | 0.009 | 0.961 | 0.003 | –0.002 | 0.014 | –0.002 | 0.960 |
| | 0.948 | –6.750 | –1.332 | –3.887 | 2.909 | 5.114 | 94.507 | 1.450 | –0.809 | 1.053 | –0.798 | |
| Spread+1 | –3.938 | 0.257 | –0.643 | 0.062 | –0.038 | –0.115 | 0.014 | 0.945 | –0.005 | 0.114 | 0.003 | 0.954 |
| | –0.855 | 0.479 | –0.473 | 0.171 | –0.043 | –3.898 | 0.194 | 56.545 | –0.259 | 1.180 | 0.187 | |
| F_ST+1 | 5.150 | 0.858 | –0.957 | –0.182 | –1.441 | 0.008 | –0.031 | 0.006 | 0.932 | –0.186 | 0.082 | 0.824 |
| | 0.988 | 2.046 | –1.012 | –0.403 | –2.143 | 0.438 | –0.343 | 0.469 | 40.917 | –2.292 | 5.432 | |
| F_Div+1 | –0.160 | 0.069 | –0.243 | –0.461 | 0.287 | 0.003 | 0.000 | 0.001 | 0.003 | 0.978 | 0.001 | 0.977 |
| | –0.410 | 1.761 | –2.395 | –6.757 | 4.168 | 2.090 | 0.017 | 0.760 | 2.070 | 106.668 | 0.492 | |
| F_Spread+1 | –2.933 | –0.300 | –0.637 | –0.134 | 1.129 | –0.009 | 0.098 | 0.009 | –0.067 | 0.026 | 0.927 | 0.910 |
| | –0.572 | –0.805 | –0.739 | –0.317 | 1.896 | –0.551 | 1.242 | 0.762 | –3.287 | 0.303 | 62.119 | |

Cross correlations of VAR residuals

| | rtbr | xsr | xbr | f_xsr | f_xbr | bill | div | spread | f_bill | f_div | f_spread |
|---|---|---|---|---|---|---|---|---|---|---|---|
| rtbr | 1.000 | | | | | | | | | | |
| xsr | (0.033) | 1.000 | | | | | | | | | |
| xbr | 0.098 | 0.198 | 1.000 | | | | | | | | |
| f_xsr | 0.054 | 0.182 | (0.075) | 1.000 | | | | | | | |
| f_xbr | 0.016 | (0.082) | (0.162) | 0.561 | 1.000 | | | | | | |
| bill | 0.086 | (0.104) | (0.351) | 0.056 | 0.071 | 1.000 | | | | | |
| div | (0.071) | (0.425) | (0.193) | (0.149) | 0.075 | 0.144 | 1.000 | | | | |
| spread | (0.168) | (0.045) | (0.300) | 0.007 | 0.057 | (0.728) | 0.031 | 1.000 | | | |
| f_bill | (0.076) | 0.020 | (0.049) | (0.232) | (0.206) | 0.027 | 0.079 | 0.018 | 1.000 | | |
| f_div | (0.096) | (0.287) | (0.144) | (0.559) | 0.052 | 0.016 | 0.315 | 0.079 | 0.199 | 1.000 | |
| f_spread | 0.072 | 0.030 | (0.008) | 0.055 | 0.028 | (0.011) | (0.099) | (0.011) | (0.730) | (0.049) | 1.000 |

**Notes:** $rtbr_t$ = log real 3-month Treasury bill return (SA); $xsr_t$ = log excess stock return (SA); $xbr_t$ = log excess bond return (SA), ; $f\_xsr_t$ = log excess stock return (UK) in rand, $f\_xbr_t$ = log excess bond return (UK) in rand, $bill_t$ = 3-month Treasury bill yield (SA) $div_t$ = log dividend yield (SA); $spread_t$ = difference between a 10-year government bond yield and a 3-month Treasury bill yield (SA), $f\_bill_t$ = 3-month Treasury bill yield (UK) $f\_div_t$ = log dividend yield (UK); $f\_spread_t$ = difference between a 10-year government bond yield and a 3-month Treasury bill yield (UK). T-statics are given below the parameter estimate.

