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# BEHAVIOUR ANALYSIS AND ENGINEERING CALCULATION OF THE STRESS-RIBBON BRIDGE WITH EXTERNAL TENDONS

Giedrė Sandovič, Algirdas Juozapaitis

*Department of Bridges and Special Structures, Vilnius Gediminas Technical University, Saulėtekio al. 11, 10223 Vilnius, Lithuania*

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**Abstract.** The current article discusses a single-span composite steel pedestrian bridge, the load-bearing structure of which is comprised of a stress-ribbon with extended tendons. The analysis is focused on the behaviour of such structure under symmetric loads. The article reviews the general displacement thrust force of load-bearing suspension structures of such bridge as well as presents calculation analysis values of such displacements and the thrust force. In addition, it provides results of the numerical experiment and comparative analysis of the stress-ribbon structure with extended tendons.

**Keywords:** steel bridges, pedestrian bridges, suspension structures, flexible cable, stress-ribbon with extended tendons system, vertical displacements, stresses, comparative analysis.

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## Introduction

The stress-ribbon bridge is considered one of the most elegant and lightest pedestrian bridge structures. The main advantage of the suspension structure over the stress-ribbon bridge is that due to its static and dynamic behaviour peculiarities, stress-ribbon structures are used for pedestrians and cyclists rather than vehicle or railroad traffic (Strasky 2005; Schlaich, Bergerman 1992; Atanasovski, Markovski 2002). Stressed ribbon structures tend to have fixed cross-section, low initial sag ( $f_0 = \left(1/30 \div 1/60\right)L$ ) and shape similar to that of a square parabola. Span length $L$ and the initial sag $f_0$ are the main parameters of the stress-ribbon system. The rational choice of these parameters determines behaviour of the structure under symmetric and asymmetric loads (Schlaich *et al.* 2005; Strasky 2005)

It must be noted that stress-ribbon using steel bands bridge decks can be constructed from reinforced concrete or stone rafts, which are attached to load-bearing steel strips using regular joints. The choice of such heavy raft is determined by the wish to reduce displacements of the kinematic origin. The maximum sag of steel strips is set considering the exploitation requirements, so that the incline does not exceed 8% (Schlaich *et al.* 2005). The foundations of this flexible structure have to be built to withstand high tensile strains. Therefore, the massive foundations also determine the price of the stress-ribbon bridge. One of the main drawbacks of such suspension structures is their high deformability under asymmetric and local loads (Kulbach 1999; Katchurin 1969; Caetano, Cunha 2004; Troyano 2003; Schlaich *et al.* 1999; Redfield, Strasky 2002).

Recently, different construction solutions have been applied to reduce deformability of stress ribbon structures, among which suspension combined ribbon structures can be distinguished (Sandovič, Juozapaitis 2012). It has to be noted that lately, in order to increase

Corresponding author:
G. Sandovič E-mail: *giedre.sandovic@vgtu.lt*

efficiency of such systems, stress-ribbon structures with external tendons have been introduced.

The novel solution employed in stress-ribbon structures with external tendons allows reducing tensile (thrust) forces, thus reducing mass of such load-bearing structures, including cable foundations (Ogawa *et al.* 2006).

It should be noted that flexible cable is a curvilinear freely suspended pole without compressive strain, which is attached to the deck on both sides. Flexible cable is a theoretical concept, as any component has a certain cross-section height; therefore, it has a bending strain of a finite element (not equal to zero) (Moskalev 1981; Gimsing 1997; Furst *et al.* 2003; Juozapaitis *et al.* 2002, 2013; Kala 2012). It has to be noted that flexibly based elements are recommended for the design of the abovementioned suspension structures (Juozapaitis *et al.* 2006; Grigorjeva *et al.* 2004; Juozapaitis, Norkus 2004; Kulbach 2007). It is necessary to note that design and behaviour of stress-ribbon bridges with external tendons have not been properly researched so far.

