



ANALYSIS OF EARTH'S MAGNETIC FIELD CHANGES BY CORRELATION OF ITS PARAMETERS

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Abstract. Changes of Earth magnetic field parameters: field intensity and its components along axes of rectangular coordinates are analyzed in space and time. Data from magnetic observatories Belsk and Hel in Poland were used. Correlation analysis of magnetic field components was applied. Influence of respective field components on changes of geomagnetic field intensity was determined. Shift of space and time factors was used to assess the influence of covariations of geomagnetic field components on changes of covariations of field intensity.

Keywords: parameters of Earth's magnetic field and their correlation, changes of covariations of field intensity.

1. Introduction

Earth's magnetic field is a constantly changing physical field. The frequency of parameter alterations of this geomagnetic field is not stable – from fractions of a second to year-long or even longer periods, called secular variations. Information about changes of geomagnetic field, which took place between corresponding measurement epochs, are required to process the data of magnetic measurements (Czyszek, A.; Czyszek J. 2002; Marianiuk, Reda 2001; Sas-Uhrynowski *et al.* 2000; 2002). Such information is gathered in magnetic observatories.

In this paper the covariation of geomagnetic field intensity components and its influence on field intensity covariations changes is analyzed. Correlational dependence between changes of geomagnetic field intensity in time and space was determined by employing the variation of covariation of field intensity components in time. Equations were formed to calculate estimates of covariation matrixes of field intensity components using data of magnetic measurements. Accuracy of respective calculated parameters was assessed.

2. Covariation of magnetic measurements

Results of magnetic measurements: intensity of geomagnetic field and its components correlate due to the magnetic field fluctuation. Thus errors of measurement results depend on the magnetic field turbulence.

Covariation between intensities of geomagnetic field $F(t)$ determined during different epochs t_i and t_s equals (Skeivalas 2007, 2008):

$$K\{F(t_i), F(t_s)\} = M\{\delta F(t_i) \cdot \delta F(t_s)\} = M\{\delta F(t_i) \cdot h\delta F(t_i)\} = h\sigma_{F(t_i)}^2, \tag{1}$$

here K, M – symbols of covariation and mean, $\delta F(t_i) = F(t_i) - MF(t_i)$ – random error of field intensity due to the influence of magnetic field fluctuation, $\delta F(t_s) = h\delta F(t_i)$, h – coefficient, showing the change of $F(t)$ covariation when transiting from epoch t_i to epoch t_s ; $\sigma_{F(t_i)}$ – standard deviation. Measurement moments of time are considered as epochs.

Standard deviations $\sigma_{F(t_i)}$ of geomagnetic field intensity $F(t_i)$ can be expressed using standard deviations of its components F_x, F_y, F_z . Directions of components F_x and F_y match the directions of planar coordinate axes, and direction of F_z match the vertical direction. Therefore we can write

$$F(t) = \sqrt{F_x^2(t) + F_y^2(t) + F_z^2(t)},$$

and when expanding into series we receive

$$F(t) = F_0 + \frac{F_x(t)}{F(t)} \delta F_x + \frac{F_y(t)}{F(t)} \delta F_y + \frac{F_z(t)}{F(t)} \delta F_z + R,$$

or

$$\delta F(t) = a_x \delta F_x(t) + a_y \delta F_y(t) + a_z \delta F_z(t), \quad (2)$$

here F_0 – initial value of $F(t)$, R – sum of non-linear members of the series, which approaches zero; a_x, a_y, a_z – values of partial derivatives of $F(t)$ along respective components, calculated by applying known values of the components.

According to formula (2), we receive:

$$\sigma_F^2 = \frac{F_x^2}{F^2} \sigma_{F_x}^2 + \frac{F_y^2}{F^2} \sigma_{F_y}^2 + \frac{F_z^2}{F^2} \sigma_{F_z}^2 = a_x^2 \sigma_{F_x}^2 + a_y^2 \sigma_{F_y}^2 + a_z^2 \sigma_{F_z}^2. \quad (3)$$

Provided that $\sigma_{F_x} \approx \sigma_{F_y} \approx \sigma_{F_z} \approx \sigma_0$, we write $\sigma_F \approx \sigma_0$, since $a_x^2 + a_y^2 + a_z^2 = 1$.