Therefore the results indicate that access to SA stocks and bonds for investors in the UK and the US does not generate sizable intertemporal hedging demand for SA assets. Despite small hedging demand, the mean total demand for SA stocks is sizable because of the large myopic demand, which is once again possibly due to a relatively higher Sharpe ratio. The largest intertemporal hedging demand in this portfolio is for domestic stocks in the US and the UK respectively[29].

It can be observed in Figure 3 that the hedging demand for US and UK stocks increased sharply before decreasing again roughly around the time when the financial crisis started, and that the hedging demand for bonds follows the opposite path. When converted to foreign currency, hedging demand for both SA stocks and bonds are notably less volatile than hedging demand for domestic stocks and bonds in the UK and the US respectively.

**Table 9.** An investor from SA who can invest in domestic assets and assets from the US, 1960:02–2010:09

| | rtbr | xsr | xbr | f_xsr | f_xbr | bill | div | spread | f_bill | f_div | f_spread | R-square |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T-bill+1 | 0.220 | –0.006 | 0.033 | 0.008 | –0.007 | –0.000 | –0.000 | –0.001 | 0.001 | –0.003 | 0.000 | 0.361 |
| | 4.347 | –1.874 | 3.014 | 1.665 | –1.100 | –0.604 | –0.705 | –7.051 | 2.847 | –4.834 | 2.723 | |
| Stock+1 | 0.302 | 0.057 | 0.074 | 0.098 | 0.004 | –0.004 | 0.018 | 0.002 | –0.004 | 0.002 | –0.003 | 0.040 |
| | 0.506 | 1.116 | 0.498 | 1.540 | 0.054 | –1.901 | 1.844 | 1.163 | –1.329 | 0.234 | –1.051 | |
| Bond+1 | 0.138 | 0.040 | 0.261 | –0.032 | 0.039 | 0.000 | 0.005 | 0.001 | –0.001 | –0.006 | –0.000 | 0.117 |
| | 0.679 | 2.479 | 3.978 | –1.346 | 1.383 | 0.592 | 1.593 | 2.765 | –0.869 | –1.663 | –0.157 | |
| F_Stock+1 | 1.658 | –0.093 | 0.105 | 0.060 | –0.084 | –0.001 | –0.004 | 0.001 | –0.005 | 0.022 | –0.001 | 0.047 |
| | 3.244 | –2.331 | 0.762 | 1.017 | –1.055 | –0.538 | –0.516 | 0.806 | –1.662 | 2.910 | –0.616 | |
| F_bond+1 | 0.735 | –0.086 | 0.145 | –0.010 | 0.072 | 0.002 | –0.004 | –0.000 | 0.002 | 0.009 | 0.002 | 0.050 |
| | 1.867 | –2.568 | 1.383 | –0.211 | 1.013 | 1.032 | –0.678 | –0.370 | 0.754 | 1.317 | 1.052 | |
| Short-term+1 | –2.296 | –0.700 | –4.252 | 0.425 | 0.151 | 0.964 | –0.150 | 0.052 | 0.037 | –0.039 | 0.013 | 0.907 |
| | –0.507 | –1.609 | –2.903 | 1.049 | 0.194 | 38.558 | –2.388 | 3.325 | 1.211 | –0.477 | 0.664 | |
| Dividend+1 | 0.649 | –0.280 | –0.246 | –0.264 | 0.245 | 0.009 | 0.960 | 0.003 | –0.006 | 0.007 | –0.004 | 0.961 |
| | 0.840 | –6.270 | –1.742 | –3.512 | 3.116 | 4.789 | ###### | 1.486 | –1.779 | 0.680 | –1.485 | |
| Spread+1 | –1.845 | 0.214 | –0.890 | –0.095 | –0.645 | –0.115 | 0.013 | 0.947 | –0.013 | 0.112 | –0.004 | 0.955 |
| | –0.405 | 0.425 | –0.641 | –0.162 | –0.684 | –4.001 | 0.182 | 56.744 | –0.390 | 1.289 | –0.181 | |
| F_ST+1 | –4.715 | 0.060 | –0.687 | 1.207 | –2.360 | –0.010 | –0.015 | 0.017 | 0.885 | –0.044 | 0.046 | 0.788 |
| | –1.230 | 0.176 | –0.897 | 2.085 | –3.345 | –0.583 | –0.263 | 1.782 | 26.154 | –0.730 | 2.878 | |
| F_Div+1 | –0.811 | 0.041 | –0.039 | –0.011 | 0.044 | 0.003 | 0.005 | –0.000 | 0.006 | 0.985 | 0.000 | 0.988 |
| | –1.834 | 1.211 | –0.349 | –0.218 | 0.666 | 2.115 | 0.652 | –0.400 | 2.493 | ###### | 0.029 | |
| F_Spread+1 | 6.887 | 0.172 | –0.891 | –0.355 | 0.766 | –0.002 | 0.067 | –0.006 | –0.028 | 0.028 | 0.926 | 0.896 |
| | 1.690 | 0.560 | –1.009 | –0.702 | 1.197 | –0.117 | 1.140 | –0.570 | –0.952 | 0.520 | 58.508 | |

