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# THE USE OF A MODIFICATION OF THE HURWICZ'S DECISION RULE IN MULTICRITERIA DECISION MAKING UNDER COMPLETE UNCERTAINTY

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**Abstract.** The paper concerns multicriteria decision making under uncertainty with scenario planning. This topic is explored by many researchers because almost all real-world decision problems have multiple conflicting criteria and a deterministic criteria evaluation is often impossible (e.g. mergers and acquisitions, new product development). We propose two procedures for uncertain multi-objective optimization (for dependent and independent criteria matrices) which are based on the SAPO method – a modification of the Hurwicz's rule for one-criterion problems, recently presented in another paper. The new approaches take into account the decision maker's preference structure and attitude towards risk. It considers the frequency and the level of extreme evaluations and generates logic rankings for symmetric and asymmetric distributions. The application of the suggested tool is illustrated with an example of marketing strategies selection.

**Keywords:** multicriteria decision making, optimization, uncertainty, scenario planning, one-shot decision, attitude towards risk, criteria weights, normalization technique, SAPO method, marketing strategies selection, business objectives.

**JEL Classification:** C44, C61, D81, L21, L25, M31.

## 1. Introduction

The paper deals with multiple criteria decision making for cases where attribute (criterion) evaluations are uncertain. This topic is investigated by many researchers. (Durbach, Stewart 2012) provide an impressive review of possible models, methods and tools used to support uncertain multicriteria decision making (e.g. models using probabilities or probability-like quantities, models with explicit risk measures, models with fuzzy numbers, models with scenarios). The number of various contributions devoted to uncertain multiobjective optimization is evidence of the theoretical (Dominiak 2009; Eiselt, Marianov 2014; Janjic *et al.* 2013; Michnik 2013b) and practical (Aghdaie *et al.* 2013; Ginevičius, Zubrecovas 2009; Hopfe *et al.* 2013; Korhonen 2001; Lee 2012;

Michnik 2013b; Mikhaidov, Tsvetinov 2004; Montibeller *et al.* 2006; Ram *et al.* 2010; Suo *et al.* 2012; Tsaur *et al.* 2002; Wang, Elhag 2006; Watkins *et al.* 2000) importance of this topic. In this contribution two new procedures (for dependent and independent criteria matrices) designed for multicriteria decision making with scenario planning are presented. The goal of these approaches is to select optimal decisions and generate rankings of alternatives for one-shot decision problems. The proposed methods take into consideration decision makers' preferences and attitude towards risk.

The paper is organized as follows. Section 2 deals with the main features of DMU (decision making under uncertainty) with scenario planning. Section 3 briefly describes the essence of multiobjective decision making (MDM) and its discrete version (DMDM). Section 4 is devoted to methods applied to multicriteria decision making under uncertainty with scenario planning (MDMU+SP). Section 5 demonstrates how the SAPO method, a modification of the Hurwicz's decision rule which is presented in (Gaspars-Wieloch 2014d), can be used as a tool in multicriteria optimization under uncertainty. Section 6 provides a case study related to the choice of an optimal marketing strategy. Conclusions are gathered in the last part.

## 2. Decision making under uncertainty with scenarios

According to the Knightian definition (Knight 1921), decision making under uncertainty (DMU), in contrast to decision making under certainty (DMC) or risk (DMR), is characterized by a situation where future factors (quantitative and qualitative) are neither deterministic nor probabilistic at the time of the decision (complete uncertainty, uncertainty without probabilities). Actually the decision maker (DM) has to choose the appropriate alternative (decision, act, project, strategy) on the basis of some scenarios (events, states of nature) whose probabilities are not known (Chronopoulos *et al.* 2011; Dominiak 2006; Groenewald, Pretorius 2011; Render *et al.* 2006; Sikora 2008; Trzaskalik 2008; von Neumann, Morgenstern 1944). Apart from DMU, DMR and DMC, there is a forth category – decision making with partial information (DMPI) – characterized by probability distributions not known completely (Cannon, Kmietowicz 1974; Dubois, Prade 1988; Guo 2011; Kapsos *et al.* 2014; Klir, Folger 1988; Kmietowicz, Pearman 1984; Kofler, Zwiefel 1993; Michalska 2014; Weber 1987).

There are many classical decision rules designed for DMU, such as the Wald's criterion (Wald 1950), the maximax criterion presented e.g. in (Pazek, Rozman 2009), the Hurwicz's criterion (Hurwicz 1952), the Savage's criterion (Savage 1961), the maximin joy criterion (Hayashi 2008), the Bayes (Laplace's) criterion presented e.g. in (Render *et al.* 2006). The literature also offers many diverse extensions or hybrids of those methods, e.g. (Basili 2006; Basili *et al.* 2008; Basili, Chateauneuf 2011; Ellsberg 2001; Etner *et al.* 2012; Gaspars 2007; Gaspars-Wieloch 2012, 2013, 2014a, 2014c, 2014d, 2014e, 2015; Ghirardato *et al.* 2004; Gilboa 2009; Gilboa, Schmeidler 1989; Marinacci 2002; Piasecki 1990; Schmeidler 1986; Tversky, Kahneman 1992). Nevertheless, the majority of the extended rules refer to the probability calculus (for instance expected utility maximization, maximin expected utility, α-maximin expected utility, restricted Bayes/Hurwicz, prospect theory, cumulative prospect theory, Choquet expected utility), which is rather characteristic of DMR (or DMU with probabilities). Let us remind that according to the Knight's definition the uncertainty occurs when we do not know (i.e. we can not measure) the probabilities of particular scenarios: "when the uncertainty may be measured, it is called "risk" (Knight 1921).

As it was mentioned before, scenario planning (SP) can be used within the framework of DMU (Dominiak 2006; Pomerol 2001). SP is a popular and comprehensive decision support tool for considering future uncertainties. It is a technique for facilitating the process of identifying uncertain and uncontrollable factors which may influence the effects of decisions in the strategic management context. The way how scenarios should be constructed is described e.g. in (Dominiak 2006; Van der Heijden 1996).

The result of the choice made under uncertainty with scenario planning depends on two factors: which decision will be selected and which scenario will occur. DMU can be presented by means of a payoff matrix where $m$ (the number of rows) denotes the number of mutually exclusive scenarios ($S_1$, …, $S_i$, … $S_m$), $n$ (the number of columns) stands for the number of decisions ($D_1$, …, $D_j$, …, $D_n$) and $a_{ij}$ is the profit connected with scenario $S_i$ and alternative $D_j$. We assume in this paper that the distribution of payoffs related to a given decision is discrete and that the set of those profits can be a multiset (a bag). The contribution concerns searching an optimal pure strategy. A pure strategy is a solution assuming that the DM chooses and completely executes only one alternative. Meanwhile the mixed strategy allows him or her to select and perform a weighted combination of several accessible alternatives (Gaspars-Wieloch 2014b; Officer, Anderson 1968; Puppe, Schlag 2009; Sikora 2008).

We recognize both uncertainties: internal (related to DM's values and judgments) and external (related to imperfect knowledge concerning consequences of action), but in this paper we focus on the latter (Durbach, Stewart 2012; Stewart 2005).

Existing decision rules differ one from another with respect to the DM's attitude towards risk which can be measured, for instance, by means of the coefficient of pessimism ($\alpha$) or the coefficient of optimism ($\beta$). Note that in this context we do not treat risk as a situation where the probability distribution of each parameter of the decision problem is known, but we mean the possibility that some bad circumstances will happen (Dominiak 2006, 2009; Fishburn 1984).

It is worth emphasizing that some rules find application when the DM intends to perform the selected strategy only once, i.e. the decision is experienced only once (e.g. Wald's criterion, Hurwicz's criterion, Savage's criterion, maximax criterion, maximin joy criterion). Others are recommended for people contemplating realization of the chosen variant many times (Laplace's criterion). In the first case, alternatives are called one-shot (one-time) decisions. In the second case, one deals with multi-shot decisions.