The current article discusses a single span stress-ribbon with external tendons pedestrian steel bridge structure. It analyses the behaviour of such suspension structure under symmetric loads. In addition, it presents the engineering methodology for estimating the total displacement and the thrust force of supporting structures of such bridge. Besides, the article presents the numerical experiment results.

## 1. Engineering calculation of the stress-ribbon bridge with external tendons under the symmetric load

The main supporting element of the stress-ribbon pedestrian bridge with external tendons is calculated as a structure of geometrically nonlinear behaviour. The fact that general (total) cable displacements consist of kinematic and elastic displacements shall be taken into consideration (Tarvydaitė, Juozapaitis 2010).

Stress-ribbon with external tendons structure is a single span with different initial sags, two suspension cables attached to posts. A post is a straight steel joint with a solid cross-section area with $EI \approx 0$. The diagram of the stress-ribbon structure with external tendons is presented in Fig. 1.

Elastic cable is a kind of cable without bending stiffness *EI*. The flexible cable assumes square-parabola-like shape under the self-weight.

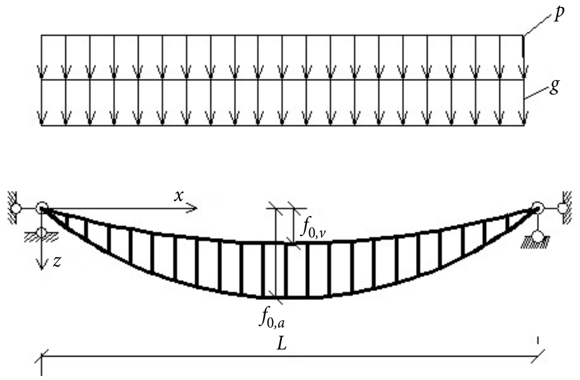


Fig. 1. Calculation diagram of the stress-ribbon with external tendons

Engineering (approximate) calculation methodology was developed for a stress-ribbon bridge with external tendons, using a flexible cable. The following calculation assumptions were assumed:

1. the shape of the stress-ribbon structure with external tendons is a square parabola;
2. posts are not affected by deformation; thus, the vertical displacement of both cables at any given layer of the structure are equal $\Delta f_v = \Delta f_a$.

An established deformations sustainability equation (Sandovič *et al.* 2011) was employed for determining the total displacements, when the length of the upper cable after elastic deformation $s_1$ equals:

$$s_1 = L + \frac{8 \cdot \left(f_{0,v} + \Delta f_v\right)^2}{3 \cdot L}, \tag{1}$$

where: $f_{0,v}$ – initial sag of the upper cable; $\Delta f_v$ – vertical displacement of the upper cable in the middle of the span; $L$ – span.

Algebraic calculations resulted in the expression for determining total upper cable displacements in the middle of the span:

$$\Delta f_v^2 + 2 f_{0,v} \cdot \Delta f_v - \frac{3 \cdot H_v \cdot L^2}{8 \cdot E \cdot A_v} = 0, \tag{2}$$

where the upper cable thrust force is:

$$H_v = \frac{p_v^* \cdot L^2}{8 \cdot \left(f_{0,v} + \Delta f_v\right)}, \tag{3}$$

where: $H_v$ – upper cable thrust force; $A_v$ – upper cable cross-section area; $p_v^*$ – upper cable load.

Having inserted Equation (3) into (2), a solution for the vertical displacement of the upper cable square equation was received:

$$\Delta f_v = -f_{0,v} + \sqrt{f_{0,v}^2 + \frac{3 \cdot L^4 \cdot p_v^*}{64 \cdot E_v \cdot A_v}}, \tag{4}$$

where: $f_{0,a}$ – initial sag of the lower cable; $A_a$ – lower cable cross-section area; $\Delta f_a$ – vertical displacement of the lower cable in the middle of the span.