[29] This is also the case in the sub-sample.

*End of Table 9*

Cross correlations of VAR residuals

| | rtbr | xsr | xbr | f_xsr | f_xbr | bill | div | spread | f_bill | f_div | f_spread |
|---|---|---|---|---|---|---|---|---|---|---|---|
| rtbr | 1.000 | | | | | | | | | | |
| xsr | (0.026) | 1.000 | | | | | | | | | |
| xbr | 0.101 | 0.198 | 1.000 | | | | | | | | |
| f_xsr | 0.029 | 0.208 | (0.112) | 1.000 | | | | | | | |
| f_xbr | 0.064 | (0.157) | (0.168) | 0.535 | 1.000 | | | | | | |
| bill | 0.075 | (0.103) | (0.352) | 0.066 | 0.080 | 1.000 | | | | | |
| div | (0.051) | (0.435) | (0.198) | (0.127) | 0.112 | 0.148 | 1.000 | | | | |
| spread | (0.162) | (0.045) | (0.299) | 0.016 | 0.057 | (0.728) | 0.030 | 1.000 | | | |
| f_bill | (0.002) | 0.077 | (0.105) | (0.021) | (0.249) | 0.067 | (0.074) | (0.006) | 1.000 | | |
| f_div | (0.031) | (0.339) | (0.078) | (0.708) | 0.057 | 0.036 | 0.190 | 0.026 | 0.053 | 1.000 | |
| f_spread | (0.043) | (0.038) | 0.034 | (0.067) | (0.102) | (0.018) | 0.020 | (0.003) | (0.727) | 0.065 | 1.000 |

**Notes:** $rtbr_t$ = log real 3-month Treasury bill return (SA); $xsr_t$ = log excess stock return (SA); $xbr_t$ = log excess bond return (SA), ; $f\_xsr_t$ = log excess stock return (US) in rand, $f\_xbr_t$ = log excess bond return (US) in rand, $bill_t$ = 3-month Treasury bill yield (SA) $div_t$ = log dividend yield (SA); $spread_t$ = difference between a 10-year government bond yield and a 3-month Treasury bill yield (SA), $f\_bill_t$ = 3-month Treasury bill yield (US) $f\_div_t$ = log dividend yield (US); $f\_spread_t$ = difference between a 10-year government bond yield and a 3-month Treasury bill yield (US). T-statics are given below the parameter estimate.

## Conclusions

In this paper, return predictability and its implications for hedging demands for stocks, bonds and bills is investigated for infinite-horizon investors with Epstein-Zin-Weil preferences in SA, the UK and the US. The results indicate that differences in return predictability across countries can lead to significant differences in the implied intertemporal hedging demand for domestic stocks and bonds in different countries. Allocations across domestic and foreign financial assets, including stocks and bonds, and domestic bills is also investigated when investors from SA can invest in the UK or the US, or when investors in the UK or the US can invest in SA. The mean intertemporal hedging demand for SA stocks remain small in all the cases considered, and the null hypothesis of zero mean hedging demand for SA stocks cannot be rejected. Furthermore, the mean hedging demand for stocks in the US and the UK are found to be significantly larger than those for SA. This is in part accounted for by the relationship between the excess stock return and the dividend yield in the respective countries. In the US, the UK and SA, stocks are seen to be the most important source of hedging of the three assets.