This paper focuses on one-shot decision problems which are commonly encountered in business, economics and social systems and have been receiving increasing research interest because of the growing dominance of service industries for which such problems are particularly applicable. Typical examples of one-shot decisions include mergers and acquisitions (M&A), emergency management for irregular events, supply chain management of products with short life cycles, new product development and private real-estate investment (Guo 2010, 2011, 2013, 2014; Liu, Zhao 2009).

## 3. Multiobjective (multicriteria) decision making

The multiobjective optimization or multiple objective decision making (MDM) is a topic discussed in many contributions because decision making is a part of our life and almost all real-world decision problems have multiple conflicting criteria. Usually those problems do not have a single global solution. Theoretical issues concerning multigoal optimization are comprehensively treated for instance in (Ehrgott 2005; Jahn 2004; Marler, Arora 2004; Michnik 2013b; Sawaragi *et al.*1985; Trzaskalik 2014).

MDM has diverse goals, such as selecting a preferred alternative, ranking alternatives from the best to the worst, sorting the alternatives into ordered classes such as "good" and "bad" (Durbach, Stewart 2012). There are various methods enabling one to find the set of satisfactory solutions and the compromise solution (i.e. the final choice among efficient solutions) for multiobjective problems, such as AHP – analytic hierarchy process (Saaty 1980; Podvezko 2009), ANP – analytic network process (Saaty 1996; Azimi *et al.* 2011), COPRAS-G method (Zavadskas, Kaklauskas 1996), TOPSIS (Hwang, Yoon 1981; Azimi *et al.* 2011), SAW – simple additive weighting method (Churchman, Ackoff 1954), goal programming (Charnes *et al.* 1955), e-constraint method (Chankong, Haimes 1983), lexicographic method (Khorram *et al.* 2010), VIKOR-S (Michnik 2013b), WINGS (Michnik 2013a) and diverse hybrids (e.g. Haeri, Tavakkoli-Moghaddam 2012; Hsu 2014; Wu *et al.* 2013).

Multicriteria methods differ according to how they (a) evaluate performances on each attribute and (b) aggregate evaluations across attributes to arrive at an overall global evaluation (Durbach, Stewart 2012; Greco *et al.* 2010).

(Marler, Arora 2004) divide multi-objective optimization concepts and methods into three categories: (a) methods with a priori articulation of preferences (MPAP), (b) methods with a posteriori articulation of preferences (MPSAP) and (c) methods with no articulation of preferences (MNAP). In MPAP the user indicates the relative importance of the objective functions or desired goals (by means of parameters which are coefficients, exponents, constraint limits) before running the optimization algorithm. The preference structure can be defined, among other things, on the basis of aspirations levels (Lotfi *et al.* 1997), utility functions (Chang 2011) and weights representing the importance of each criterion (e.g. Gaspars-Wieloch 2011). MPSAP entail selecting a single solution from a set of mathematically equivalent solutions. It means that the DM imposes preferences directly on a set of potential final solutions.

Within MDM, the set of alternatives may be either explicitly defined and discrete in number or implicitly defined via constraints in a mathematical programming formulation (Ehrgott 2005). In the first case the problem is discrete (DMDM), in the second one – we deal with the continuous version of multicriteria decision making (CMDM). This contribution is devoted to DMDM. The discrete problem consists of $n$ decisions ($D_1$, …, $D_j$, …, $D_n$), each evaluated on $p$ criteria denoted by: $C_1$, …, $C_k$, …, $C_p$. There are also $n \times p$ evaluations of alternatives ($D_j$) in terms of particular criteria ($C_k$), according to some suitable performance measures. Let us denote them by $a_j^k$.

The multi-objective optimization may be analyzed for the deterministic (MDMC – multi-criteria decision making under certainty) or non-deterministic (MDMU, MDMR, MDMPI) case. In the second situation the evaluation of alternatives is complicated by their performance on at least some attributes not being known with certainty (Durbach, Stewart 2012). In this paper we consider the case of MDMU.

## 4. Multicriteria decision making with scenario planning

Many MDM models and methods are based on essentially deterministic evaluations of the consequences of each action in terms of each criterion, possibly subjecting final results and recommendations to a degree of sensitivity analysis. In many situations, such an approach can be justified when the primary source of complexity in decision making relates to the multicriteria nature of the problem rather than to the stochastic nature of individual consequences. Nevertheless, situations do arise, especially in strategic planning problems, when uncertainty is as critical as the issue of conflicting management goals. In such cases, approaches designed for MDMU become necessary, for instance (a) the multi-attribute utility theory, (b) pairwise comparisons of probability distributions, (c) the use of surrogate risk measures (quantiles, variances) as additional decision criteria, (d) models combining fuzzy numbers with the analytic hierarchy process, (e) fuzzy TOPSIS, and (f) the integration of MDMU and scenario planning (SP) (Ben Amor *et al.* 2007; Dominiak 2009; Durbach 2014; Durbach, Stewart 2012; Keeney, Raiffa 1993; Liu *et al.* 2011; Michnik 2013b; Stewart 2005; Triantaphyllou, Lin 1996; Urli, Nadeau 2004; Watkins *et al.* 2000; Xu 2000; Yu 2002; Zaras 2004). (Durbach, Stewart 2012) state that uncertainties become increasingly so complex that the elicitation of measures such as probabilities, belief functions or fuzzy membership functions becomes operationally difficult for DMs to comprehend and virtually impossible to validate. Therefore, in such contexts it is useful to construct scenarios which describe possible ways in which the future might unfold. Hence, the last example mentioned in the previous paragraph (MDMU+SP: multicriteria decision making under uncertainty with scenario planning) will be considered in this paper. When MDMU+SP is taken into account, the problem can be discrete (the number of possible decisions is finite and countable) or continuous (the set of decisions is given through constraints), but here, as already mentioned, we only discuss the discrete type.

The discrete version of MDMU+SP (DMDMU+SP) consists of *n* decisions ($D_1$, …, $D_j$, …, $D_n$), each evaluated on *p* criteria denoted by: $C_1$, …, $C_k$, …, $C_p$ and on *m,* scenarios ($S_1$, …$S_i$, … $S_m$). Hence, the problem can be presented by means of *p* payoff matrices (one for each criterion) and $p \times n \times m$ evaluations. Each payoff matrix contains $n \times m$ evaluations, say $a_{ij}^{\ k}$, which signify the performance of criterion $C_k$ resulting from the choice of decision $D_j$ and the occurrence of scenario $S_i$ .

According to (Durbach, Stewart 2012; Michnik 2013b) MDMU+SP models can be divided into two classes. The first one (A) includes two-stage models in which evaluations of particular alternatives are estimated in respect of scenarios and criteria in two separate stages. Class A contains two subclasses: A-CS and A-SC. Subclass A-CS denotes the set of approaches considering decisions separately in each scenario before and setting a $n \times m$ table giving the aggregated (over attributes/criteria) performance of alternative $D_j$ under scenario $S_{i.}$ These evaluations are then aggregated over scenarios. In subclass A-SC the order of aggregation is reversed – performances are generated across scenarios and then measures are calculated over criteria. The second class (B) consists of one-stage procedures considering combinations of scenarios and attributes (scenario-criterion pairs) as distinct meta-criteria. There is currently no consensus on the best way to solve uncertain multigoal problems.

Within the framework of the discrete multicriteria optimization with scenarios, researchers have already proposed, among others, the following techniques:

1. additive aggregation giving a scenario-based utility (Stewart 2005);
2. multiattribute value modelling (Goodwin, Wright 2001);
3. results aggregation over scenarios using a relative likelihood (Korhonen 2001);
4. maximization of the worst performance across scenarios (Montibeller *et al.* 2006; Ram *et al.* 2010);
5. dominance relation based on Wald's rule (Dominiak 2006);
6. hierarchy and quasi-hierarchy approach – when the DM is able to formulate his preferences in the form of order of criteria (Dominiak 2006);
7. distance function between alternatives and different reference points, such as the ideal pessimistic or ideal optimistic point – when the DM can describe weights of criteria (Dominiak 2006; Michnik 2013b);
8. interactive approach based on the Interactive Multiple Goal Programming and potency matrices with criteria evaluation of the ideal optimistic (max-max), ideal pessimistic (max-min) and current pessimistic (min-min) solution (Dominiak 2006);
9. interactive approach applying Monte Carlo simulation (Dominiak 2009);
10. combination of Hurwicz's rule with MDM (Ravindran 2008; Michnik 2013b).