3. Coordinate expressions of covariations

Here we will write the formula of covariation of geomagnetic field intensity $F(t)$ using its component expression:

$$K\{F(t_i), F(t_s)\} = K\{\delta F(t_i), \delta F(t_s)\} = M\left\{\begin{pmatrix} a_{x_i} \delta F_{x_i} + a_{y_i} \delta F_{y_i} + a_{z_i} \delta F_{z_i} \\ a_{x_s} \delta F_{x_s} + a_{y_s} \delta F_{y_s} + a_{z_s} \delta F_{z_s} \end{pmatrix}\right\} = M\left\{\begin{pmatrix} \delta F_{x_i} + \delta F_{y_i} + \delta F_{z_i} \\ \delta F_{x_s} + \delta F_{y_s} + \delta F_{z_s} \end{pmatrix}\right\}. \quad (4)$$

Values of coefficients $a_{x_i}, a_{y_i}, a_{z_i}$ ($i = 1, \dots, s$) are less than one, i.e. $a_{x_i} < 1, a_{y_i} < 1, a_{z_i} < 1$.

Then formula (4) obtains the following form:

$$K\{F(t_i), F(t_s)\} \leq K(F_{x_i} F_{x_s}) + K(F_{y_i} F_{y_s}) + K(F_{z_i} F_{z_s}) + 2K(F_{x_i} F_{y_s}) + 2K(F_{x_i} F_{z_s}) + 2K(F_{y_i} F_{z_s}); \quad (5)$$

here $K(F_{x_i} F_{y_s}) = K(F_{x_s} F_{y_i}), K(F_{x_i} F_{z_s}) = K(F_{x_s} F_{z_i}), K(F_{y_i} F_{z_s}) = K(F_{y_s} F_{z_i})$.

By considering, that covariation between components of $F(t)$ is approximately equal, we write:

$$K\{F(t_i), F(t_s)\} \leq 9(F_{\alpha_i}, F_{\alpha_s}), \quad (6)$$

here $K(F_{x_i}, F_{x_s}) \approx K(F_{y_i}, F_{y_s}) \approx \dots \approx K(F_{x_i}, F_{y_s}) \approx K(F_{\alpha_{i0}}, F_{\alpha_{s0}}), \alpha \rightarrow (x, y, z)^T$.

We use matrix expression of measurements results of components in s epochs to calculate the estimates of covariations of geomagnetic field intensity $F(t)$:

$$F_\alpha = (F_{\alpha_1}, F_{\alpha_2}, \dots, F_{\alpha_s}) = (F_\alpha(t_1), F_\alpha(t_2), \dots, F_\alpha(t_s)), \\ F_{\alpha_i} = (F_{x_i}, F_{y_i}, F_{z_i})^T, \quad i = 1, 2, \dots, s$$

and

$$F_\alpha = \begin{pmatrix} F_{x_1} & F_{x_2} & \dots & F_{x_s} \\ F_{y_1} & F_{y_2} & \dots & F_{y_s} \\ F_{z_1} & F_{z_2} & \dots & F_{z_s} \end{pmatrix}. \quad (7)$$

Estimate \bar{K}_{F_α} of covariation matrix of $F(t)$ component vector F_α along coordinates is equal

$$\bar{K}'_{F_\alpha} = \frac{1}{s} \delta F_\alpha \cdot \delta F_\alpha^T. \quad (8)$$

Here $\delta F_{\alpha_i} = (\delta F_{x_i}, \delta F_{y_i}, \delta F_{z_i})^T, \delta F_{x_i} = F_{x_i} - \frac{1}{s} \sum_1^s F_{x_i}, \delta F_{y_i} = F_{y_i} - \frac{1}{s} \sum_1^s F_{y_i}, \delta F_{z_i} = F_{z_i} - \frac{1}{s} \sum_1^s F_{z_i}, i = 1, 2, \dots, s$.