Analogically, an equation for estimating the vertical displacement of the lower cable is received:

$$\Delta f_a = -f_{0,a} + \sqrt{f_{0,a}^2 + \frac{3 \cdot L^4 \cdot p_a^*}{64 \cdot E_a \cdot A_a}}. \quad (5)$$

Constant and periodic loadings affecting the stress-ribbon structure with external tendons are distributed between the upper and lower cables through the posts:

$$g + p = (g_v + p_v) + (g_a + p_a); \quad (6)$$

$$g + p = p_v^* + p_a^*, \quad (7)$$

where: $g$ – constant loading; $g$ – periodic loading; $p_a^*$ – loading of the lower cable.

The parity of the upper and the lower cables' total displacements in the middle of the span, $\Delta f_v = \Delta f_a$, allows determining the load on the upper and lower cables:

$$p_v^* = \frac{(g+p) \cdot E_v A_v \cdot \left(f_{0,v} + \Delta f_v\right)^2}{E_a A_a \left(f_{0,a} + \Delta f_a\right)^2 + E_v A_v \left(f_{0,v} + \Delta f_v\right)^2}; \quad (8)$$

$$p_a^* = \frac{(g+p) \cdot E_a A_a \cdot \left(f_{0,a} + \Delta f_a\right)^2}{E_a A_a \left(f_{0,a} + \Delta f_a\right)^2 + E_v A_v \left(f_{0,v} + \Delta f_v\right)^2}, \quad (9)$$

$p_a^*$ was expressed from expression (7):

$$p_a^* = g + p - p_v^*. \quad (10)$$

The thrust force of the lower cable is:

$$H_a = \frac{p_a^* \cdot L^2}{8 \cdot \left(f_{0,a} + \Delta f_a\right)}. \quad (11)$$

The calculations are carried out by means of approximation (iterations). In the first iteration, in Equations (8) and (9), $\Delta f_v = \Delta f_a = 0$. As vertical displacements of the upper and lower cables are equal ($\Delta f_v = \Delta f_a$), only the vertical displacement of the upper cable $\Delta f_v$ is estimated. The share of loading on the upper cable equals:

$$p_v^* = \frac{(g+p) \cdot E_v A_v \cdot \left(f_{0,v} + \Delta f_v\right)^2}{E_a A_a \left(f_{0,a} + \Delta f_a\right)^2 + E_v A_v \left(f_{0,v} + \Delta f_v\right)^2}. \quad (12)$$

The iteration calculations are carried out until the vertical displacements received in the final iteration suffice the selected accuracy condition.

The obtained calculation results were presented in Figs 2–4.

## 2. Comparative analysis

The comparative analysis of the stress-ribbon structure with external tendon calculations based on the engineering (approximate) methodology was carried out. The span of the structure is 40 m, initial sag of the upper cable $f_{0,v} = 0.8$ m, initial sag of the lower cable $f_{0,a} = 4$ m.

The cables with initial sag are described according to the square parabola. Three variants of the suspended stress-ribbon structure with external tendons were considered: 1) when cross-section area of the upper and lower cables is equal, i.e. $A_v = A_a = 30$ cm$^2$; 2) when cross-section area of the upper cable is twice as big as that of the lower cable, i.e. $A_v = 40$ cm$^2$, and $A_a = 20$ cm$^2$; 3) when the cross-section area of the lower cable is twice as big as that of the upper cable i.e. $A_v = 20$ cm$^2$, and $A_a = 40$ cm$^2$. Finite element software Cosmos/M was employed for the numerical analysis. The diagram of the stress-ribbon structure with external tendons is presented in Fig. 2. The calculation results were presented in Figs 3–5.