Some of these methods are a little criticized – for instance the approach used by (Korhonen 2001) due to the fact that the set of scenarios does not constitute a complete probability space. According to (Durbach, Stewart 2012), one should not aggregate scenario "probabilities". In the case of dominance relation, hierarchy, quasi-hierarchy and distance function described in (Dominiak 2006) only the worst evaluations of particular alternatives are taken into account, which means that those procedures are rather devoted to a radical pessimist and that they do not fit the solution to the DM's attitude towards risk (understood as the possibility that some bad circumstances will happen). Furthermore, some rules assume that the occurrence of a given scenario with respect to criterion $C_k$ does not mean that this state of nature will be the true one in terms of another criterion (Ravindran 2008). In such approaches, evaluations from one payoff matrix do not depend on evaluations from other matrices (payoff matrices are totally independent), which is rather rarely found in real decision problems. On the other hand, interactive approaches proposed by (Dominiak 2006, 2009) are much desired since they are very flexible – they can be used without any a priori knowledge about DM's preferences and can also be applied when criteria are on the ordinal scale.

In Section 5, a new approach for generating rankings of decisions under uncertainty will be presented. We will notice that this procedure has many advantages. Firstly, it adjust the recommended solution not only to the DM's preference structure concerning particular criteria, but also to the DM's attitude towards risk (measured by the coefficient of pessimism). Secondly, if necessary, it allows to treat matrices related to particular attributes as dependent. Thirdly, it enables to make a multi-criteria analysis even in problems with objectives defined in different dimensions and scales. Fourthly, the new method copes with asymmetric distributions of (aggregated) evaluations.

## 5. The SAPO method for multicriteria decision making under uncertainty

The procedure proposed in this Section refers to a two-stage model (see Section 4). We will consider two cases (I and II). In the first case (Section 5.1.) we assume that payoff matrices related to particular attributes are dependent. Thus, for instance, evaluation $a_{ij}^{k}$ can be only connected with evaluations $a_{ij}^{1}$, …, $a_{ij}^{k-1}$, $a_{ij}^{k+1}$, …, $a_{ij}^{p-1}$ and $a_{ij}^{p}$. Those values describe the performance of each criterion by decision $D_j$ provided that scenario $S_i$ happens. There is no possibility that, for a given alternative, evaluations concerning particular criteria, come from different scenarios. This assumption implies the necessity of using A-CS model. In the second case (Section 5.2) we treat values related to one criterion as independent of evaluations of other criteria. That means that evaluation $a_{ij}^{k}$ may be connected with any evaluation $a_{ij}^{1}$ ($i$ = 1, …, $m$), any evaluation $a_{ij}^{2}$ ($i$ = 1, …, $m$), … and any evaluation $a_{ij}^{p}$ ($i$ = 1, …, $m$). Those values describe the performance of each criterion by decision $D_j$ assuming that any scenario occurs for criteria $C_1$, …, $C_{k-1}$, $C_{k+1}$, …, $C_{p.}$ The second case (II) allows us to apply A-SC model.

In both approaches, we will take advantage of an aggregate objective function (see SAW, Section 3) and a modification of the Hurwicz's decision rule, which is described and justified in (Gaspars-Wieloch 2014d). This modification is called "SAPO method". In contrast to the original version of the Hurwicz's criterion, the SAPO method copes with asymmetric distribution of payoffs. Even in that case, it recommends logic rankings and provides answers reflecting the DM's preferences. The procedure aforementioned is designed for one-criterion decision problems and its goal is to find an optimal pure strategy. The essence of the SAPO method is to multiply the coefficient of optimism by all payoffs belonging to a suitably defined range of good results (not only the highest payoff) and to multiply the coefficient of pessimism by all values belonging to a properly set range of bad results (not only the worst payoff), according to the level of risk aversion and on the basis of some additional bounds or deviation degrees. The SAPO rule takes into account both the level and the frequency of extreme values.

The first steps of the procedure combining SAW with SAPO are common in both cases (A-CS and A-SC):

1. Present the multicriteria problem by means of $p$ tables containing $n\times m$ evaluations $a_{ij}^k$ (where $i = 1, \ldots, m$ and $j = 1, \ldots, n$).
2. If criteria are defined in different scale or/and different units, use a normalization technique for each attribute separately (Equation 1 for maximized targets, Equation 2. for minimized targets), (Gaspars-Wieloch 2012, 2015). Otherwise, go to step 3.

$$a(n)_{ij}^k = \frac{a_{ij}^k - \min\limits_{\substack{i=1,\ldots,m\\ j=1,\ldots,n}}\left\{a_{ij}^k\right\}}{\max\limits_{\substack{i=1,\ldots,m\\ j=1,\ldots,n}}\left\{a_{ij}^k\right\} - \min\limits_{\substack{i=1,\ldots,m\\ j=1,\ldots,n}}\left\{a_{ij}^k\right\}} \quad k=1,\ldots,p;\ i=1,\ldots,m;\ j=1,\ldots,n\,; \tag{1}$$

$$a(n)_{ij}^k = \frac{\max\limits_{\substack{i=1,\ldots,m\\ j=1,\ldots,n}}\left\{a_{ij}^k\right\} - a_{ij}^k}{\max\limits_{\substack{i=1,\ldots,m\\ j=1,\ldots,n}}\left\{a_{ij}^k\right\} - \min\limits_{\substack{i=1,\ldots,m\\ j=1,\ldots,n}}\left\{a_{ij}^k\right\}} \quad k=1,\ldots,p;\ i=1,\ldots,m;\ j=1,\ldots,n\,. \tag{2}$$

3. Define weights for each target (criterion): $w^k$ ($k = 1, \ldots, p$) and declare the coefficient of pessimism ($\alpha$).

### 5.1. The SAPO procedure based on A-CS model (case I)

Let us analyze further steps of the SAPO(CS) approach – in the first place, criteria are aggregated within scenarios and then obtained values are calculated over scenarios:

4. Compute an aggregated measure for each pair of decision $D_j$ and scenario $S_i$ (for each combination ($D_j$,$S_i$)) and generate a table with $n\times m$ aggregated measures $A_{ij}$ or $A(n)_{ij}$ (Equation 3 for problems that do not require normalization, Equation 4 for problems with normalized evaluations):

$$A_{ij} = \sum_{k=1}^{p} w^k \cdot a_{ij}^k \quad i = 1,...,m; j = 1,...,n\,; \tag{3}$$

$$A(n)_{ij} = \sum_{k=1}^{p} w^k \cdot a(n)_{ij}^k \quad i = 1,...,m; j = 1,...,n\,. \tag{4}$$

5. Present the aggregated measures as a non-increasing sequence $Sq_j = (A_{1j},...,A_{sj},...,A_{mj})$ containing $m$ terms (where $m$ still denotes the number of scenarios, $s$ is the number of the term in the sequence and $A_{1j} > A_{mj}$) for each alternative $D_j$. If the normalization technique was applied in step 2, then, instead of values $A_{sj}$, use measures $A(n)_{sj}$ in step 5 and all further steps.
6. Generate the subsequence of good results ($SSq_j^{\max}$) and the subsequence of bad results ($SSq_j^{\min}$) for each alternative:

$$SSq_j^{\max} = \{A_{sj} \in Sq_j : (A_{1j} - d^{\max}(A_{1j} - A_{mj}) \le A_{sj} \le A_{1j}) \wedge \left(\left|SSq_j^{\max}\right| \le C\right) \wedge (A_{sj} \to \max)\} \quad j = 1,...,n\,, \tag{5}$$

$$SSq_j^{\min} = \{A_{sj} \in Sq_j : (A_{mj} \le A_{sj} \le A_{mj} + d^{\min}(A_{1j} - A_{mj})) \wedge \left(\left|SSq_j^{\min}\right| \le C\right) \wedge (A_{sj} \to \min)\} \quad j = 1,...,n\,, \tag{6}$$