The effects of the financial crisis on intertemporal hedging demand were seen to be the most pronounced in the US, whilst the effects were limited in SA. It was seen that between the end of 2007 and the beginning of 2010 hedging demand for US and UK stocks increased significantly before falling again, and the hedging demand for bonds decreased before increasing again. Overall the results indicate that UK and US stocks

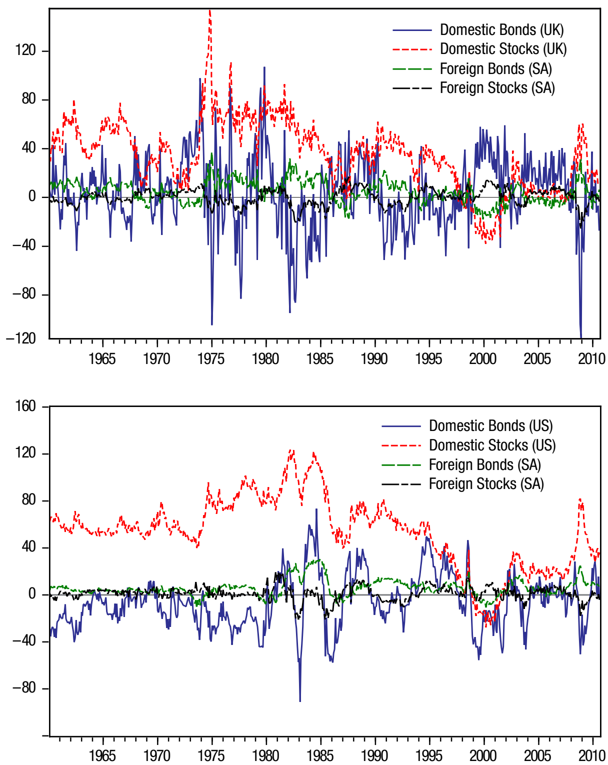


**Fig. 3.** The historical intertemporal hedging demands for domestic stocks, domestic bonds, foreign stocks and foreign bonds for investors in the UK and the US who can also invest in assets in SA

provide attractive hedging instruments for domestic investors from the two countries respectively, but not to investors from SA. SA stocks and bonds were not found to provide attractive hedging instruments for either domestic investors or international investors.

This paper includes the first empirical research related to intertemporal hedging demand for SA, to our knowledge, hence there are numerous avenues still unexplored that could improve the understanding of this topic. One approach could be to include more asset classes. The majority of strategic asset allocations, including this paper, consist mainly of allocations to stocks, bonds and cash – three traditional asset classes. However, the risk-return characteristics of strategic asset allocation can be improved by including investment in other asset classes with low correlations to the current set of asset classes.

## References


Ang, A.; Papanikolaou, D; Westerfield, M. 2013. Portfolio choice with illiquid assets, *The National Bureau of Economic Research.* Working Paper No. 19436 [online]. Available from Internet: http://www.nber.org/papers/w19436

Balduzzi, P.; Lynch, A. W. 1999. Transaction cost and predictability: some utility cost calculations, *Journal of Financial Economics* 52(1): 47–78. http://dx.doi.org/10.1016/S0304-405X(99)00004-5

Barberis, N. C. 2000. Investing for the long run when returns are predictable, *Journal of Finance* 55(1): 225–264. http://dx.doi.org/10.1111/0022-1082.00205

Barberis, N. C.; Huang, M.; Santos, T. 2000. Prospect theory and asset prices, *Quarterly Journal of Economics* 116(1): 1–54. http://dx.doi.org/10.1162/003355301556310

Bekkers, N.; Doeswijk, R. Q.; Lam, T. W. 2009. Strategic asset allocation: determining the optimal portfolio with ten asset classes, *The Journal of Wealth Management* 12(3): 61–77. http://dx.doi.org/10.3905/JWM.2009.12.3.061