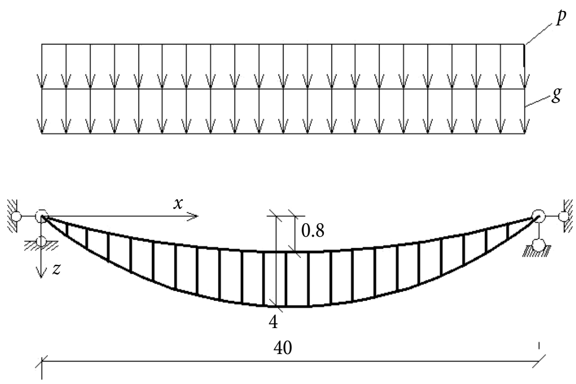


Fig. 2. Calculation diagram of the stress-ribbon structure with external tendons

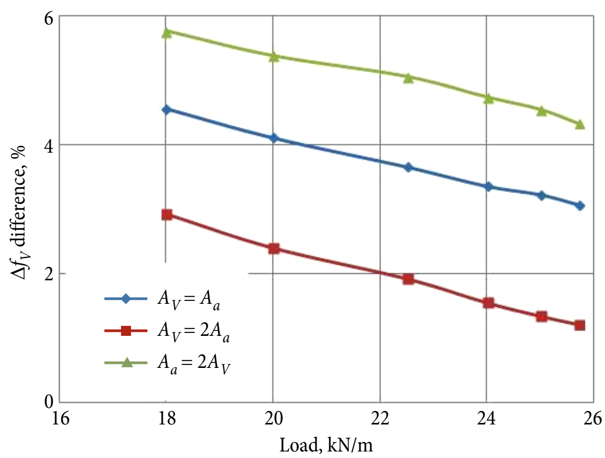


Fig. 3. Percentage difference of the upper cable vertical displacement calculations in the middle of the stress-ribbon structure with external tendon span

As indicated in Fig. 3, the difference between the vertical displacements received using FEM program *Cosmos/M* and according to engineering formulas does not exceed 6%. The highest errors were obtained when the cross-section area of the lower cable of the stress-ribbon structure with external tendons was twice bigger than that of the upper cable, i.e. $A_v = 20$ cm$^2$, and $A_a = 40$ cm$^2$. It has to be noted that with the growth of symmetric load, the errors decrease.

The differences of the upper and lower cable thrust force in the stress-ribbon with external tendons, differences between values established using FEM program *Cosmo/M* and estimated using engineering formulas are presented in Figs 4 and 5. Accordingly, thrust force values determined using FEM program *Cosmos/M* and estimated according to engineering formulas practically coincide. The highest errors (about 2%) were obtained when calculating thrust force values of the upper cable.

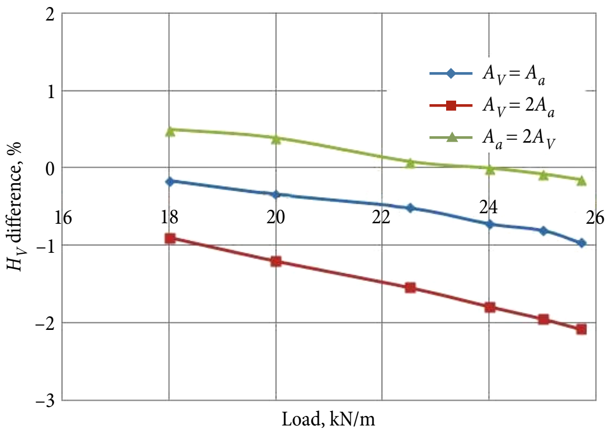


Fig. 4. The difference between thrust force calculations of the upper cable in the middle of the stress-ribbon structure with external tendon span in percent

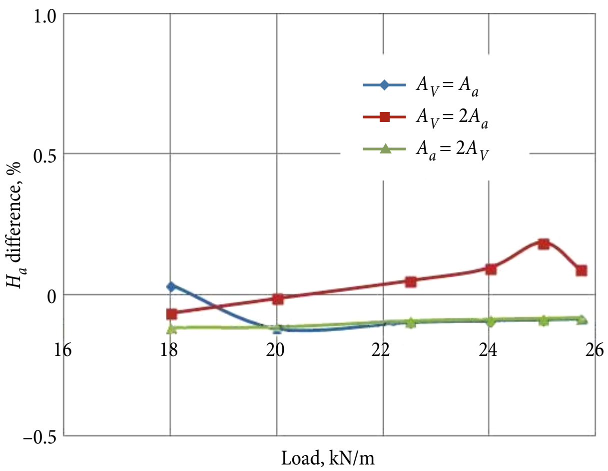


Fig. 5. The difference between thrust force calculations of the lower cable in the middle of the stress-ribbon structure with external tendon span in percent

The comparative analysis of calculations revealed sufficient accuracy of the presented engineering methodology.