where $d^{\max}$ and $d^{\min}$ signify the allowable degrees of deviation from the highest ($A_{1j}$) and the lowest ($A_{mj}$) aggregated measure, respectively. The deviation degrees can be set separately for each decision and then, instead of $d^{\max}$ and $d^{\min}$, parameters $d_j^{\max}$ and $d_j^{\min}$ are applied. The deviation degrees are set arbitrarily by the DM. These parameters ought to satisfy the following conditions:

$$d^{\max} + d^{\min} < 1\,, \tag{7}$$

$$d^{\max}, d^{\min} \ge 0\,. \tag{8}$$

$|SSq_j^{\max}|$ and $|SSq_j^{\min}|$ denote final cardinalities of both subsequences. $C$ is the maximal number of good and bad results computed according to constraint (9):

$$C = \max\{1, \lceil m \cdot \min\{\alpha, 1-\alpha\} \rceil\}\,. \tag{9}$$

Equation (5) allows the DM to include in subsequence $SSq_j^{\max}$ only the elements of the whole sequence which belong to the range determined by $d^{\max}$. For example, if $d^{\max} = 0.2$, $A_{1j} = 20$, $A_{mj} = 5$, then the elements of $SSq_j^{\max}$ should satisfy the following constraint $A_{sj} \in [20 - 0.2 \cdot (20 - 5); 20] = [17; 20]$. Note that the final cardinality of $SSq_j^{\max}$ is additionally limited by $C$ which depends on the pessimism and optimism indices. Closer to 0 and 1 they are, fewer elements subsequence $SSq_j^{\max}$ can contain. Such a relation may be explained by the fact that more radical the decision maker is, more likely, in his or her opinion, one of the extreme values is. If $m = 10$ and $\alpha = 0.2$, $SSq_j^{\max}$ may constist of at the most two elements which, due to the last part of Equation (5), must be the highest. Thanks to parameters $d^{\max}$ and $\alpha$ the DM is able to set a subsequence $SSq_j^{\max}$ which, from his or her point of view, is composed of appropriate values, because formula (5) considers both the subjective evaluation of good values and the DM's risk aversion. Equation (6) has an analogical interpretation.

7. Compute averages of good and bad results for each decision:

$$AV_j^{\max} = \frac{1}{\left|SSq_j^{\max}\right|} \sum_{A_{sj} \in SSq_j^{\max}} A_{sj} \quad j = 1,...,n, \tag{10}$$

$$AV_j^{\min} = \frac{1}{\left|SSq_j^{\min}\right|} \sum_{A_{sj} \in SSq_j^{\min}} A_{sj} \quad j = 1,...,n. \tag{11}$$

8. Calculate the SAPO(CS) measure ($S^{CS}_j$) for each alternative:

$$S_j^{CS} = \alpha \cdot \frac{m+1-\left|SSq_j^{\min}\right|}{m} AV_j^{\min} + (1-\alpha) \cdot \frac{m-1+\left|SSq_j^{\max}\right|}{m} AV_j^{\max} \quad j = 1,...,n. \tag{12}$$

Parameters $m, \left|SSq_j^{\min}\right|, \left|SSq_j^{\max}\right|$ inserted in condition (12) enable one to take into consideration the size of both subsequences, i.e. the frequency of extreme aggregated measures. $S^{CS}_j$ is proportional to the number of good values, i.e. the final cardinality of $SSq_j^{\max}$, and inversely proportional to the number of bad values, i.e. the final cardinality of $SSq_j^{\min}$, because a given alternative is more attractive when it contains many high aggregated measures and few low results. Fractions $\frac{m+1-\left|SSq_j^{\min}\right|}{m}$ and $\frac{m-1+\left|SSq_j^{\max}\right|}{m}$ are equal to 1 when particular subsequences consist of one term. If $|SSq_j^{\min}|$ increases, then the first fraction is less than 1, but greater than 0.5. Weight $\frac{m+1-\left|SSq_j^{\min}\right|}{m}$ is a kind of punishment for the alternative whose number of bad results is high because such a distribution of measures is not desirable for the decision maker. On the other hand, if $|SSq_j^{\max}|$ increases, then the second fraction is greater than 1, but less than 1.5. Weight $\frac{m-1+\left|SSq_j^{\max}\right|}{m}$ is a bonus for the alternative whose number of good results is high since such a value distribution is much desired.

9. Select the decision according to Equation (13):

$$D_j^{CS*} = \arg\max_j S_j^{CS}. \tag{13}$$

### 5.2. The SAPO procedure based on A-SC model (case II)

Now, let us check how the remaining steps of the SAPO(SC) procedure should be formulated. We assume that evaluations from particular matrices are independent.

4. Present evaluations as a non-increasing sequence $Sq_j^k = (a_{1j}^k,...,a_{sj}^k,...,a_{mj}^k)$ containing $m$ terms (where $m$ denotes the number of scenarios, $s$ is the number of the term in the sequence and $a^k_{1j} > a^k_{mj}$) for each alternative $D_j$ and within each crite-

rion. If the normalization technique was applied in step 2, then, instead of values $a^k_{sj}$, use measures $a(n)^k_{sj}$ in step 4 and all further steps.

5. Generate the subsequence of good results ($SSq_j^{k,\max}$) and the subsequence of bad results ($SSq_j^{k,\min}$) for each alternative and within each criterion:

$$SSq_j^{k,\max} = \{a_{sj}^k \in Sq_j^k : (a_{1j}^k - d^{\max}(a_{1j}^k - a_{mj}^k) \le a_{sj}^k \le a_{1j}^k) \wedge \left(\left|SSq_j^{k,\max}\right| \le C\right)$$
$$\wedge (a_{sj}^k \to \max)\} \quad k = 1,...,p; j = 1,...,n \,, \tag{14}$$

$$SSq_j^{k,\min} = \{a_{sj}^k \in Sq_j^k : (a_{mj}^k \le a_{sj}^k \le a_{mj}^k + d^{\min}(a_{1j}^k - a_{mj}^k)) \wedge \left(\left|SSq_j^{k,\min}\right| \le C\right)$$
$$\wedge (a_{sj}^k \to \min)\} \quad k = 1,...,p; j = 1,...,n \,, \tag{15}$$

where $d^{\max}$ and $d^{\min}$ signify the allowable degrees of deviation from the highest ($a^k_{1j}$) and the lowest ($a^k_{mj}$) evaluation, respectively. The deviation degrees can be set separately for each decision (or for each criterion / and for each criterion) and then, instead of $d^{\max}$ and $d^{\min}$, parameters $d_j^{\max}$ and $d_j^{\min}$ ($d^{k,\max}$ and $d^{k,\min}$ / $d_j^{k,\max}$ and $d_j^{k,\min}$) are applied. As in case I, the deviation degrees are set arbitrarily by the DM and satisfy Equations (7)–(8). $|SSq_j^{k,\max}|$ and $|SSq_j^{k,\min}|$ denote final cardinalities of both subsequences and $C$ is calculated following Equation (9).

6. Compute averages of good and bad results for each decision within each attribute:

$$av_j^{k,\max} = \frac{1}{\left|SSq_j^{k,\max}\right|} \sum_{a_{sj}^k \in SSq_j^{k,\max}} a_{sj}^k \quad k = 1,...,p; j = 1,...,n \,, \tag{16}$$

$$av_j^{k,\min} = \frac{1}{\left|SSq_j^{k,\min}\right|} \sum_{a_{sj}^k \in SSq_j^{k,\min}} a_{sj}^k \quad k = 1,...,p; j = 1,...,n \,. \tag{17}$$

7. Calculate the criterion-dependent SAPO measure for each alternative:

$$S_j^k = \alpha \cdot \frac{m + 1 - \left|SSq_j^{k,\min}\right|}{m} av_j^{k,\min} + (1-\alpha) \cdot \frac{m - 1 + \left|SSq_j^{k,\max}\right|}{m} av_j^{k,\max} \quad k = 1,...,p; j = 1,...,n \,. \tag{18}$$

$S^k_j$ is proportional to the number of good evaluations connected with $D_j$, and inversely proportional to the number of bad evaluations related to $D_j$. Fractions $\frac{m + 1 - \left|SSq_j^{k,\min}\right|}{m}$ and $\frac{m - 1 + \left|SSq_j^{k,\max}\right|}{m}$ have a similar interpretation as in case I.