Expression (8) in expanded form is written as:

$$\bar{K}'_{F_\alpha} = \begin{pmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{pmatrix} = \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{pmatrix}, \quad (9)$$

here $K_{xx} = \sigma_x'^2, K_{yy} = \sigma_y'^2, K_{zz} = \sigma_z'^2, \sigma_x', \sigma_y', \sigma_z'$ – estimates of standard deviations.

Estimate K'_{F_α} of covariation matrix of $F(t)$ component vector F_α along epochs is calculated as:

$$K'_{F_\alpha} = \frac{1}{3} \delta F_\alpha^T \cdot \delta F_\alpha. \quad (10)$$

Formula (10) in expanded form is written as

$$K'_{F_\alpha} = \begin{pmatrix} K_{t_1 t_1} & K_{t_1 t_2} & \dots & K_{t_1 t_s} \\ K_{t_2 t_1} & K_{t_2 t_2} & \dots & K_{t_2 t_s} \\ \dots & \dots & \dots & \dots \\ K_{t_s t_1} & K_{t_s t_2} & \dots & K_{t_s t_s} \end{pmatrix}, \quad (11)$$

here $K_{t_i t_j} = K(F_{\alpha_i}, F_{\alpha_j}) = K\{F_\alpha(t_i), F_\alpha(t_j)\}$.

Estimate of covariation of geomagnetic field intensities $F(t_i)$ and $F(t_s)$, considering expressions (6) and (8), is not higher than the sum of members of covariation matrix K'_{F_α} (9):

$$K'\{F(t_i), F(t_s)\} \leq \sum_{i,j=1}^3 \bar{K}'_{F_{\alpha,ij}}. \quad (12)$$

By applying formulas (1), (3) and (12), we can write

$$h \sigma_0^2 \leq \sum_{i,j} \bar{K}'_{F_{\alpha,ij}} \quad (13)$$

and further

$$h \leq \frac{1}{\sigma_0^2} \sum_{i,j} \bar{K}'_{F_{\alpha,ij}}. \quad (14)$$

So the coefficient h indicates the relative change of covariations of geomagnetic field intensity, when transitioning from epoch t_i to epoch t_s , during the measurements.

According to the presented formulas, the computer program *Kormag.m* was developed using operators of software package *Matlab*. Data from Belsk and Hel magnetic observatory (October, 2000) was used to perform calculations. Results of measurements were registered

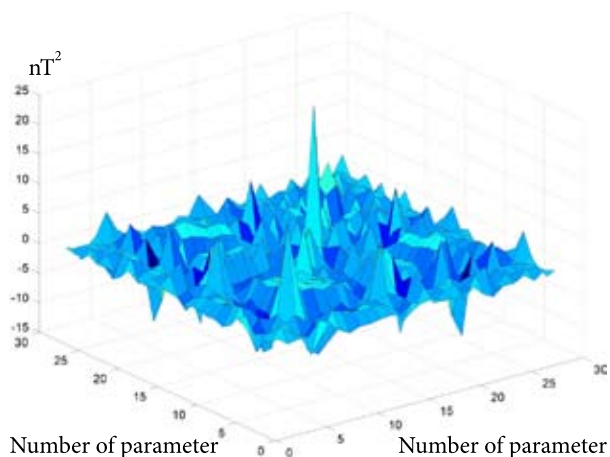


Fig. 1. Graphical view of covariation matrix of $F(t)$ vectors from Belsk2 and Hel30 observatories