## 3. Comparative behaviour analysis of the stress-ribbon bridge with external tendons

The comparative behaviour analysis of the stress-ribbon and stress-ribbon with external tendons under symmetric loading (Fig. 1) was carried out (Fig. 6).

Each bearing cable of the span consisted of linear finite elements. Evenly distributed load was replaced with added concentrated force in points (nods). In order to maintain the same initial rise of the deck, the initial upper cable sag of the stress-ribbon bridge with external tendons was considered equal to the initial sag of the stress-ribbon, i.e. $f_{0,v} = f_0 = 0.8$ m, whereas sag of the lower cable equalled $f_{0,a} = 4.0$ m. Posts were deployed every 2 m. The overall cross-section area of the stress-ribbon bridge with external tendons is equal to the stress-ribbon using steel bands cross-section area, i.e. $A_v + A_a = A = 60$ cm$^2$ – the same weight of the structures is maintained.

The obtained calculation results were presented in Figs 7 and 8.

As indicated in Fig. 7, vertical displacements of the stress-ribbon bridge with external tendons are approximately four times lower when the cross-section area of the upper and lower cables is the same. It has

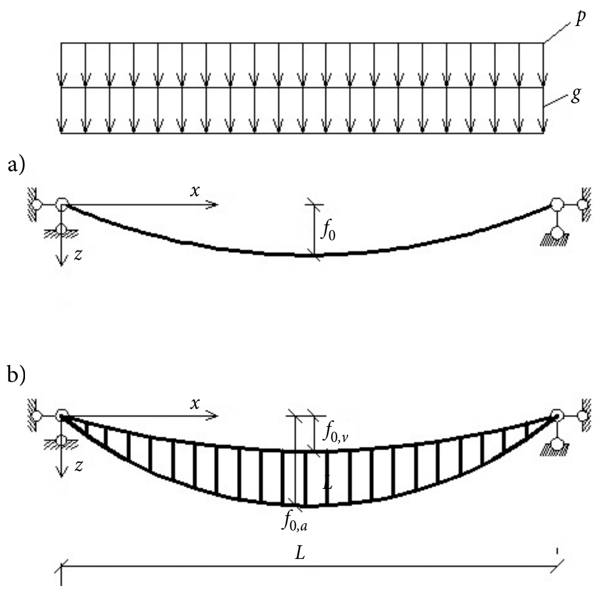


Fig. 6. Calculation diagram of the suspension structure: a) stress-ribbon using steel bands; b) stress-ribbon with external tendons

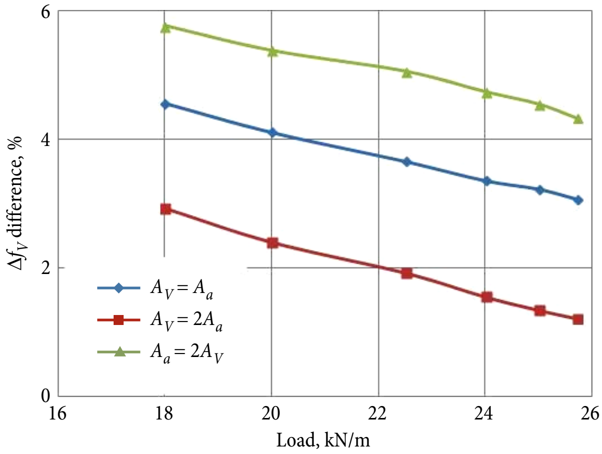


Fig. 7. The difference of the vertical displacement (in times) in the middle of the stress-ribbon structure and stress-ribbon structure with external tendon span