8. Compute an aggregated measure, the SAPO(SC) measure, for each decision $D_j$:

$$S_j^{SC} = \sum_{k=1}^{p} w^k \cdot S_j^k \quad j = 1,...,n \,. \tag{19}$$

9. Select the decision according to Equation (20):

$$D_j^{SC*} = \arg\max_j S_j^{SC} \,. \tag{20}$$

Note that the number of scenarios considered for particular attributes can be different in case II, since events from each matrix are totally independent. If such a situation takes place, it is recommended to apply separate notation for each set of scenarios. For instance, $m(k)$ may denote the number of states of nature assigned to attribute $C_k$ and $\{S^k{}_1, \dots, S^k{}_i, \dots, S^k{}_{m(k)})\}$ may be the set of scenarios connected with this criterion. SAPO(CS) and SAPO(SC) are methods with a priori articulation of preferences.

## 6. Case study

We will illustrate the use of the approach described in the previous section on the basis of a case of marketing strategies (activities) selection and by referring to a very interesting paper concerning the so-called 4P marketing model including product, price, promotion and place (Ginevičius *et al.* 2013). Results gathered by (Ginevičius *et al.* 2013) enable one to assess the effectiveness of marketing strategies thanks to a multicriteria evalutation procedure based on the sum of products of criteria values and their weights properly estimated. Authors of that contribution develop a hierarchical structure of criteria describing enterprise marketing system. They distinguish 8 criteria of product (e.g. innovations, product design, quality, brand), 7 criteria of price (e.g. initial price, terms of payment, price differentiation), 7 criteria of promotion (e.g. advertising, increase of sales, corporate identity) and 5 criteria of place (e.g. place of sale, sales online). All those criteria are integrated into one generalized quantity and are applied in order to quantitatively evaluate marketing activities. Note that if a company is trying to choose the best strategy among a set o potential activities, all criteria values are known before the choice made by the enterprise, since this data constitutes initial parameters of particular strategies. This set of criteria describes factors which are planned and controlled by the company.

Now, let us analyze possible further steps performed by a fictitious enterprise, say enterprise "E", which has already evaluated potential marketing activities (connected with a new product development) and selected 4 strategies with the highest measures proposed in (Ginevičius *et al.* 2013). Assume that enterprise "*E*" contemplates realization of one of these activities, but it aims to choose the strategy which will maximize the future annual profit and the market share. Increasing these criteria is one of the most important objectives of business. Hence, we have two new criteria (apart from 27 initial attributes enumerated by the authors mentioned), but, this time, they are a little beyond the control of the company. They depend not only on the decisions made by the company, but also on some macroeconomic, microeconomic and environmental factors (demand, fashion, population structure, tax mechanism, competitors' strategies, misfortune, weather and so on). For such criteria (profit and market share) the exact estimation is rather complicated. Therefore, instead of using deterministic parameters, possible states of nature may be predicted, for instance by experts. In our example we assume that there are 5 possible scenarios. Forecasted (step 1) and normalized (step 2, Equation 1) criteria values for each pair $(D_j,S_i)$ are given in Table 1. Enterprise "E" declares attribute weights, e.g. $w_1 = 0.4$, $w_1 = 0.6$, and the level of pessimism, e.g.

$\alpha$ = 0.43 (step 3). Now, one should choose the appropriate procedure: SAPO(CS) or SAPO(SC). Profits and market shares certainly depend on each other, thus one cannot consider scenario evaluations independently, which means that the first approach is correct. We calculate aggregated measures (step 4, Equation 4, Table 2, rows 1–5), which are very similar to formulas applied by (Ginevičius *et al.* 2013). Then, non-increasing sequences of normalized synthetic values can be generated (step 5, Table 2, row 6). In order to define subsequences of good and bad values (step 6), we have to calculate $C = \max\left\{1, \left\lceil 5 \cdot \min\left\{0.43, 0.57\right\}\right\rceil\right\} = 3$ and set deviation degrees, e.g. $d^{\max} = d^{\min} = 0.3$. The use of Equations 5–6 enables us to find the elements of both subsequences (Table 2, rows 7–8). For instance, $SSq_1^{\max} = \left\{0.600, 0.554, 0.532\right\}$ since $\left|SSq_1^{\max}\right| \le C = 3$ and $0.600 - 0.3 \cdot \left(0.600 - 0.326\right) \le 0.600, 0.554, 0.532 \le 0.600$.

Table 1. Annual profit (mln euro) and market share (%): initial ($a^k_{ij}$) and normalized ($a(n)^k_{ij}$) values (Source: created by the author)

| No | Annual profit ($C_1$) | | | | Market share ($C_2$) | | | |
|---|---|---|---|---|---|---|---|---|
| | D1 | D2 | D3 | D4 | D1 | D2 | D3 | D4 |
| S1 | 2.5/0.38 | 4.0/0.84 | 4.5/1.00 | 3.0/0.53 | 20/0.29 | 22/0.41 | 15/0.00 | 21/0.35 |
| S2 | 1.3/0.00 | 2.5/0.38 | 3.5/0.69 | 3.0/0.53 | 32/1.00 | 18/0.18 | 19/0.24 | 17/0.12 |
| S3 | 1.6/0.09 | 3.0/0.53 | 4.3/0.94 | 2.0/0.22 | 29/0.82 | 19/0.24 | 16/0.06 | 18/0.18 |
| S4 | 1.7/0.13 | 3.0/0.53 | 2.0/0.22 | 2.5/0.38 | 28/0.76 | 15/0.00 | 23/0.47 | 24/0.53 |
| S5 | 1.5/0.06 | 3.5/0.69 | 4.2/0.91 | 4.0/0.84 | 30/0.88 | 17/0.12 | 16/0.06 | 24/0.53 |

Table 2. Aggregated measures $A(n)_{ij}$ (Source: created by the author)

| No | D1 | D2 | D3 | D4 |
|---|---|---|---|---|
| S1 | 0.326 | 0.585 | 0.400 | 0.424 |
| S2 | 0.600 | 0.256 | 0.416 | 0.283 |
| S3 | 0.532 | 0.354 | 0.410 | 0.193 |
| S4 | 0.509 | 0.213 | 0.370 | 0.468 |
| S5 | 0.554 | 0.346 | 0.398 | 0.655 |
| $Sq_j$ | 0.60;0.55;0.53;0.51;0.33 | 0.58;0.35;0.35;0.26;0.21 | 0.42;0.41;0.40;0.40;0.37 | 0.66;0.47;0.42;0.28;0.19 |
| $SSq^{max}_j$ | 0.600; 0.554; 0.532 | 0.585 | 0.416; 0.410 | 0.655 |
| $SSq^{min}_j$ | 0.326 | 0.256; 0.213 | 0.370 | 0.283; 0.193 |
| $AV^{max}_j$ | 0.562 | 0.585 | 0.413 | 0.655 |
| $AV^{min}_j$ | 0.326 | 0.234 | 0.370 | 0.238 |
| $S^{CS}_j$ | 0.589 | 0.414 | 0.442 | 0.445 |

Now, let us compute the averages of good and bad results (step 7, Equations 10–11, Table 2, rows 9–10) and the SAPO(CS) measure (step 8, Equations 12, Table 2, row 11). For instance: $S_1^{CS} = 0.43 \cdot \left(\frac{5+1-1}{5}\right) \cdot 0.326 + 0.57 \cdot \left(\frac{5-1+3}{5}\right) \cdot 0.562 = 0.589$. According to Equation 13 (step 9) the first marketing strategy (D1) should be selected. The ranking is: D1, D4, D3, D2. Note that the coefficient of pessimism has a significant impact on the final decision. If $\alpha$ = 0.90, the ranking is as follows: D3, D1, D4, D2. Of course, the change of criteria weights also influences the order of activities (when $w_1$ = 0.6, $w_1$ = 0.4, the order is D3, D4, D2, D1 for $\alpha$ = 0.43 and D3, D1, D2, D4 for $\alpha$ = 0.90).