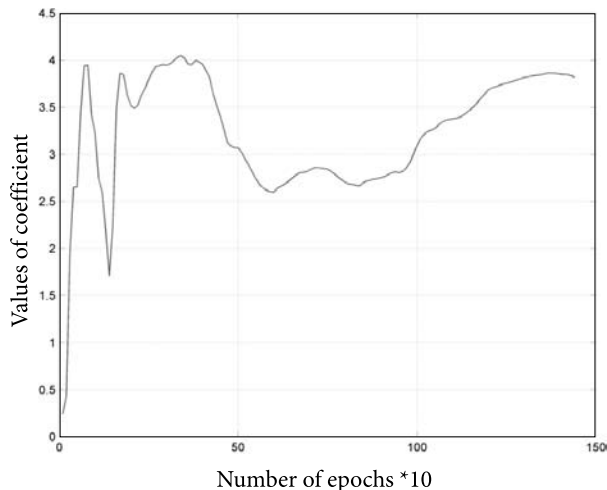


Fig. 4. Values of covariation variation coefficient h of vector $F(t)$ from Hel30 observatory

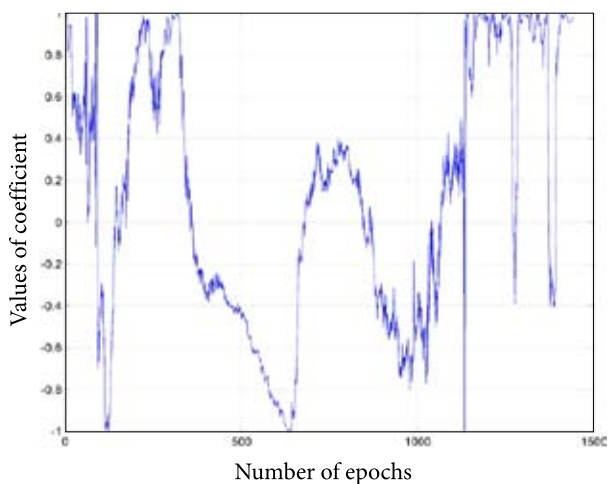


Fig. 2. Epoch-based variation of correlation coefficients of $F_x(t), F_y(t), F_z(t)$ vectors, calculated from 2nd day data from Belsk2 observatory

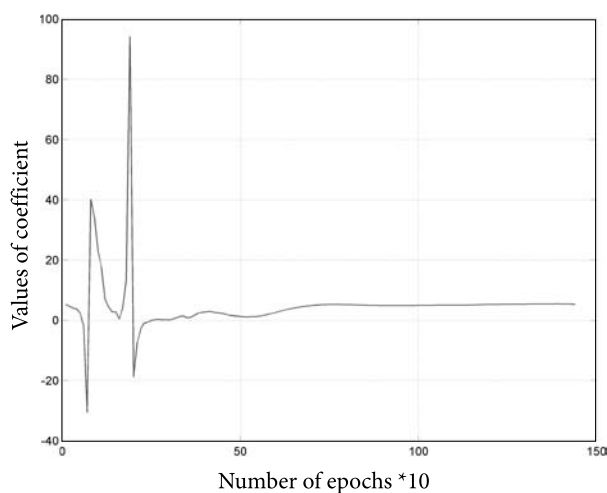


Fig. 5. Values of covariation variation coefficient h of $F(t)$ vectors from data of Belsk2 and Belsk30 observatories

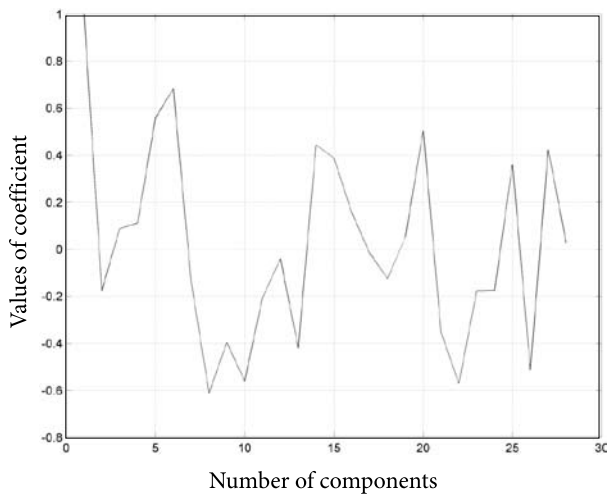


Fig. 3. Values of correlation coefficients of $F(t)$ vectors calculated each 100 epochs using data from Belsk2 and Hel30 observatories