Bhamra, H. S.; Uppal, R. 2006. The role of risk aversion and intertemporal substitution in dynamic consumption-portfolio choice with recursive utility, *Journal of Economic Dynamics and Control* 30: 967–991. http://dx.doi.org/10.1016/j.jedc.2005.04.001

Brandt, M. W. 1999. Estimating portfolio and consumption choice: a conditional Euler equations approach, *Journal of Finance* 54(5): 1609–1646. http://dx.doi.org/10.1111/0022-1082.00162

Brennan, M. J.; Schwartz, E. S; Lagnado, R. 1997. Strategic asset allocation, *Journal of Economic Dynamics and Control* 21(8–9): 1377–1403. http://dx.doi.org/10.1016/S0165-1889(97)00031-6

Campbell, J. Y. 1991. A variance decomposition for stock returns, *Economic Journal* 101(405): 157–179. http://dx.doi.org/10.2307/2233809

Campbell, J. Y.; Cochrane, J. H. 1999. By force of habit: a consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107(2): 205–251. http://dx.doi.org/10.1086/250059

Campbell, J. Y.; Viceira, L. M. 1999. Consumption and portfolio decisions when expected returns are time varying, *The Quarterly Journal of Economics* 114(2): 433–495. http://dx.doi.org/10.1162/003355399556043

Campbell, J. Y.; Viceira, L. M. 2000. Consumption and portfolio decisions when expected returns are time varying, Erratum, *The Quarterly Journal of Economics* [online]. Available from Internet http://scholar.harvard.edu/files/campbell/files/consumption_erratum_2000.pdf

Campbell, J. Y.; Viceira, L. M. 2001. Who should buy long-term bonds?, *The American Economic Review* 91(1): 99–127. http://dx.doi.org/10.1257/aer.91.1.99

Campbell, J. Y.; Viceira, L. M. 2002. *Strategic asset allocation: portfolio choice for long-term investors.* Oxford, UK: Oxford University Press. http://dx.doi.org/10.1093/0198296940.001.0001

Campbell, J. Y.; Chan, Y. L.; Viceira, L. M. 2003. A multivariate model of strategic asset allocation, *Journal of Financial Economics* 67(1): 41–80. http://dx.doi.org/10.1016/S0304-405X(02)00231-3

Epstein, L. G.; Zin, S. E. 1989. Substitution, risk aversion and the temporal behavior of consumption and asset returns: a theoretical framework, *Econometrica* 57(4): 937–969. http://dx.doi.org/10.1086/261750

Epstein, L. G.; Zin, S. E. 1991. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: an empirical investigation, *Journal of Political Economy* 99(2): 263–286.

Harvey, C. 1991. The world price of covariance risk, *Journal of Finance* 46(1): 111–157. http://dx.doi.org/10.1111/j.1540-6261.1991.tb03747.x

Hodrick, R. J. 1992. Dividend yields and expected stock returns: alternative procedures for inference and measurement, *Review of Financial Studies* 5(3): 357–386.
http://dx.doi.org/10.1093/rfs/5.3.357

Kandel, S.; Stambaugh, R. F. 1996. On the predictability of stock returns: an asset-allocation perspective, *Journal of Finance* 51(2): 385–424.
http://dx.doi.org/10.1111/j.1540-6261.1996.tb02689.x

Kim, T. S.; Omberg, E. 1996. Dynamic nonmyopic portfolio behavior, *Review of Financial Studies* 9(1): 141–161. http://dx.doi.org/10.1093/rfs/9.1.141

Liu, J. 2006. *Portfolio selection in stochastic environments.* Oxford, UK: Oxford University Press.

Lynch, A. W. 2001. Portfolio choice and equity characteristics: characterizing the hedging demands induced by return predictability, *Journal of Financial Economics* 62: 67–130.
http://dx.doi.org/10.1016/S0304-405X(01)00074-5

Markowitz, H. M. 1952. Portfolio selection, *Journal of Finance* 7(1): 77–91.