## 7. Conclusions

Many rules for the discrete version of uncertain multicriteria decision making with SP have been already developed. Methods proposed in this paper: SAPO(CS) and SAPO(SC) for dependent and independent criteria matrices respectively – are based on the concept presented in (Ravindran 2008), i.e. they consider the DM's risk aversion and they refer to SAW and the Hurwicz's rule. Nevertheless, in the new approaches the additive weighting method is combined with a modification of the Hurwicz's criterion – the SAPO method, which leads to logic rankings even in problems with asymmetric payoff distributions, because it analyzes the frequency of good and bad results related to particular decisions. SAPO(CS) and SAPO(SC) allow the DM to define precisely his or her preferences concerning the importance of considered goals, the attitude towards risk and the evaluation of extreme results related to particular alternatives. They do not require any information about the probabilities of states of nature, which is certainly a significant advantage. Both procedures can be successfully applied in any business or management multi-objective problem provided that future economic consequences are presented by means of scenario planning which is a comprehensive decision support tool for considering uncertainties.

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## References

Aghdaie, M. H.; Zolfani, S. H.; Zavadskas, E. K. 2013. Market segment evaluation and selection based on application of fuzzy AHP and COPRAS-G methods, *Journal of Business Economics and Management* 14(1): 213–233. http://dx.doi.org/10.3846/16111699.2012.721392

Azimi, R.; Fouladgar, M. M.; Zavadskas, E. K.; Basiri, M. H. 2011. Ranking the strategies of mining sector through ANP and TOPSIS in a SWOT framework, *Journal of Business Economics and Management* 12(4): 670–689. http://dx.doi.org/10.3846/16111699.2011.62655

Basili, M. 2006. A rational decision rule with extreme events, *Risk Analysis* 26: 1721–1728. http://dx.doi.org/10.1111/j.1539-6924.2006.00826.x

Basili, M.; Chateauneuf, A.; Fontini, F. 2008. Precautionary principle as a rule of choice with optimism on windfall gains and pessimism on catastrophic losses, *Ecological Economics* 67: 485–491. http://dx.doi.org/10.1016/j.ecolecon.2007.12.030

Basili, M.; Chateauneuf, A. 2011. Extreme events and entropy: a multiple quantile utility model, *International Journal of Approximate Reasoning* 52: 1095–1102. http://dx.doi.org/10.1016/j.ijar.2011.05.005

Ben Amor, S.; Jabeur, K.; Martel, J. 2007. Multiple criteria aggregation procedure for mixed evaluations. *European Journal of Operational Research* 181(3): 1506–1515. http://dx.doi.org/10.1016/j.ejor.2005.11.048

Cannon, C. M.; Kmietowicz, Z. W. 1974. Decision theory and incomplete knowledge, *Journal of Management Studies* 11(3): 224–232. http://dx.doi.org/10.1111/j.1467-6486.1976.tb00531.x

Chang, C.-T. 2011. Multi-choice goal programming with utility functions, *European Journal of Operational Research* 215: 439–445. http://dx.doi.org/10.1016/j.ejor.2011.06.041

Chankong, V.; Haimes, Y. Y. 1983. *Multiobjective decision making: theory and methodology*. New York: Elsevier Science.

Charnes, A.; Cooper, W.; Ferguson, R. 1955. Optimal estimation of executive compensation by linear programming, *Management Science* 1: 138–151. http://dx.doi.org/10.1287/mnsc.1.2.138

Chronopoulos, M.; De Reyck, B.; Siddiqui, A. 2011. Optimal investment under operational flexibility, risk aversion, and uncertainty, *European Journal of Operational Research* 213: 221–237. http://dx.doi.org/10.1016/j.ejor.2011.03.007

Churchman, C. W.; Ackoff, R. L. 1954. An approximate measure of value, *Journal of Operations Research of America* 2(1): 172–187.

Dominiak, C. 2006. Multicriteria decision aid under uncertainty, in T. Trzaskalik (Ed.). *Multiple criteria decision making' 05*. Katowice: Publisher of The Karol Adamiecki University of Economics in Katowice, 63–81.

Dominiak, C. 2009. Multi-criteria decision aiding procedure under risk and uncertainty, in T. Trzaskalik (Ed.). *Multiple criteria decision making' 08*. Katowice: Publisher of The Karol Adamiecki University of Economics in Katowice, 61–88.

Dubois, D.; Prade, H. 1988. *Possibility theory*. New York: Plenum Press.

Durbach, I. N. 2014. Outranking under uncertainty using scenarios, *European Journal of Operational Research* 232(1): 98–108. http://dx.doi.org/10.1016/j.ejor.2013.06.041

Durbach, I. N.; Stewart, T. J. 2012. Modeling uncertainty in multi-criteria decision analysis, *European Journal of Operational Research* 223(1): 1–14. http://dx.doi.org/10.1016/j.ejor.2012.04.038

Ehrgott, M. 2005. *Multicriteria optimization*. 2nd ed. Berlin – Heidelberg: Springer.

Eiselt, H. A.; Marianov, V. 2014. Multicriteria decision making under uncertainty: a visual approach, *International Transactions in Operational Research* 21(4): 525–540. http://dx.doi.org/10.1111/itor.12073

Ellsberg, D. 2001. *Risk, ambiguity and decision*. New York: Garland Publishing.

Etner, J.; Jeleva, M.; Tallon, J.-M. 2012. Decision theory under ambiguity, *Journal of Economic Surveys* 26(2): 234–270. http://dx.doi.org/10.1111/j.1467-6419.2010.00641.x

Fishburn, P. C. 1984. Foundations of risk measurement. I. Risk or probable loss, *Management Science* 30: 396–406. http://dx.doi.org/10.1287/mnsc.30.4.396

Gaspars, H. 2007. Alokacja zasobu w warunkach niepewności: modele decyzyjne i procedury obliczeniowe [Ressource allocation under uncertainty: choice models and computational procedures], *Operations Research and Decisions* 17(1): 5–27. (in Polish).

Gaspars-Wieloch, H. 2011. Metakryterium w ciągłej wersji optymalizacji wielocelowej – analiza mankamentów metody i próba jej udoskonalenia [The aggretage objective function in the continuous version of the multicriteria optimization – analysis of the shortcomings of the method and attempt at improving it], in E. Konarzewska-Gubała (Ed.). *Zastosowania badań operacyjnych. Zarządzanie projektami, decyzje finansowe, logistyka*. Wrocław: Wydawnictwo Uniwersytetu Ekonomicznego we Wrocławiu, 313–332 (in Polish).

Gaspars-Wieloch, H. 2012. Limited efficiency of optimization methods in solving economic decision problems [Ograniczona skuteczność metod optymalizacyjnych w rozwiązywaniu ekonomicznych problemów decyzyjnych], *Ekonomista* 2012/3: 303–324 (in Polish).

Gaspars-Wieloch, H. 2013. On a decision rule supported by a forecasting stage based on the decision maker's risk aversion, in L. Zadnik Stirn, J. Zerovnik, J. Povh, S. Drobne, A. Lisec (Eds.). *SOR'13 Proceedings, The 12th International Symposium on Operational Research in Slovenia*, 25–27 September 2013, Dolenjske Toplice, Slovenia, Slovenian Society INFORMATIKA, Section for Operational Research, 53–59.

Gaspars-Wieloch, H. 2014a. Propozycja hybrydy reguł Hurwicza i Bayesa w podejmowaniu decyzji w warunkach niepewności [A hybrid of the Hurwicz and Bayes rules in decision making under uncertainty], in T. Trzaskalik (Ed.) *Modelowanie preferencji a ryzyko 2014.* Studia Ekonomiczne. Zeszyty Naukowe Uniwersytetu Ekonomicznego w Katowicach 178, Katowice: Wydawnictwo Uniwersytetu Ekonomicznego w Katowicach, 74–92. (in Polish).