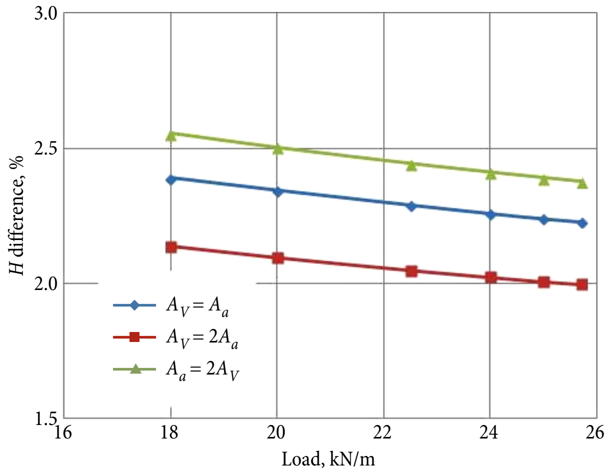


Fig. 8. The difference of the thrust force of the stress-ribbon structure and stress-ribbon structure with external tendons (in times)

to be noted that with the increase in symmetric loading, the efficiency of the stress-ribbon structure with external tendons is insignificantly reduced.

While conducting comparative thrust force analysis of the stress-ribbon bridge using steel bands and stress-ribbon bridge with external tendons, the sum of upper and lower cable thrust force of the stress-ribbon bridge with external tendons is considered, i.e. $H_v + H_a$. Fig. 8 reveals that the thrust force of stress-ribbon structures with external tendons is twice lower than stress-ribbon using steel bands.

## Conclusions

1. The conducted behaviour analysis of stress-ribbons using steel bands and stress-ribbons with external tendons revealed that the thrust force in the stress-ribbon with external tendons is twice lower than in stress-ribbons using steel bands. The total vertical displacement in the middle of the span is three times lower in the stress-ribbons with external tendons than in the stress-ribbons using steel bands.
2. The comparative analysis of the engineering calculation methodology revealed that accuracy of the engineering calculation methodology is sufficient.

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## DVIJUOSČIO LANKSTAUS KABAMOJO PLIENO TILTO ELGSENOS ANALIZĖ IR INŽINERINIS SKAIČIAVIMAS

**G. Sandovič, A. Juozapaitis**

**Santrauka.** Straipsnyje aptariama vieno tarpatramio pėsčiųjų kombinuotų tiltų lanksti dvijuostė kabamoji konstrukcija. Analizuojama tokios konstrukcijos elgsena veikiant simetrinėms apkrovoms. Apžvelgiami tokių tiltų laikančiųjų kabamųjų konstrukcijų bendrieji poslinkiai ir skėtimo jėgos, pateikiamos tokių poslinkių ir skėtimo jėgų skaičiavimo analizinės išraiškos. Pateikiami skaitinio eksperimento rezultatai, bei vienajuostės ir dvijuostės kabamųjų konstrukcijų lyginamoji analizė.

**Reikšminiai žodžiai:** tiltai, kabamosios konstrukcijos, vertikalūs poslinkiai, lankstus lynas, lyginamoji analizė.

**Giedrė SANDOVIČ.** Assistant at the Department of Bridges and Special Structures, Vilnius Gediminas Technical University (VGTU), Lithuania. BSc (2006, Civil Engineering) and MSc (2008, Civil Engineering) received at VGTU. Research interests: non-linear analysis of the cables, optimal structural design.

**Algirdas JUOZAPAITIS.** Associate Professor at the Department of Bridges and Special Structures, Vilnius Gediminas Technical University (VGTU), Lithuania. PhD at VGTU. Research interests: steel bridges and special steel structures, optimal shape determining of structures, geometrical non-linear analysis of structures.