Merton, R. C. 1969. Lifetime portfolio selection under uncertainty: the continuous-time case, *The Review of Economics and Statistics* 51(3): 247–257. http://dx.doi.org/10.2307/1926560

Merton, R. C. 1973. An intertemporal capital asset pricing model, *Econometrica* 41(5): 867–887.
http://dx.doi.org/10.2307/1913811

Moody, Lynn & Lieberson, Inc. 2009 Investment review and outlook, Investment council [online], [cited 20 July 2011]. Available from Internet: www.moodylynn.com/investment_outlook/4Q_2008.pdf

Rapach, D. E.; Wohar, M. E. 2009. Return predictability and the implied intertemporal hedging demands for stocks and bonds: international evidence, *Journal of International Money and Finance* 28: 427–453. http://dx.doi.org/10.1016/j.jimonfin.2008.12.004

Samuelson, P. A. 1969. Lifetime portfolio selection by dynamic stochastic programming, *The Review of Economics and Statistics* 51(3): 239–246. http://dx.doi.org/10.2307/1926559

Su, Y.; Lau, M. C. K. 2010. Strategic asset allocation and intertemporal hedging demands: with commodities as an asset class, *MPRA Paper* 26337. University Library of Munich, Germany.

Tobin, J. 1958. Liquidity preference as behaviour towards risk, *Review of Economic Studies* 25(67): 65–86. http://dx.doi.org/10.2307/2296205

# APPENDIX

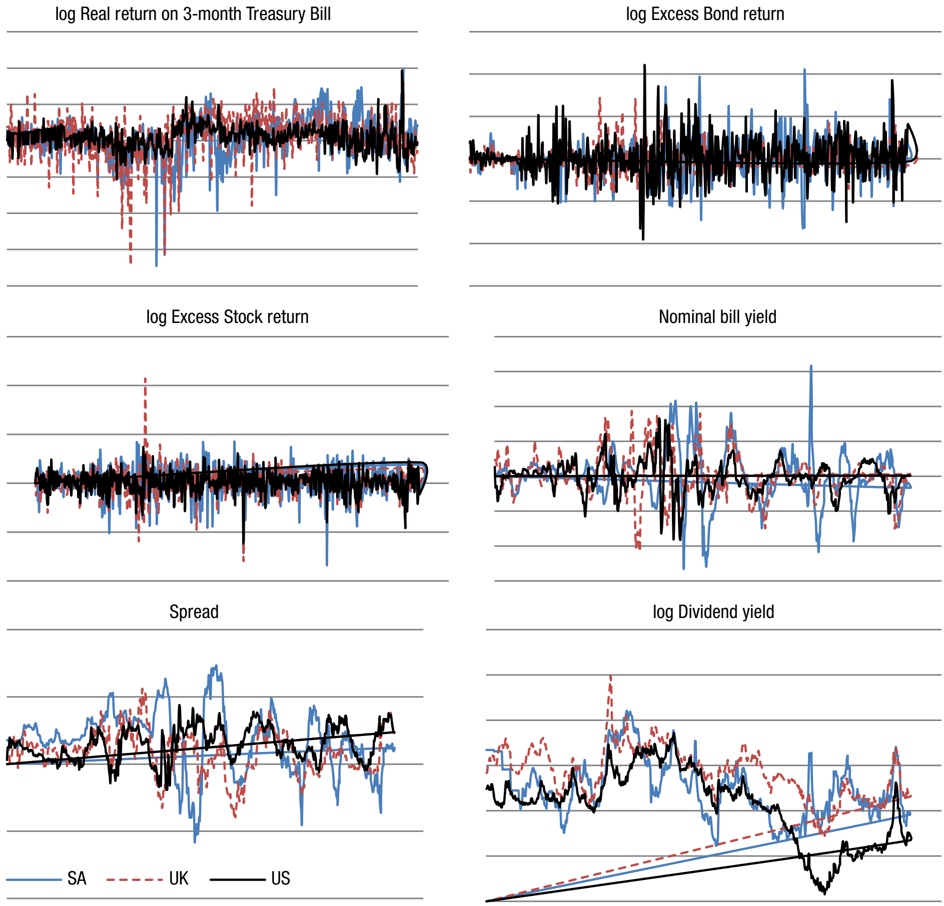


**Fig. A.** Variables included in the vector of state variables

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