Gaspars-Wieloch, H. 2014b. On a decision for mixed strategy searching under uncertainty on the basis of the coefficient of optimism, *Procedia – Social and Behavioral Sciences.* 110: 923–931. Elsevier. http://dx.doi.org/10.1016/j.sbspro.2013.12.938

Gaspars-Wieloch, H. 2014c. On a decision rule supported by a forecasting stage based on the decision maker's coefficient of optimism, *Central European Journal of Operations Research*. Springer. http://dx.doi.org/10.1007/s10100-014-0364-5

Gaspars-Wieloch, H. 2014d. Modifications of the Hurwicz's decision rules, *Central European Journal of Operations Research* 22(4): 779–794. http://dx.doi.org/10.1007/s10100-013-0302-y

Gaspars-Wieloch, H. 2014e. Modification of the maximin joy criterion for decision making under uncertainty, *Quantitative Methods in Economics* XV. Warsaw (in press).

Gaspars-Wieloch, H. 2015. The impact of the structure of the payoff matrix on the final decision made under uncertainty, *Asia-Pacific Journal of Operational Research* (in press).

Ghirardato, P.; Maccheroni, F.; Marinacci, M. 2004. Differentiating ambiguity and ambiguity attitude, *Journal of Economic Theory* 118: 133–173. http://dx.doi.org/10.1016/j.jet.2003.12.004

Gilboa, I. 2009. *Theory of decision under uncertainty*. Cambridge, New York: Cambridge University Press.

Gilboa, I.; Schmeidler, D. 1989. Maxmin expected utility with non-unique prior, *Journal of Mathematical Economics* 18: 141–153. http://dx.doi.org/10.1016/0304-4068(89)90018-9

Ginevičius, R.; Podvezko, V.; Ginevičius, A. 2013. Quantitative evaluation of enterprise marketing activities, *Journal of Business Economics and Management* 14(1): 200–212. http://dx.doi.org/10.3846/16111699.2012.731143

Ginevičius, R.; Zubrecovas, V. 2009. Selection of the optimal real estate investment project basing on multiple criteria evaluation using stochastic dimensions, *Journal of Business Economics and Management* 10(3): 261–270. http://dx.doi.org/10.3846/1611-1699.2009.10.261-270

Goodwin, P.; Wright, G. 2001. Enhancing strategy evaluation in scenario planning: a role for decision analysis, *Journal of Management Studies* 38(1): 1–16. http://dx.doi.org/10.1111/1467-6486.00225

Groenewald, M. E.; Pretorius, P. D. 2011. Comparison of decision making under uncertainty investment strategies with the money market, *Journal of Financial Studies and Research* http://dx.doi.org/10.5171/2011.373376

Guo, P. 2014. One-shot decision theory: a fundamental alternative for decision under uncertainty, *Human-Centric Decision-Making Models for Social Sciences* 2014: 33–55. http://dx.doi.org/10.1007/978-3-642-39307-5

Guo, P. 2013. One-shot decision making with regret, in (*ICIST*) *International Conference on Information Science and Technology*, 23–25 March 2013, Yangzhou, China, 493–495. http://dx.doi.org/10.1109/ICIST.2013.6747596

Guo, P. 2011. One-shot decision theory, *IEEE Transactions on Systems, Man, and Cybernetics*, Part A 41(5): 917–926. http://dx.doi.org/10.1109/TSMCA.2010.2093891

Guo, P. 2010. Private real estate investment analysis within one-shot decision framework, *International Real Estate Review* 13(3): 238–260.

Haeri, A.; Tavakkoli-Moghaddam, R. 2012. Developing a hybrid data mining approach based on multi-objective particle swarm optimization for solving a traveling salesman problem, *Journal of Business Economics and Management* 13(5): 951–967. http://dx.doi.org/10.3846/16111699.2011.643445

Hayashi, T. 2008. Regret aversion and opportunity dependence, *Journal of Economic Theory* 139(1): 242–268. http://dx.doi.org/10.1016/j.jet.2007.07.001

Hopfe, C. J.; Augenbroe, G. L. M.; Hensen, J. L. M. 2013. Multicriteria decision making under uncertainty in building performance assessment, *Building and Environment* 69: 81–90. http://dx.doi.org/10.1016/j.buildenv.2013.07.019

Hsu, J.-C. 2014. A hybrid multiple criteria decision-making model for investment decision making, *Journal of Business Economics and Management* 15(3): 509–529. http://dx.doi.org/10.3846/16111699.2012.722563

Hurwicz, L. 1952. *A criterion for decision making under uncertainty*. Technical Report, 355. Cowles Commission.

Hwang, C. L.; Yoon, K. 1981. *Multiple attribute decision making methods and applications: a state of the art survey*. New York: Springer-Verlag.

Jahn, J. 2004. *Vector optimization theory, applications and extensions*. Berlin: Springer.

Janjic, A.; Andjelkovic, A.; Docic, M. 2013. Multiple criteria decision making under uncertainty based on stochastic dominance, in *Proceedings of the 2013 International Conference on Applied Mathematics and Computational Methods in Engineering*, 16–19 July 2013, Rhodes Island, Greece, 86–91.

Kapsos, M.; Christofides, N.; Rustem, B. 2014. Worst-case robust Omega ratio, *European Journal of Operational Research* 234(2): 499–507. http://dx.doi.org/10.1016/j.ejor.2013.04.025

Keeney, R.; Raiffa, H. 1993. *Decisions with multiple objectives: preferences and value tradeoffs*. Cambridge: Cambridge University Press.

Khorram, E.; Zarepisheh, M.; Ghaznavi-Ghosoni, B. 2010. Sensitivity analysis on the priority of the objective functions in lexicographic multiple objective linear programs, *European Journal of Operational Research* 207: 1162–1168. http://dx.doi.org/10.1016/j.ejor.2010.05.016

Klir, G. J.; Folger, T. 1988. *Fuzzy sets, uncertainty and information*. Englewood Cliffs, New Jersey: Prentice-Hall.

Kmietowicz, Z. W.; Pearman, A. D. 1984. Decision theory, linear partial information and statistical dominance, *Omega* 12: 391–399. http://dx.doi.org/10.1016/0305-0483(84)90075-6

Knight, F. H. 1921. *Risk, uncertainty, profit.* Hart. Boston MA: Schaffner & Marx, Houghton Mifflin Co.

Kofler, E.; Zweifel, P. 1993. One-shot decisions under linear partial information, *Theory and Decision* 34: 1–20. http://dx.doi.org/10.1007/BF01076102

Korhonen, A. 2001. Strategic financial management in a multinational financial conglomerate: a multiple goal stochastic programming approach, *European Journal of Operational Research* 128: 418–434. http://dx.doi.org/10.1016/S0377-2217(99)00366-5

Lee, Y.-H. 2012. A fuzzy analytic network process approach to determining prospective competitive strategy in China: a case study for multinational biotech pharmaceutical enterprises, *Journal of Business Economics and Management* 13(1): 5–28. http://dx.doi.org/10.3846/16111699.2011.620165

Liu, M.; Zhao, L. 2009. Optimization of the emergency materials distribution network with time windows in anti-bioterrorism system, *International Journal of Innovative Computing, Information and Control* 5 (11A): 3615–3624.

Liu, Y.; Fan, Z.; Hang, Y. 2011. A method for stochastic multiple criteria decision making based on dominance degrees, *Information Sciences* 181(19): 4139–4153. http://dx.doi.org/10.1016/j.ins.2011.05.013

Lotfi, V.; Yoon, Y. S.; Zionts, S. 1997. Aspiration-based search algorithm (ABSALG) for multiple objective linear programming problems: theory and comparative tests, *Management Science* 43: 1047–1059. http://dx.doi.org/10.1287/mnsc.43.8.1047

Marinacci, M. 2002. Probabilistic sophistication and multiple priors, *Econometrica* 70: 755–764. http://dx.doi.org/10.1111/1468-0262.00303

Marler, R.; Arora, J. 2004. Survey of multiobjective methods for engineering, *Structural and Multidisciplinary Optimization* 26(6): 369–395. http://dx.doi.org/10.1007/s00158-003-0368-6

Michalska, E. 2014. Zastosowanie wskaźnika Omega w podejmowaniu decyzji przy niepełnej informacji liniowej [Application of the Omega ratio in decision making with partial information], in J. B. Gajda (Ed.). *Metody i zastosowania badań operacyjnych 2013*. Łódź: Wydawnictwo Uniwersytetu Łódzkiego (in press).