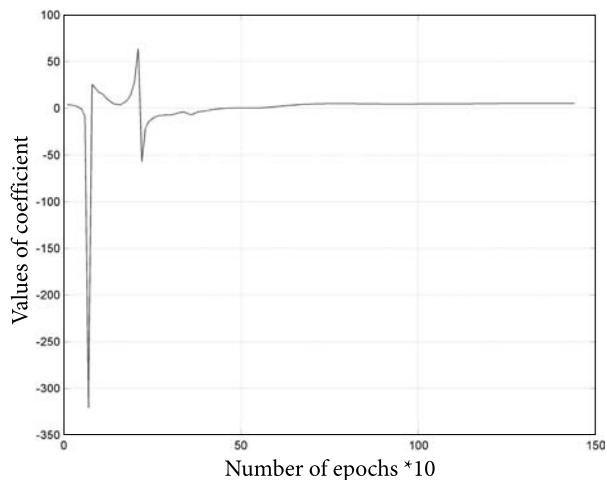


Fig. 6. Values of covariation variation coefficient h of $F(t)$ vectors from data of Belsk2 and Hel30 observatories

each minute. Summaries of the experiments are shown in diagrams.

General view of covariation matrixes of measurement data $F(t)$ vectors from Belsk2 and Hel2 observatories, when covariations were calculated each interval of 100 epochs (Fig. 1). Numbers included in the names of observatories indicate days of a month.

Epoch-based variation of correlation coefficients of $F_x(t)$, $F_y(t)$, $F_z(t)$ vectors, calculated from one day data from Belsk2 observatory is in Fig. 2.

Values of correlation coefficients of $F(t)$ vectors calculated each 100 epochs using data from Belsk2 and Hel30 observatories is in Fig. 3.

Values of covariation variation coefficient h of $F(t)$ vectors from observatory Hel30, calculated according to covariations of respective components $F_x(t)$, $F_y(t)$ and $F_z(t)$ each 10 epochs, are in Fig. 4.

Values of covariation variation coefficient h of $F(t)$ vectors from data of Belsk2 and Belsk30 observatories, calculated according to covariations of respective components $F_x(t)$, $F_y(t)$, $F_z(t)$ each 10 epochs are in Fig. 5.

Values of covariation variation coefficient h of $F(t)$ vectors from data of Belsk2 and Hel30 observatories, calculated according to covariations of respective components $F_x(t)$, $F_y(t)$, $F_z(t)$ each 10 epochs are in Fig. 6.

4. Conclusions

1. The results of magnetic field parameter measurement in single epochs strongly correlate due to turbulence of magnetic field and measurement errors, therefore covariation matrix should be used when processing them.
2. Formulas were derived for determining relative change of covariations of geomagnetic field intensity $F(t)$, when transiting from one epoch to other during measurements, using covariations of $F(t)$ components $F_x(t)$, $F_y(t)$, $F_z(t)$, determined according to single epochs.
3. It was found, according to the results of investigation, that the relative change of covariations of geomagnetic field parameter $F(t)$ of both observatories Belsk and Hel, according to epochs of own observatory, vary in the range of $1,5 < h < 4$, when this change is determined according to $F(t)$ components.
4. Relative change of covariations of magnetic field parameter $F(t)$, which was determined according to the data of different days of single observatory and according to the data of different observa-

tories is close to zero ($h \approx 0$), when components F_x , F_y , F_z are used to calculate $F(t)$. However, values of coefficient h significantly vary in initial epochs, apparently due to the small amount of available data.

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