Michnik, J. 2013a. Weighted influence non-linear gauge system (WINGS) – an analysis method for the systems of interrelated components, *European Journal of Operation00al Research.* http://dx.doi.org/10.1016/j.ejor.2013.02.007

Michnik, J. 2013b. *Wielokryterialne metody wspomagania decyzji w procesie innowacji* [Multicriteria methods supporting decisions in innovation process]. Katowice: Wydawnictwo Uniwersytetu Ekonomicznego w Katowicach.

Mikhaidov, L.; Tsvetinov, P. 2004. Evaluation of services using a fuzzy analytic hierarchy process, *Applied Soft Computing Journal* 5(1): 23–33.

Montibeller, G.; Gummer, H.; Tumidei, D. 2006. Combining scenario planning and multi-criteria decision analysis in practice, *Journal of Multi-criteria Decision Analysis* 14: 5–20. http://dx.doi.org/10.1002/mcda.403

Officer, R. R.; Anderson, J. R. 1968. Risk, uncertainty and farm management decisions, *Review of Marketing and Agricultural Economics* 36(01). 17 p.

Pazek, K.; Rozman, C. 2009. Decision making under conditions of uncertainty in agriculture: a case study of oil crops, *Poljoprivreda* 15(1): 45–50.

Piasecki, K. 1990. *Decyzje i wiarygodne prognozy* [Decisions and reliable forecasts]. Poznań: Akademia Ekonomiczna w Poznaniu.

Podvezko, V. 2009. Application of AHP technique, *Journal of Business Economics and Management* 10(2): 181–189. http://dx.doi.org/10.3846/1611-1699.2009.10.181-189

Pomerol, J. C. 2001. Scenario development and practical decision making under uncertainty, *Decision Support Systems* 31(2): 197–204. http://dx.doi.org/10.1016/S0167-9236(00)00131-7

Puppe, C.; Schlag, K. 2009. Choice under complete uncertainty when outcome spaces are state dependent, *Theory and Decision* 66: 1–16. http://dx.doi.org/10.1007/s11238-007-9082-y

Ravindran, A. R. 2008. *Operations research and management science handbook*, Boca Raton, London, New York: CRS Press.

Render, B.; Stair, R. M.; Hanna, M. E. 2006. *Quantitative analysis for management.* Upper Saddle River, New Jersey: Pearson Prentice Hall.

Ram, C.; Montibeller, G.; Morton, A. 2010. Extending the use of scenario planning and MCDA for the evaluation of strategic options, *Journal of Operational Research Society* 62(5): 817–829. http://dx.doi.org/10.1057/jors.2010.90

Savage, L. 1961. The foundations of statistics reconsidered, in *Studies in Subjective Probability.* New York: Wiley, 173–188.

Sawaragi, Y.; Nakayama, H.; Tanino, T. 1985. Theory of multiobjective optimization, *Mathematics in Science and Engineering* 176, 296 p. Orlando: Academic Press. http://dx.doi.org/10.1137/1028177

Saaty, T. L. 1980. *The analytic hierarchy process*. New York: McGraw Hill.

Saaty, T. L. 1996. *Decision making with dependence and feedback: analytic network process*. Pittsburgh: RWS Publications.

Schmeidler, D. 1986. Integral representation without additivity, *Proceedings of the American Mathematical Society* 97: 255–261. http://dx.doi.org/10.1090/S0002-9939-1986-0835875-8

Sikora, W. (Ed.). 2008. *Badania operacyjne* [Operations research]. Warszawa: Polskie Wydawnictwo Ekonomiczne.

Stewart, T. J. 2005. Dealing with uncertainties in MCDA, multiple criteria decision analysis: state of the art surveys, *International Series in Operations Research & Management Scienc*e 78: 445–466.

Suo, M. Q.; Li, Y. P.; Huang, G. H. 2012. Multicriteria decision making under uncertainty: an advanced ordered weighted averaging operator for planning electric power systems, *Engineering Applications of Artificial Intelligence* 25(1): 72–81. http://dx.doi.org/10.1016/j.engappai.2011.08.007

Triantaphyllou, E.; Lin, C. 1996. Development and evaluation of five fuzzy multiattribute decision-making methods, *International Journal of Approximate Reasoning* 14(4): 281–310. http://dx.doi.org/10.1016/0888-613X(95)00119-2

Trzaskalik, T. 2008. *Wprowadzenie do badań operacyjnych z komputerem* [Introduction to operations research with computer]. 2nd ed. Warszawa: Polskie Wydawnictwo Ekonomiczne.

Trzaskalik, T. 2014. *Wielokryterialne wspomaganie decyzji* [Multicriteria decision aiding]. Warszawa: Polskie Wydawnictwo Ekonomiczne.

Tsaur, S.; Chang, T.; Yen, C. 2002. The evaluation of airline service quality by fuzzy MCDM, *Tourism Management* 23(2): 107–115. http://dx.doi.org/ 10.1016/S0261-5177(01)00050-4

Tversky, A.; Kahneman, D. 1992. Advances in prospect theory: cumulative representation of uncertainty, *Journal of Risk and Uncertainty* 5: 297–323. http://dx.doi.org/10.1007/BF00122574

Urli, B.; Nadeau, R. 2004. PROMISE/scenarios: an interactive method for multiobjective stochastic linear programming under partial uncertainty, *European Journal of Operational Research* 155(2): 361–372. http://dx.doi.org/10.1016/S0377-2217(02)00859-7

Van der Heijden, K. 1996. *Scenarios: the art of strategic conversation*. Chichester: John Wiley and Sons.

von Neumann, J.; Morgenstern, O. 1944. *Theory of games and economic behavior.* Princeton, New York: Princeton University Press.

Wald, A. 1950. *Statistical decision functions*. New York: Wiley.

Wang, Y.; Elhag, T. 2006. Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment, *Expert Systems with Applications* 31(2): 309–319. http://dx.doi.org/10.1016/j.eswa.2005.09.040

Watkins, D. W. Jr.; McKinney, D. C.; Lasdon, L. S.; Nielsen, S. S.; Martin, Q. W. 2000. A scenario-based stochastic programming model for water supplies from the Highland Lakes, *International Transactions in Operational Research* 7: 221–230. http://dx.doi.org/10.1111/j.1475-3995.2000.tb00195.x

Weber, M. 1987. Decision making with incomplete information, *European Journal of Operational Research* 28: 44–57. http://dx.doi.org/10.1016/0377-2217(87)90168-8

Wu, C.-M.; Hsieh, C. L.; Chang, K.-L. 2013. A hybrid multiple criteria decision making model for supplier selection, *Mathematical Problems in Engineering* 2013. 8 p. http://dx.doi.org/10.1155/2013/324283

Xu, R. 2000. Fuzzy least-squares priority method in the analytic hierarchy process, *Fuzzy Sets and Systems* 112(3): 395–404. http://dx.doi.org/10.1016/S0165-0114(97)00376-X

Yu, C. 2002. A GP-AHP method for solving group decision-making fuzzy AHP problems, *Computers and Operations Research* 29(14): 1969–2001. http://dx.doi.org/10.1016/S0305-0548(01)00068-5

Zaras, K. 2004. Rough approximation of a preference relation by a multi-attribute dominance for deterministic, stochastic and fuzzy decision problems, *European Journal of Operational Research* 159(1): 196–206. http://dx.doi.org/10.1016/S0377-2217(03)00391-6

Zavadskas, E. K.; Kaklauskas, A. 1996. Determination of an efficient contractor by using the new method of multi criteria assessment, in D. A. Langford, A. Retik (Eds.). *International Symposium for "The Organization and Management of Construction". Shaping Theory and Practice. Vol. 2: Managing the Construction Project and Managing Risk.* CIB W 65. London, Weinheim, New York, Tokyo, Melbourne, Madras, London: E and FN SPON, 94–104.

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